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Resonating with Students in the Undergraduate Physics Laboratory: Comprehending Acoustic Vibrations

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Abstract: Acoustic vibrations are studied for several objects through the application of computational and optical diagnostic techniques. Computational studies are carried out using the eigenfrequency analysis option in the COMSOL structural mechanics application mode, whereas experimental optical studies utilize real-time stroboscopic holography. The two approaches provide complementary descriptions of the vibrational modes of a handbell, coffee cup and tuning fork. Together, COMSOL and optical diagnostics provide a powerful blending of computational modeling and experimental metrology that can provide students of applied physics, optics, and engineering with a fuller understanding on acoustic vibrations and an enriched upper level lab experience.

Keywords: Acoustic vibrations, eigenfrequency analysis, modal analysis, real-time stroboscopic holography, interferometry.

1. Introduction

Shattering of a crystal glass with sound resonance is a favorite demonstration of the first year physics or engineering student. Unfortunately, many first year students find the demonstration entertaining, but fail to appreciate the richness of the underlying physics. It is often in the upper level lab experience that these students eventually have a “Eureka moment” and the fuller understanding of the shattering glass is obtained [1]. In this paper, we combine computational and optical diagnostics of a handbell, coffee cup, and tuning fork as a means of more fully comprehending the vibrational modes of these resonating structures. The experimental studies are based on stroboscopic holography and carried out as an advanced lab project in the optics course. Computational eigenfrequency studies of the systems are carried out with COMSOL as a complementary means of vibrational analysis.

2. Approach

2.1 Computational Modeling: Eigenfrequency Analysis with COMSOL

Vibration analyses are carried out for three objects (a handbell, coffee cup, and tuning fork) using the eigenfrequency analysis option in the COMSOL structural mechanics application mode. Solutions are based on the equations of linear elasticity for the solid objects

$$\rho \frac{\partial^2 \vec{u}}{\partial t^2} - \nabla \cdot \bar{\bar{\sigma}} = \vec{F}$$

where ρ is the material density, $\vec{u}$ is the deformation vector, $\bar{\bar{\sigma}}$ is the symmetric stress tensor, and $\vec{F}$ is the body force vector. Each resulting eigenvalue relates to a resonance frequency f as

$$\lambda = 4\pi^2 f^2$$

and the corresponding eigenvector defines the shape for the mode of vibration at that frequency.

2.2 Optical Diagnostics: Real-time Stroboscopic Interferometry

In real-time holographic interferometry a transmission holographic image of a static scene or object is first recorded, the hologram is returned precisely to its original location for the subsequent playback, and then this holographic image is brought into interference with the dynamic object. If the object is subject to some type of strain (or is given to a refractive change), then fringes from interference between the holographic image and the changing object will allow one to quantify the motion or phenomena in real time and digitally video record the results. For periodic disturbances at acoustic frequencies, one can still follow the fringe patterns if the playback laser is strobed (typically by an acousto-optic modulator - AOM) at close to the same frequency as the phenomenon being observed [2-3]. Therefore this technique allows one to quantify and record the displacements of an entire set of resonances over the surface of the object and record the results for subsequent careful analysis and comparison to computational results. Reference fringes play a key role in such an analysis and are typically created by a vertical or horizontal tip of one mirror, and the motions of these reference fringes then convey the details of the disturbance in space and time.

3. Results

3.1 Vibrational Modes of a Handbell

The vibrational modes of a symmetric handbell have been extensively studied by Rossing et. al [4-5] using interferometric holographic techniques. These studies effectively identify the relative frequencies and modal patterns for the symmetric object. The symmetric modes of vibration are said to be "degenerate" because they do not have a single preferred orientation for the symmetric bell. A COMSOL eiqenfrequency analysis is carried out for a symmetric handbell with a fully clamped handle. The resulting vibrational pattern is shown in Figure 1 for the (3,1) mode. This result is compared to a holographic image of the (3,1) mode that was produced as part of an undergraduate optics lab project at Bethel University (see Figure 2).

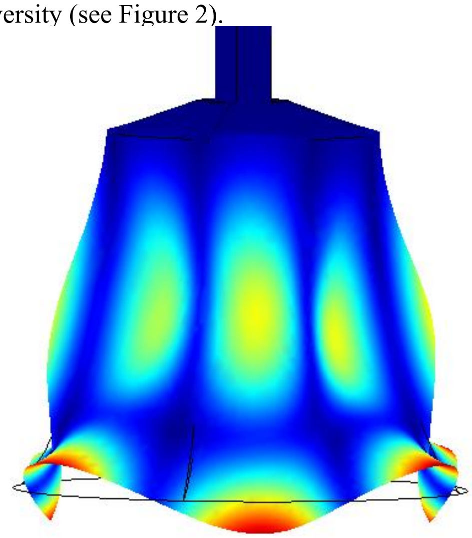


Figure 1. Degenerate vibrational mode (3,0) of a handbell: COMSOL eigenfrequency analysis.

