COMSOL MULTIPHYSICS®

Eigenmodes of a Room

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COMSOL

Eigenmodes of a Room

Resonance can at times be a problem in everyday life. The low bass notes from the music system or home theater in the living room can shake the windows and make the floor vibrate. This happens only for certain frequencies—the eigenfrequencies of the room.

It is only in the low-frequency range that the eigenfrequencies are well separated. In the mid- and high-frequency ranges, the eigenfrequencies are packed so closely, with less than a halftone between them, that the individual resonances are insignificant for music and other natural sounds. Nevertheless, the music experience is affected by the acoustics of the room.

When designing a concert hall, it is extremely important to take the resonances into account. For a clear and neutral sound, the eigenfrequencies should be evenly spaced. For the home theater or music system owner, who cannot change the shape of the living room, another question is more relevant: Where should the speakers be located for the best sound?

Model Definition

For example, take a room with the dimensions 5 by 4 by 2.6 meters equipped with a TV set, two speakers, and a couch. To illustrate the effects on the music, compute all eigenfrequencies below 100 Hz together with the corresponding eigenmodes. The eigenmode shows the sound intensity pattern for its associated eigenfrequency. From the characteristics of the eigenmodes, you can draw some conclusions as to where the speakers should be placed.

DOMAIN EQUATIONS

Sound propagating in free air is described by the wave equation:

$$-\Delta p + \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0$$

where p is the pressure, and c is the speed of sound. If the air is brought into motion by a harmonically oscillating source, for example, a loudspeaker, only one frequency f exists in the room. For that reason it makes sense to look for a time-harmonic solution on the form

$$p = \hat{p}e^{i\omega t}$$

The wave equation then simplifies to the Helmholtz equation for p, the amplitude of the acoustic disturbances:

$$\Delta\hat{p} + \frac{\omega^2}{c^2}\hat{p} = 0$$

BOUNDARY CONDITIONS

This model assumes that all boundaries—walls, floor, ceiling, and furniture —are perfectly rigid (sound hard boundaries).

ANALYTIC COMPARISON

It is possible to solve the simpler case of an empty room analytically. Each eigenfrequency corresponds to an integer triple (i, l, m):

$$f_{i,l,m} = \frac{c}{2}\sqrt{\left(\frac{i}{L_x}\right)^2 + \left(\frac{l}{L_y}\right)^2 + \left(\frac{m}{L_z}\right)^2}$$

The eigenmodes can be divided into three distinct classes:

- Eigenfrequencies with only one index different from zero give rise to axial modes, that is, plane standing waves between two opposite walls.
- If one index is zero, the mode is tangential.
- If all indices are different from zero, the mode is oblique.

Theoretical eigenvalues for a room without furniture are found in the following table.

MODE INDEX	$\omega^2/10^5$	MODE INDEX	$\omega^2/10^5$
0,0,0	0	0,1,1	2.444
1,0,0	0.465	2,1,0	2.584
0,1,0	0.726	0,2,0	2.902
1,1,0	1.191	1,1,1	2.908
0,0,1	1.718	1,2,0	3.367
2,0,0	1.858	2,0,1	3.575
1,0,1	2.182	3,0,0	4.180

Results and Discussion

The relevant quantity when it comes to placing the loudspeakers is the amplitude of the standing pressure wave. A sound source excites an eigenmode the most if it is placed in one of the pressure antinodes for the mode. Conversely, with the source in a pressure node, the eigenmode remains silent.

All modes have local maxima in the corners of an empty room so speakers in the corners excite all eigenfrequencies. This simulation predicts eigenmodes that strongly resemble those of the corresponding empty room. The higher the frequency, the more the placing of the furniture matters. For instance, some of the high-frequency eigenmodes are located behind the couch.

In the strictest sense, the results of this simulation only apply to a room with perfectly rigid walls and nonabsorbing furniture. The prediction that speakers placed in the corners of the room excite many eigenmodes and give a fuller and more neutral sound, however, holds for real-life rooms.

