Modeling and Analysis of a Feedback-Controlled Active Magnetic Levitation System Using COMSOL Multiphysics Finite Element Software

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Abstract: Magnetic levitation systems have been studied in the context of high-speed transportation as maglev trains, high speed machinery as magnetic bearings, and other similar engineering applications. In this paper, a three dimensional arbitrary shaped object is modeled and analyzed through COMSOL. Mechanically the levitated object has three degrees of freedom- two along the x and y axes and the third one rotational about the z axis. The x, y and Θ are the controlled variables, while the control input is the current fed to the field coil placed above the object.

A basic requirement in simulation and analysis of the system is computation of the electromagnetic fields and forces for various positions of the disc. For levitation systems, especially those involving more complicated geometries, this task has to be carried out through numerical FE or FE – BE simulation. In this work it was achieved using the basic COMSOL multiphysics software.

Keywords: Magnetic Levitation, FEM, Curve Fitting

1. Introduction

Magnetic levitation systems have been studied in the context of high speed transportation as MagLev trains, high-speed machinery as magnetic bearings, and other similar engineering applications. A basic requirement in simulation and analysis of the dynamics of a levitation system is computation of the electromagnetic fields and forces for various positions of the levitated object. For levitation systems, especially those involving more complicated geometries, analytical approaches are not feasible and only numerical approaches [1] are relevant, in particular, finite element (FE) or Boundary Element (BE) or their combination . In this work it was achieved using the basic COMSOL multiphysics software. Based on this basic core, few different schemes for modeling and analysis of the overall dynamics of the closed loop control system is illustrated.

In general, various such schemes are possible [2-3]. They are of varying complexity, yielding increasingly accurate analyses of the dynamics and involving in different cases use of analytical approximate models generated from the numerical results. In this paper, a numerical modeling strategy using only the basic COMSOL multiphysics software is illustrated. Other possible schemes, using the additional rigid body mechanics module of COMSOL, or COMSOL multiphysics interfaced with MATLAB, are also possible and relatively straightforward.. Finally, the approaches discussed and relevant results can be readily applied to similar systems with more complicated geometries in two or three dimensions as well as more complicated problem characteristics like material nonlinearity. The work thus develops a framework of modeling and analyzing generic electromagnetic levitating systems using the COMSOL multiphysics FE software at its core.

2. System

The system consists of an disc shaped iron piece suspended by a levitating coil wound over an iron core. The proposed method is illustrated here for motion in one dimensions, i.e. For vertical displacement. The engineering problem requires stabilization of the levitated disc at a levitated position using the current in the coil as a control input.

An axisymmetric formulation of the problem is described in the Fig 1., with 20 thick conductors wound around an iron core forming the coil, and the disc suspended below the coil.

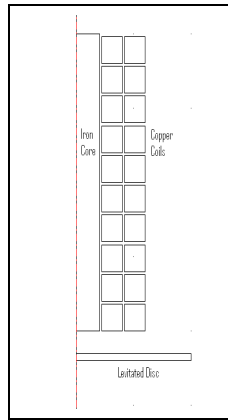The finite element mesh as generated through COMSOL for the above geometry is shown along with the geometry.

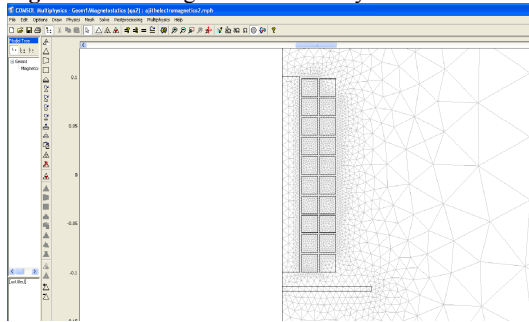**Figure 1.** The magnetic levitation system

Figure 2. System after meshing

2.1 Modeling of the System

In general, a body moving on a two dimensional region as the disc would have three degrees of motion, two translational and one rotational. In that case, the levitated motion is described by the following 6 nonlinear state space dynamic equations, where x, and y are the coordinates of the centroid of the disc while θ is the orientation with the horizontal.