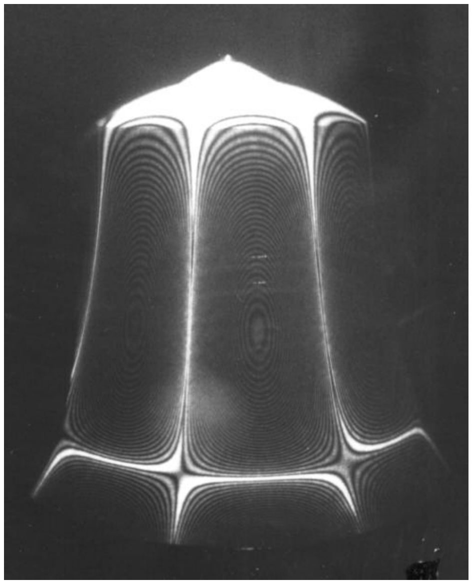


Figure 2. Degenerate vibrational mode (3,0) of a handbell: Snapshot from stroboscopic holography.

3.2 Vibrational Modes of a Coffee Cup

A degenerate mode of a symmetric cup will split into two distinct non-degenerate modes when a handle is added [6]. The asymmetry provided by the handle is the mechanism for this splitting. It is easy to demonstrate to a group of first-year physics students that a standard coffee cup has at least two frequencies and vibrational modes. The distinct frequencies heard when tapping the handle and the tapping the side of the cup 45-degrees from the handle provide all of the evidence necessary to verify that two modes with preferred orientations exist. However, it is not intuitively obvious the first-year students that many other modes of vibration are present in the coffee cup. COMSOL computations and optical diagnostics studies are useful in identifying these other vibrational modes. Figure 3 depicts the (3,0) mode for a ceramic coffee as predicted by COMSOL and imaged in an optics lab project at Bethel University.

3.3 Vibrational Modes of a Tuning Fork

A two-pronged tuning fork is an effective tuning device because it exhibits a pure musical tone at a known frequency. However, shortly after being striked, the tuning fork emits a variety of overtones. Each of these overtones corresponds

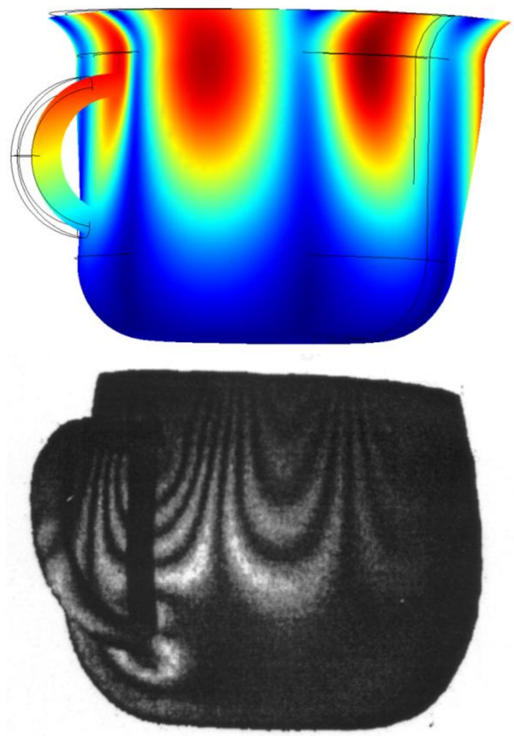


Figure 3. Vibrational mode (3,0) of a coffee cup: COMSOL result (top) and snapshot from stroboscopic holography (bottom).

to a different vibrational pattern. Rossing et. al characterized the acoustics and various bending and torsional modes of vibration for a tuning for that was freely suspended from the stem [7].

Stroboscopic holography was used in an undergraduate optics lab project at Bethel University to analyze the frequencies and vibrational modes for a steel tuning fork with a clamped stem. A FFT spectrum analysis provides a means for identifying the dominant natural frequencies of the tuning fork. The FFT graph is shown in Figure 4 and clearly indicates the presence of at least nine natural frequencies. Frequencies corresponding to two of the symmetric bending modes and to the first in-phase and out-of-phase torsional modes are highlighted in the FFT graph. The first spike identifies the preferred mode of vibration at a frequency of 273 Hz. This preferred mode is a bending mode, with the two tines vibrating symmetrically (i.e., mirrored) about the symmetry plane between the tines.