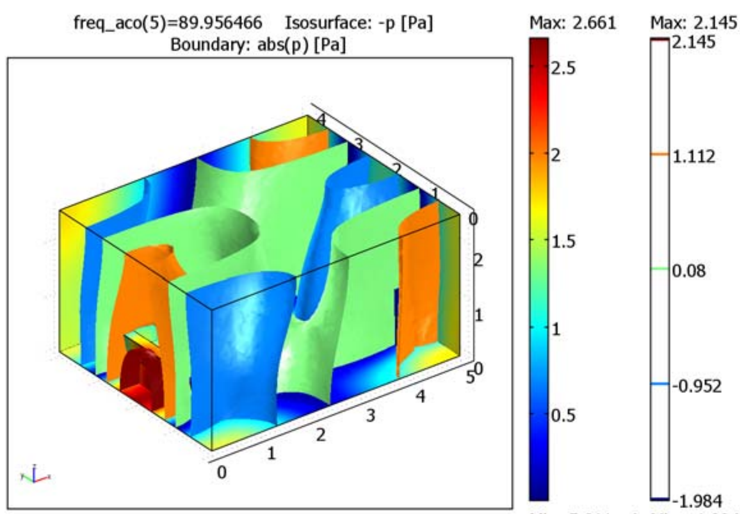

Figure 1: The sound pressure distribution for $\omega = 569$ *rad/s. The (negative of) the real part of the pressure is visualized as an isosurface plot, and the absolute value of the pressure as a boundary plot.*

Model Library path: `COMSOL_Multiphysics/Acoustics/eigenmodes_of_room`

Modeling Using the Graphical User Interface

MODEL NAVIGATOR

1 Click **Multiphysics**.

2 Select **3D** in the **Space dimension** list.

3 In the list of application modes, select **Acoustics** (in the **COMSOL Multiphysics** folder if your license includes additional modules), and in the **Acoustics** folder, select **Eigenfrequency analysis**.

4 Click **Add**.

5 Add geometries according to the following table by clicking the **Add Geometry** button.

NAME	SPACE DIMENSION	INDEPENDENT VARIABLES
Floor	2D	x y z
Table	2D	x y z
TV	2D	x y z

6 Click **OK** to close the **Model Navigator**.

GEOMETRY MODELING

1 Click the **Geom1** tab.

2 Click the **Block** button to create a block with the following dimensions that you enter in the **Length** area; when done, click **OK**.

PARAMETER	VALUE
X	5
Y	4
Z	2.6

3 Click to clear the **Highlight Face** button on the Draw toolbar for a wireframe view.

Now create the cross section of the geometry on the floor of the room. The floor is by default an xy-plane at $z = 0$.

1 Go to the **Floor** work plane and click the **Zoom Extents** button on the Main toolbar.
2 From the **Options** menu, open the **Axes/Grid Settings** dialog box.
3 Click the **Grid** tab, clear the **Auto** check box, and change the grid according to the following table; when done, click **OK**.

GRID	
x spacing	`0.2`
Extra x	`1.7 2.1 4.5 4.7`
y spacing	`0.2`
Extra y	`1.7 2.3`

4 Draw a rectangle with top left corner at (0.4, 3) and bottom right corner at (1.2, 1).
5 Draw eight squares with side 0.1 and upper left corners at (1.6, 1.7), (1.6, 2.4), (2.1, 1.7), (2.1, 2.4), (4.4, 1.7), (4.4, 2.4), (4.7, 1.7), and (4.7, 2.4).
6 From the **Draw** menu, choose **Extrude**.
7 In the **Extrude** dialog box, select all objects and set the **Distance** parameter to `0.4`.
8 Click **OK**.

