Dynamic Model for Catenary Mooring: Experimental Validation of the Wave Induced Load

Luca Martinelli[*,1], Alberto Spiandorello[2], Alberto Lamberti[1] and Piero Ruol[2]
[1]DICAM, University of Bologna, [2]IMAGE, University of Padova
*Corresponding author: V.le Risorgimento 2, 40136 Bologna, luca.martinelli@mail.ing.unibo.it

Abstract: This note presents a model for the dynamic simulation of a catenary mooring line (or a submarine power cable). In general, mooring lines are subject to a direct wave load (e.g. drag, inertia) in addition to the induced load due to movement of the vessel to which they are linked. Specific aim of this note is to present, calibrate and validate the numerical response to direct wave action, by comparisons with physical model tests.

Tests were carried out at the wave flume of the Maritime Laboratory of IMAGE Department, Padova University, within the framework of a Master thesis. Wave induced loads were measured at the fairlead of a compliant chain, with the peculiarity that the fairlead was hinged to a fixed point, the chain being simply subject to the load induced by regular (slightly non-linear) waves.

The equations describing the dynamic movements of the chain in Comsol Multiphysics are written in weak form.
It is concluded that the model well represents the tests when the drag coefficient is equal to 1.2-1.4 and applied to an area equal to the maximum apparent width.

Keywords: Drag, Dynamic load, Mooring, Morrison Equations, Power Umbilical, Weak form.

1. Introduction

Mooring systems of floating Wave Energy Converters (WEC) are characterized for instance in Harris et al. (2004) and - more extensively – in Fitzgerald (2008).

They may be classified in “Passive”, “Reactive” and “Active”.

“Passive” moorings (possibly catenary anchor leg) have mainly station keeping purposes and are typical of devices based on the overtopping principle.

“Reactive” moorings (possibly spread mooring with taut lines) provide a reaction force which is directly used by the power take off system and are typical of devices based on the wave activated body concept.

“Active” moorings (possibly spread moorings with chains), have an optimal rigidity to tune the dynamic movement of the WEC and maximize efficiency.

In the latter case, optimal performance can be reached only if the dynamic behavior of the WEC is known with accuracy. Lumped mass models of the WEC that concentrate the mooring system effect into stiffness and damping coefficients cannot be used, since such analysis ignores the additional modes of oscillations caused by the presence of many added masses, each with many degrees of freedom.

In practice, the dynamic of each line must be investigated.

Note that there are many sources of nonlinearities in the problem: part of them are related to the material and geometry of the system, and are well known in naval architecture; part of them are rather new, and are related to the non linear behavior of the wave induced load which is much more pronounced than for ships and vessels (so that many commonly accepted simplifying assumptions are not justified for WEC applications).

Design poses many challenges, non only because the number of elements under analysis explodes, but also because overdesign is not an option (the aim being an optimization).

A number of recent project focus on mooring design of WECs and their deliverables may be used as reference to this problem: Fp7 project CORES (Components for Ocean Energy Renewable Energy Systems) funded by the European Community and the project SDWED (Structural Design of Wave Energy Devices) funded by the Danish Council for Strategic Research.

Several models of the chain dynamic are available.

Brown and Mavrakos (1999) compared the results of a number of codes for cable analysis in both frequency and time domain. The analysis includes friction due to movement between the cable and the sea floor. Elasticity of the chain and synthetic cable are well represented. However, the analysis is done in a still water velocity field so that the wave excitation forces on the cable are not included. Johanning and Smith (2006) have compared experimental results for tension and damping of a 7m catenary using a time domain model (OrcaFlex), showing a good representation of the line dynamic under non-linear conditions.

Two kinds of approaches can be followed, that assume as dependent variables either the line deformation or the line position. Gobat and Grosenbaugh (2006) follow the first approach. The second approach (Martinelli et al., 2010) is more suited to deal with interaction with the bed Comsol Multiphysics, and it was therefore preferred.

The load induced by the wave orbital movement on the chain is usually evaluated by means of the Morrison Equation. The drag coefficient in the formula has been frequently computed for a fixed cylinder (characterized by its diameter) at any Reynolds numbers. For a chain, the characteristic dimension is reasonably the total width and the extension of the laminar regime should be somewhat smaller, given the chain irregular geometry.

In order to study the processes involved into mooring dynamic, the Authors investigate simultaneously physical models in wave flumes and numerical simulations at the same scale, and are therefore interested at low Reynolds numbers.

