COMSOL MULTIPHYSICS®

3D Thermally Loaded Beam

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3D Thermally Loaded Beam

Introduction

In the following example you build and solve a 3D beam model using the 3D Euler Beam application mode. This model shows how to model a thermally induced deformation of a beam. Temperature differences are applied across the top and bottom surfaces as well as the left and right surfaces of the beam. The deformation is compared with the value given by a theoretical solution given in Ref. 1.

Model Definition

GEOMETRY

The geometry consists of one beam. The beam cross-section area is A and the area moment of inertia I. The beam is L long, and the Young's modulus is E.

- Beam length $L = 3$ m.
- The beam has a square cross section with a side length of 0.04 m giving an area of $A = 1.6 \cdot 10^{-3}$ m^2 and an area moment of inertia of $I = 0.04^4/12$ m^4.

MATERIAL

- Young's modulus $E = 210$ GPa.
- Poisson's ratio $\nu = 0.3$.
- Thermal expansion coefficient $\alpha = 11 \cdot 10^{-6}/°\text{C}$.

CONSTRAINTS

- Displacements in x, y, and z direction are constrained to zero at $x = 0$, $y = 0$, and $z = 0$.
- Rotation around x-axis are constrained to zero at $x = 0$, $y = 0$, and $z = 0$ to prevent singular rotational degrees of freedom.
- Displacements in the x and y direction are constrained to zero at $x = 3$, $y = 0$, and $z = 0$.

THERMAL LOAD

Figure 1 on page 2 shows the surface temperature at each corner of the cross section. The temperature varies linearly between each corner. The deformation caused by this

temperature distribution is modeled by specifying the temperature differences across the beam in the local y and z directions.

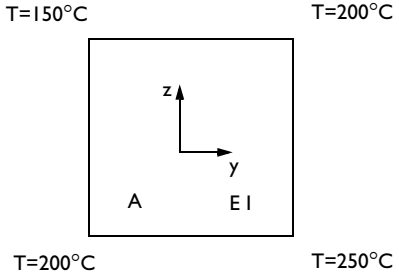

Figure 1: Geometric properties and thermal loads at corners.

Results and Discussion

Based on Ref. 1, you can compare the maximum deformation in the global z direction with analytical values for a simply supported 2D beam with a temperature difference between the top and the bottom surface. The maximum deformation (according to Ref. 1) is:

$$w = \frac{\alpha L^2}{8t}(T_2 - T_1)$$

where t is the depth of the beam (0.04 m), T_2 is the temperature at the top and T_1 at the bottom.

The following table shows a comparison of the maximum global z-displacement, calculated with COMSOL Multiphysics, with the theoretical solution.

W	COMSOL MULTIPHYSICS (MAX)	ANALYTICAL
	15.5 mm	15.5 mm

Figure 2 shows the global z-displacement along the beam.

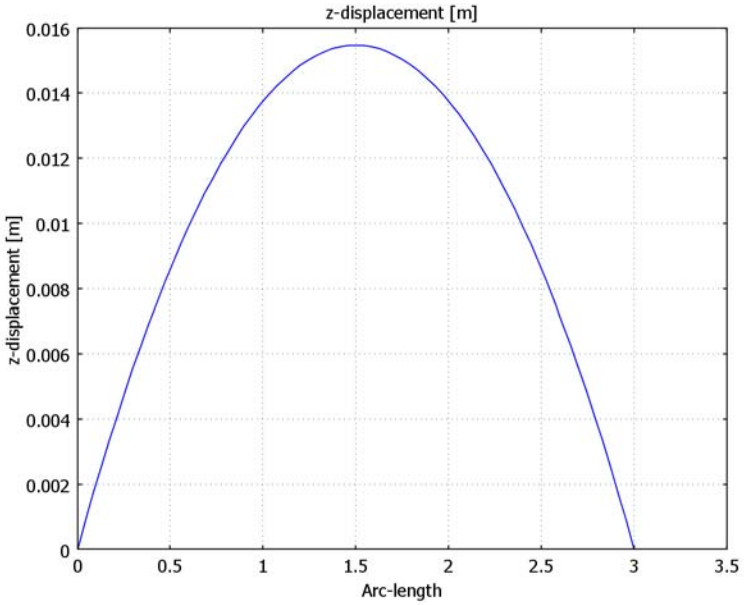

Figure 2: z -displacement along the beam

The analytical values for the maximum total camber can be calculated by:

$$\delta = \sqrt{w^2 + v^2}$$

where v is the maximum deformation in the global y direction which is calculated in the same way as w.

A comparison of the camber calculated with COMSOL Multiphysics and the analytical values are shown in the table below.

TOTAL CAMBER	COMSOL MULTIPHYSICS	ANALYTICAL
	22 mm	21.9 mm

Figure 3 shows the total camber along the beam.

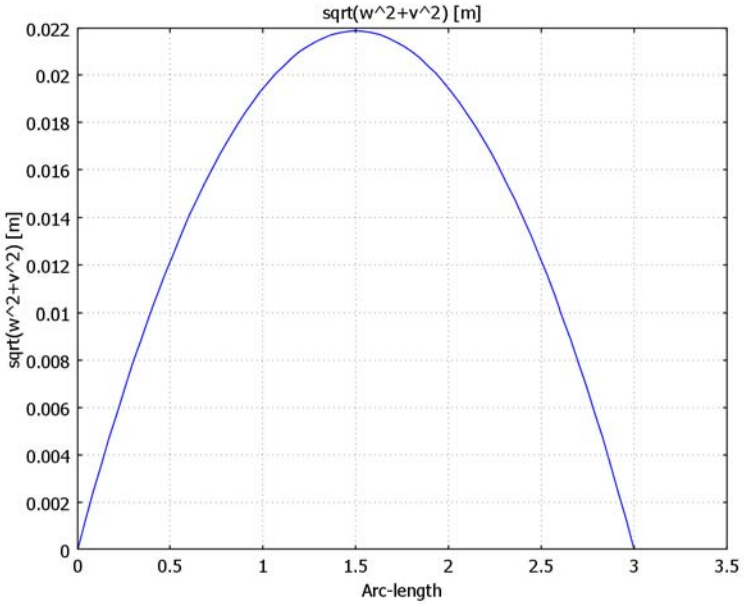

Figure 3: Camber along the beam

Reference

1. Young, W, *Roark's Formulas for Stress & Strain*, McGraw-Hill, 1989.