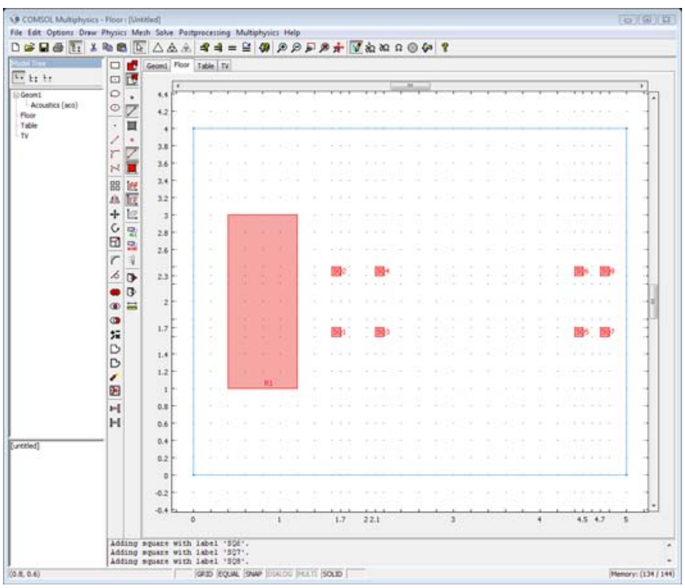

Figure 2: The geometry of the Floor work plane.

Now proceed to create the cross section of the geometry at the level of the table.

1 Go to the **Table** work plane.
2 Go to the **Work-Plane Settings** dialog box and change the **z** coordinate to `0.4`. When done, click **OK**.
3 Click the **Zoom Extents** button on the Main toolbar.
4 In the **Axes/Grid Settings** dialog box, click the **Grid** tab and clear the **Auto** check box. Then change the grid spacing according to the following table; when done, click **OK**.

GRID	
x spacing	0.2
y spacing	0.2

5 Draw a rectangle with top left corner at (1.6, 2.6) and bottom right corner at (2.2, 1.4).
6 Make sure the rectangle is selected and choose **Extrude** from the **Draw** menu. Set the distance to `0.1` and click **OK**.
7 Return to the **Table** work plane.
8 Draw a rectangle with top left corner at (0.4, 3) and bottom right corner at (1.2, 1), that is, following the blue projected contour.
9 Draw another rectangle with top left corner at (0.6, 2.8) and bottom right corner at (1.2, 1.2).
10 Select the two last rectangles and click the **Difference** button.

11 With the new composite object still selected, open the **Extrude** dialog box. Set **Distance** to `0.4` and **Displacement x** to `-0.1` and click **OK**.

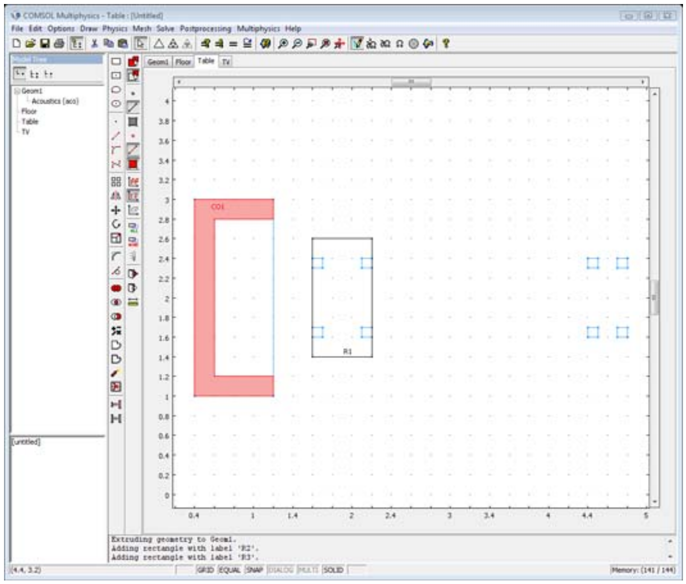

Figure 3: The geometry of the Table work plane.

Finally, create the geometry in the cross section of the TV.