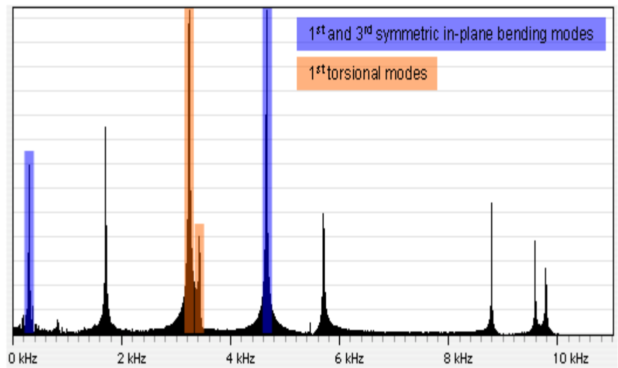
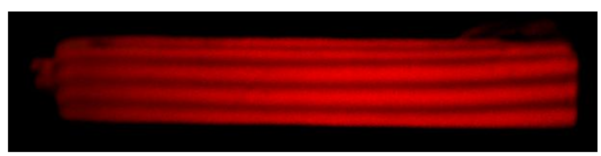


Figure 4. FFT sound spectrum for a steel tuning fork with a clamped stem.

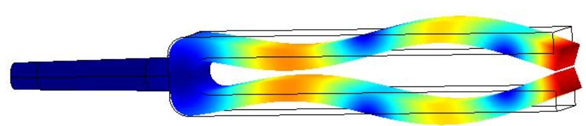
An eigenfrequency analysis is carried out for the steel tuning fork with COMSOL with the stem of the fork pinned. The various modes of vibration are compared with results from the stroboscopic holography studies. Comparisons for the first and third symmetric/bending modes are shown in Figures 5 and 6.

COMSOL (278 Hz)

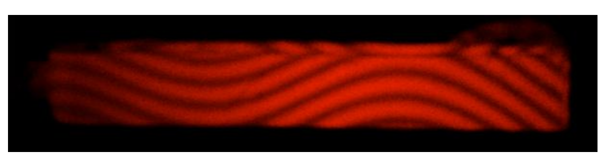


Stroboscopic holography (273 Hz)

Figure 5: First in-plane symmetric vibrational mode of the tuning fork.



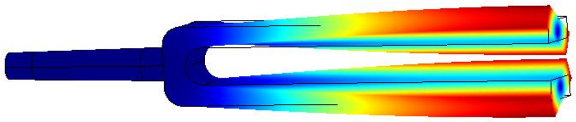
COMSOL (4560 Hz)



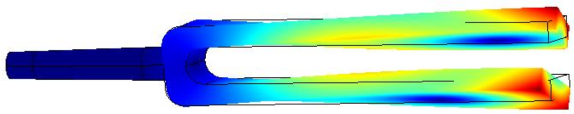
Stroboscopic holography (4647 Hz)

Figure 6: Third symmetric vibrational mode of a tuning fork.

The first in-phase and out-of-phase torsional modes of vibration are depicted in Figure 6.



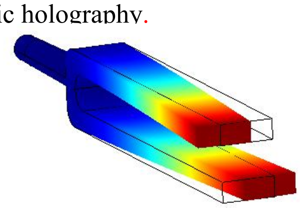
Out-of-phase torsional mode (3240 Hz)



In-phase torsional mode (3462 Hz)

Figure 6: First torsional modes of the tuning fork.

The COMSOL eigenfrequncy analysis provides descriptions for vibrational behavior of the tuning fork that can serve to complement the descriptions obtained by stroboscopic holography. COMSOL can also serve to uncover vibrational modes that might not be evident through experiment procedures. For example, the FFT graph in Figure 4 fails to recognize the out-of-plane bending modes for the tuning fork predicted in the eigenfrequency analysis. These modes are not detected because the excitation force was originally applied perpendicular to the wide faces of the tuning fork. Follow-on holographic studies, with a periodic driving force perpendicular to the narrow faces of the tuning fork, excited out-of-plane bending modes. Figure 7 depicts the first out-of-plane bending modes as predicted by COMSOL and imaged with stroboscopic holography.



COMSOL (582 Hz)

Stroboscopic holography (530 Hz)

Figure 7: First out-of-plane bending mode of the tuning fork.

4. Conclusion

The ringing of a handbell, coffee cup, or tuning fork can serve as simple demonstrations on acoustic vibrations for the first year physics or engineering student. However, the underlying physics for each of these demonstrations is much richer than the first impression may indicate. In this paper we have shown how COMSOL and optical diagnostics can be used in the upper level lab experience to convey a much fuller understanding on acoustic vibrations.

5. References

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6. Acknowledgements

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