$$\frac{d}{dt}x = v_x = f_1(x, y, \theta, v_x, v_y, \omega) \dots\dots(1)$$

$$\frac{d}{dt}y = v_y = f_2(x, y, \theta, v_x, v_y, \omega) \dots\dots(2)$$

$$\frac{d}{dt}\theta = \omega = f_3(x, y, \theta, v_x, v_y, \omega) \dots\dots(3)$$

$$\frac{d}{dt}v_x = (1/m)F_x(x, y, \theta, J)$$

$$= f_4(x, y, \theta, v_x, v_y, \omega) \dots\dots(4)$$

$$\frac{d}{dt}v_y = g - (1/m)F_y(x, y, \theta, J)$$

$$= f_5(x, y, \theta, v_x, v_y, \omega) \dots\dots(5)$$

$$\frac{d}{dt}\omega = (1/I_z)\mathrm{T}_z(x, y, \theta, J)$$

$$= f_6(x, y, \theta, v_x, v_y, \omega) \dots\dots(6)$$

In the above the forces F on the right hand sides are nonlinear functions of the state variables, making the overall dynamic system nonlinear.

2.1.1 Linearization about an Operating Point

For a linear system analysis of the system, it needs to be linearized about an operating point of choice, preferably an equilibrium point. Following standard small-signal linearization process about an arbitrary operating point, a linear state-space system [4] is obtained with

$$A = \begin{bmatrix} \dfrac{\partial f_1}{\partial x} & \dfrac{\partial f_1}{\partial y} & \cdots & \dfrac{\partial f_1}{\partial \omega} \\ \dfrac{\partial f_2}{\partial x} & \dfrac{\partial f_2}{\partial y} & \cdots & \dfrac{\partial f_2}{\partial \omega} \\ . & . & . & . \\ . & . & . & . \\ \dfrac{\partial f_6}{\partial x} & \dfrac{\partial f_6}{\partial y} & \cdots & \dfrac{\partial f_6}{\partial \omega} \end{bmatrix} \quad B = \begin{bmatrix} \dfrac{\partial f_1}{\partial J} \\ \dfrac{\partial f_2}{\partial J} \\ . \\ . \\ \dfrac{\partial f_6}{\partial J} \end{bmatrix}$$

A and B are the Jacobean matrices with respect to the six states and the input respectively. The above sixth-order state space system represents the dynamic motion of the levitated object moving on a plane with three mechanical degrees of freedom..
For the system considered here, with only one motion along y axis (vertical) under consideration, the corresponding state space system is

$$\frac{d}{dt}\begin{bmatrix} \delta\,y \\ \delta\,v_y \end{bmatrix} = \begin{bmatrix} \dfrac{\partial f_1}{\partial y} & \dfrac{\partial f_2}{\partial v} \\ \dfrac{\partial f_5}{\partial y} & \dfrac{\partial f_5}{\partial v} \end{bmatrix}\begin{bmatrix} \delta\,y \\ \delta\,v_y \end{bmatrix} + \begin{bmatrix} \dfrac{\partial f_1}{\partial J} \\ \dfrac{\partial f_5}{\partial J} \end{bmatrix}\delta$$

For complicated geometries, evaluation of the nonlinear forces F and hence the functions f on the right hand side is not possible analytically. Consequently the Jacobian A and B are also impossible to be evaluated analytically. It is at this stage that COMSOL is used to evaluate the above forces and their derivatives in the Jacobian numerically. The system is modeled using COMSOL and the nonlinear forces F and torque experienced by the body are calculated. The derivatives of the forces are computed by computing the forces for slightly perturbed positions of the disc and dividing the incremental change in forces by corresponding change in the position.

Here, the operating point is selected at a distance of 2.2 cm from the coil as an arbitrary point where the disc needs to be balanced. Current Density, J has the value of 10^6A/m^2 at the operating point. The mass of the disc is 1.3 Kilograms. The attractive force toward the coil at the operating point is 25.96N. Substituting after numerically finding the element values in COMSOL, we get:

$$\frac{d}{dt}\begin{bmatrix} \delta\,y \\ \delta\,v_y \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 2000 & 0 \end{bmatrix}\begin{bmatrix} \delta\,y \\ \delta\,v_y \end{bmatrix} + \begin{bmatrix} 0 \\ 54.5 \end{bmatrix}\delta$$

Here δJ has the unit of MA/m^2. Eigen values of A matrix are $20\sqrt{5}$ and $-20\sqrt{5}$. Hence the system is unstable at the given operating point of $y = 2.2$ cm. The system (A, B) is controllable. Hence we can stabilize the system using pole placement.