1 Go the **TV** work plane.

2 Go to the **Work-Plane Settings** dialog box. Select the **y-z** plane and set the **x** coordinate to `4.4`.

3 Click **OK**.

4 Click the **Projection of All 3D Geometries** button and after that the **Zoom Extents** button.

5 Use **Axes/Grid Settings** to change the grid as in the following table; when done, click **OK**.

GRID	
x spacing	`0.2`
y spacing	`0.2`

6 Draw the TV set as a rectangle with top left corner at (1.6, 1.0) and bottom right corner at (2.4, 0.4).

7 Draw the left speaker as a rectangle with top left corner at (0.8, 1.0) and bottom right corner at (1.2, 0.0).

8 Draw the right speaker with the same size, but with top left corner at (2.8, 1.0).

9 Select all objects and open the **Extrude** dialog box. Set distance to `0.4`, then click **OK**.

10 In the 3D geometry, select all objects and click the **Difference** button.

Now the room is completed and should look like in Figure 4.

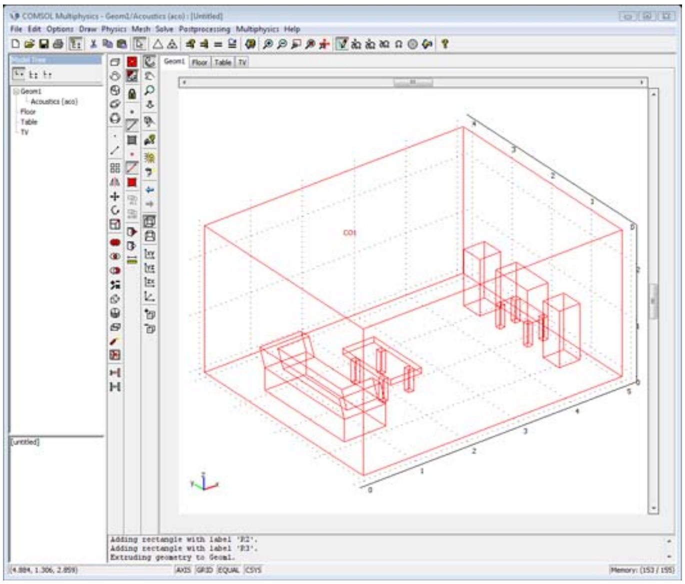

Figure 4: The 3D geometry.

PHYSICS SETTINGS

Boundary Conditions

Use the **Sound hard boundary (wall)** boundary condition for all boundaries.

Subdomain Settings

Use the default subdomain settings.

MESH GENERATION

Click the **Initialize Mesh** button to initialize the mesh.

COMPUTING THE SOLUTION

1 Make sure that **Eigenfrequency** is selected from the **Solver** list in the **Solver Parameters** dialog box.

2 To get 6 eigenfrequencies around 90 Hz enter `6` in the **Desired number of eigenfrequencies** edit field, this is default, and `90` in the **Search for eigenfrequencies around** edit field.

3 Click **OK** to close the **Solver Parameters** dialog box.

4 Click the **Solve** button to compute the solution.

POSTPROCESSING AND VISUALIZATION

The default plot style is a slice plot showing the sound pressure in five equally spaced slices. Try looking at some of the eigenmodes. The first eigenvalue in the list is very small. Its true value is zero, corresponding to the solution without any sound. The next few are axial and tangential modes. To get a better view of the more complicated eigenmodes, you can do a combined surface and boundary plot (see Figure 1).

1 In the **Suppress Boundaries** dialog box in the **Options** menu, suppress Boundaries 1, 2, and 4.

2 On the **General** page of the **Plot Parameters** dialog box, clear the **Slice** check box and select the **Isosurface**, **Boundary**, and **Geometry edges** check boxes in the **Plot type** area. In the **Eigenfrequency** list, select the eigenvalue of about `90.6`.

3 Click the **Boundary** tab and type `abs(p)` in the **Expression** edit field.

4 Click the **Isosurface** tab and type `-p` in the **Expression** edit field.

5 Click the **Scene Light** and **Perspective Projection** buttons in the Camera toolbar.

6 Click **OK**.

This particular mode is concentrated behind the couch. Try moving around the room and looking at other eigenmodes. It is possible that you can identify some of them with the exact solutions for the case of an empty room.