Aim of this note is to present a dynamic model for a complaint chain, validate it by means of physical model tests and calibrate the drag coefficient in the Morrison Equation at laboratory scale, i.e. for Reynold numbers of order 1000.

The following Sections describe: the physical model tests, including the results; the governing equations given in weak form, with some details concerning the application to Comsol; the numerical results and the agreement with the experimental measurements; the conclusive discussion.

2. Physical model tests

Tests were carried out at the Maritime Laboratory of IMAGE Department, Padova University. The laboratory is equipped with a flume with wave generator equipped with active wave absorber. The flume is 36.0 m long, x 1.00 m wide x 1.30 m high.

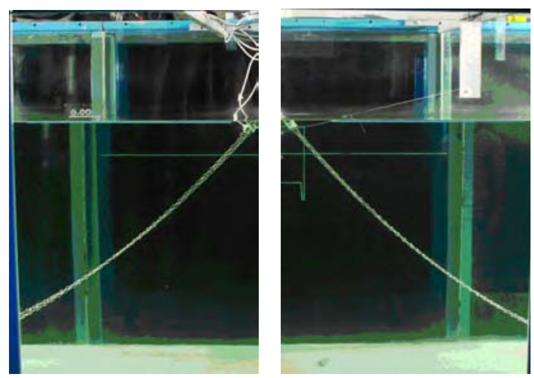


Figure 1. Left and right chains (load cell 1 and 2 respectively).

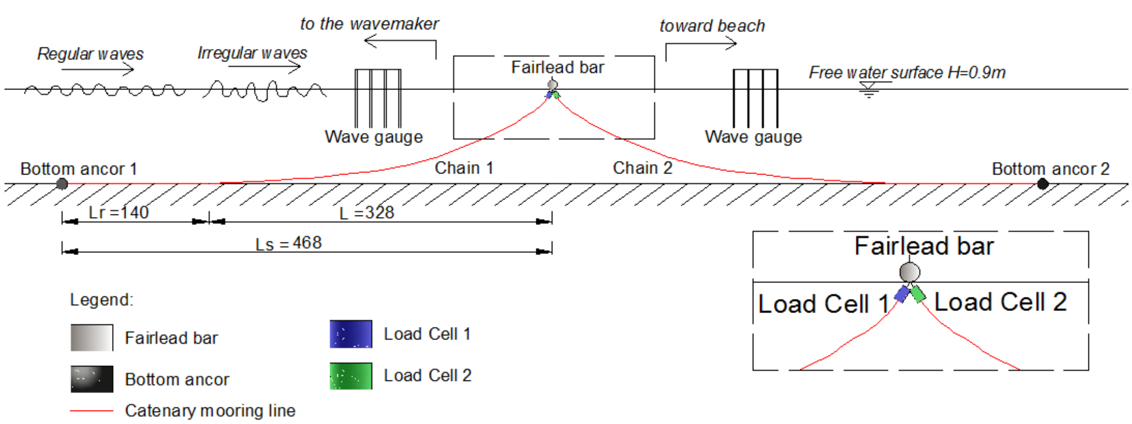


Figure 2 Set up of chains in the flume and particular of Load Cells position.

Two steel chains, made of welded stainless steel, are placed parallel to the flume axis. The the fairlead is hinged to a bar at the water surface. One chain (chain1, Figure 1 left) has the anchor position toward the wavemaker and the other toward the absorbing beach (chain 2, Figure 1 right).

Three regular waves were generated. Wave characteristics and results of the measured load are given in Table 1 and Figure 3.

Table 1 Wave characteristics for regular tests, amplitude of force signal.

Type	T [s]	H [cm]	Fmax	Fmin
Reg1	0,8	10,45	0,153	0,137
Reg2	0,98	14,06	0,154	0,128
Reg3	1,22	14	0,154	0,125

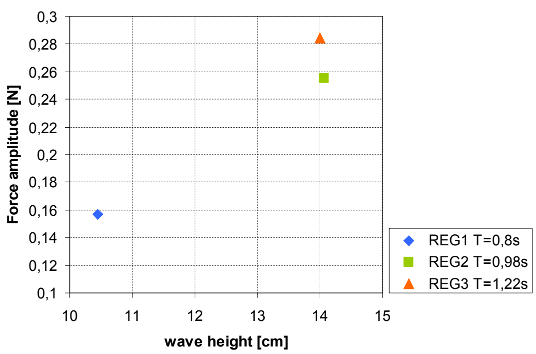


Figure 3 Amplitude of the load oscillation Vs incident wave height.