2.2 Stabilization by Pole Placement

We measure the position and velocity in a feedback loop with feedback gain $K=[K_1, K_2]$.

$$\delta J = -K\overline{x} \dots\dots\dots\dots\dots\dots\dots\dots\dots (\vee)$$

Where $\overline{x}$ is the state vector $\begin{bmatrix} \delta\,y \\ \delta\,v_y \end{bmatrix}$

Now the system equations are:

$$\frac{d}{dt}\delta\,\overline{x} = (A - BK)\overline{x} \dots\dots\dots\dots\dots\dots (\wedge)$$

For a feedback gain of K = [37.064, 0.165] the new closed loop poles are at -4 and -5. Hence the closed loop system is stable.
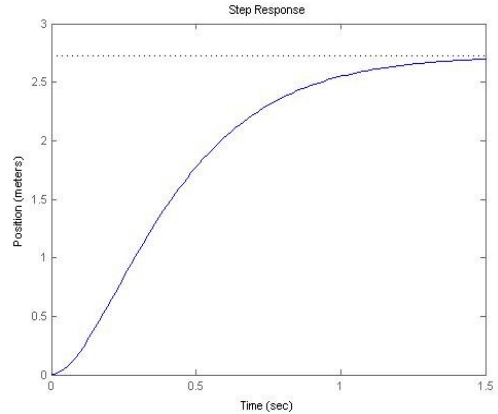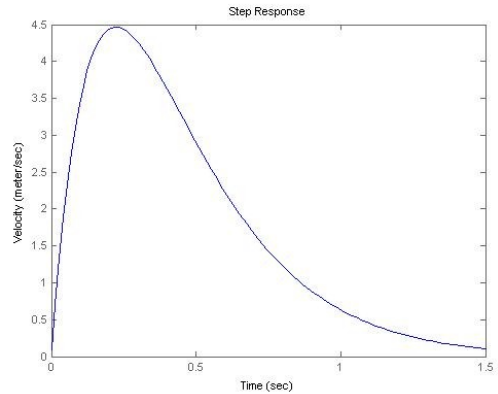

Figure 3. Step Response of Position

Figure 4. Step Response of Velocity

2.3 An alternative method: Curve Fitting

An alternative approach to generating the dynamic state space model can also be used, based on curve fitting. In this method, the electromagnetic forces acting on the levitated body are computed for its various positions in the desired region and plotted. It yields a set of points corresponding to the discrete increments in position. Now we use the method of curve

fitting to get the best fit for the curve using polynomial. This can be used to design the requisite controller.
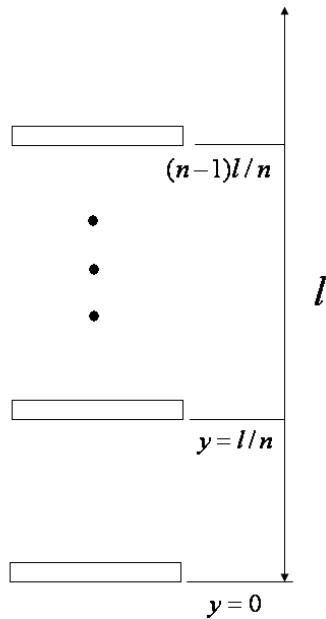

Figure 5. The forces are calculated for discrete values of y in the desired range of motion

F_y is plotted with respect to the discrete values of y. The interpolating polynomial is calculated for these set of values. Smaller the increments in y, better the polynomial approximation.

Now this analytical expression can be used to obtain the derivatives in the Jacobian and hence the overall state space system. This method will be the implemented with COMSOL in the near future, while the first method of numerical evaluation of the forces and derivatives is being illustrated here.

3. Implementation using COMSOL Multiphysics

The object was modeled as an axis-symmetric cup shaped structure with a cylinder in the middle made of aluminium. The core was modeled a cylinder made of iron and the coils around it as of rectangular cross section made of copper. The forces acting on the object were computed for alternating currents. The updated forces for incremental displacements in x, y and θ directions were calculated in COMSOL and then exported to MATLAB where the difference between the new force and the original force is

divided with corresponding change in distance/angle to calculate the Jacobean matrix.

4. Conclusions

The work presents a framework for modeling and analyzing generic electromagnetic levitation systems using the COMSOL multiphysics FE software at its core. It shows the utility of a general purpose finite element software to modeling systems for control applications, in addition to regular FE applications lie analysis of solutions, design etc. it is felt that this will enhance the scope of use of such numerical tools in control systems engineering for electromagnetic and related systems.

5. References

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6. Acknowledgements