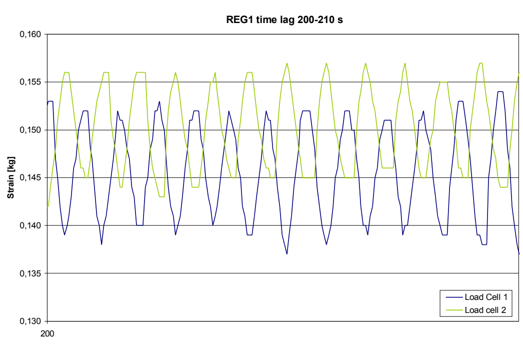


Figure 4 Example of record of load cell readings. Test Reg1 in Table 1

Chains are defined by the following characteristics: dry weight w=70.0 gr/m; total length L_{tot} = 5.00 m; horizontal distance from anchor to fairlead L_s = 4.68 m; length of chain lying down on the ground L_r = 1.4m; horizontal length of suspended chain L = 3.28m; length of suspended chain s = 3.6m.

Stress along the axis direction is measured close to the fairlead by impermeable strain gauges (full bridge).

Figure 4 shows an example of measured load.

4. Numerical model

4.1 Domain equation

The domain is 1D, and is the chain itself and the length of the domain is therefore equal to the chain length at rest. The independent variables are s and t, i.e. the abscissa along the chain and time.

The dependent variables are X and Y, defined as the horizontal and vertical movement respectively, the axial deformation ε and the angle φ formed by the tangent with the horizontal.

The differential equations of motion have been derived in many publications, see as for instance Goodman and Breslin (1976) or Aamo and Fossen (2000).

The domain equations in terms of displacements become:

$$m\frac{\partial^2 X}{\partial t^2} = \frac{\partial H}{\partial s} + f_x(1+\varepsilon)$$

$$m\frac{\partial^2 Y}{\partial t^2} = \frac{\partial V}{\partial s} + f_v(1+\varepsilon)$$

$$\frac{\partial^2 X}{\partial t\partial s}\cos\varphi + \frac{\partial^2 Y}{\partial t\partial s}\sin\varphi = \frac{\partial \varepsilon}{\partial t}$$

$$-\frac{\partial^2 X}{\partial t\partial s}\sin\varphi + \frac{\partial^2 Y}{\partial t\partial s}\cos\varphi = (1+\varepsilon)\frac{\partial \varphi}{\partial t}$$

The first two equation force the momentum balance along the horizontal and vertical direction, the last two are congruence equations. The constitutive law is used to define the stress along the cable, that is then projected along the horizontal and vertical direction:

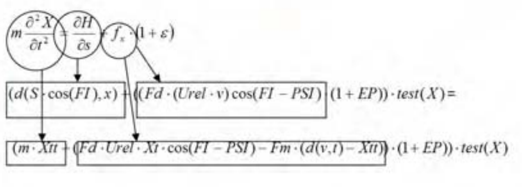
The constitutive laws are:

$H = EA\varepsilon\cos\varphi$, $V = EA\varepsilon\sin\varphi$

E being the Young modulus of steel.

Two auxiliary dependent variables are added, the reaction force R at the bottom, a mere elastic, and the angle of the apparent orbital velocity ψ, defined on the basis of the wave and cable velocity.

As a guideline for the implementation, it is shown in the following how to insert the weak and dweak terms:



4.2 Wave load

In order to include the wave action, the orbital velocity up to second order are inserted as global expressions:

$v = v_1 + v_2$ (horizontal velocity of waves)

$w = w_1 + w_2$ (vertical velocity of waves)

$v_1 = \omega A \cosh(k(z+D))/\sinh(k_3 D)\cos(k X - \omega t)$

$v_2 = 3/16\ \omega k (2A)^2 \cosh(2k(z+D))/$
$\sinh(k D)^4 \cos(2kX - 2\omega t)$

$w_1 = \omega A \sinh(k(z+D))/\sinh(k_3 D)\sin(kX - \omega t)$

$w_2 = 3/16\ \omega k (2A)^2 \sinh(2k_3(z+D))/$
$\sinh(kD)^4 \sin(2kX - 2\omega t)$

ω, k= wave frequency, wave number;

A = wave amplitude;

z = Y-D

D= depth (0.9 m)

The Morrison Equations needed to evaluate the load induced by the horizontal and vertical velocity are:

$$F_{Mx} = 0.5 \cdot C_D \cdot d \cdot |\overline{u}| \cdot (v - \dot{X}) + \rho \cdot C_M \cdot Ac \cdot \left(\frac{\partial v}{\partial t} - \ddot{X}\right)$$

$$F_{My} = 0.5 \cdot C_D \cdot d \cdot |\overline{u}| \cdot (w - \dot{Y}) + \rho \cdot C_M \cdot Ac \cdot \left(\frac{\partial w}{\partial t} - \ddot{Y}\right)$$

where d is the maximum width of the chain links, i.e. the 7 mm, and not their thickness (2 mm), Ac is the volume per unit length of the chain, and the absolute velocity is obviously

$$|\overline{u}| \cdot = \sqrt{(v - \dot{X})^2 + (w - \dot{Y})^2}$$

Note that the drag component has the direction of the normal to the line and the inertial term has the direction of the corresponding velocity.

4.3 Initial, boundary conditions and mesh

Initial conditions are supplied indirectly. First, a known simple configuration is supplied, consisting in a catenary fully raised from the bed. Then, the fairlead is slowly moved (lowered and shifted) to the true initial position, so that the geometry and the stress are computed by the code itself. During this stage, occupying the computational time t between 0 and 25 s, no waves are present. After 5 seconds the wave load is gradually applied and after 5 more seconds the regular (non-linear) wave maintains its full amplitude.

The Boundary conditions are applied only to the end points of chain. X,Y are given both at the anchor point and at the fairlead, where they are hinged.

Mesh element size used in the computation is 0.01 m. Simulations with grid size of 0.001 m proved the stability of the results.

The convergence tolerance and the equation scaling must account for the fact that the deformation needs to be very accurate.

5. Numerical results

Results are given in terms of axial load, obtained as $S = EA\varepsilon$, at the fairlead.

Figure 5 shows S for the first wave, Reg1 (see Tab. 1). The initial load is computed in absence of waves and corresponds to the static condition.

Agreement between numerical and experimental results shows that the laboratory set up is accurately described. When the regular wave load is applied, the load at the fairlead obviously oscillates: Average is approximately equal to the initial load, since non-linear steady drift on the chain is negligible.

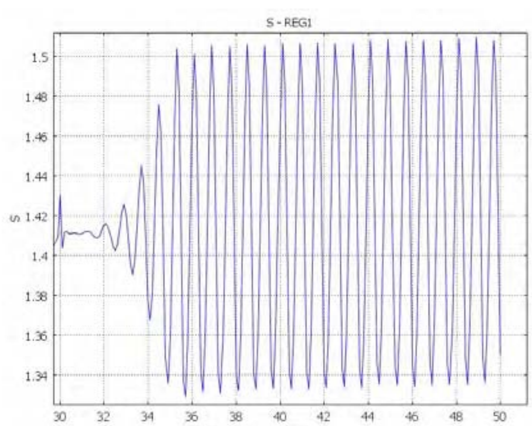


Figure 5 Simulated load along Chain 1, Test Reg1 in Table 1.

Figure 6 shows a comparison of mean load between numerical and experimental results and optimal agreement is obtained.

Excursion between maximum and minimum load gives a measure of the effect of waves. Figure 7 shows a comparison of load excursion between numerical and experimental results. The obtained good agreement is the result of a calibration procedure. The image presents the result of a calibration carried out on the Test Reg1, that determined a drag coefficient of 1.4. A value of 1.2 gives similar results.

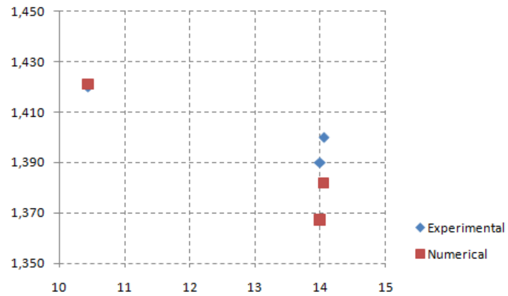


Figure 6 Comparison between average measured load.

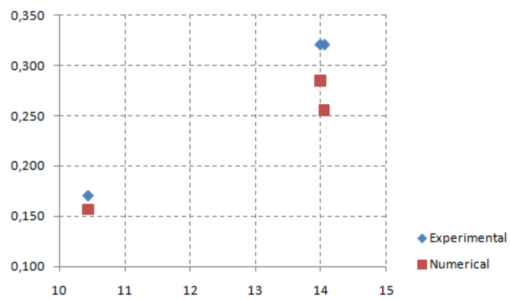


Figure 7 Comparison between amplitude of load oscillation

6. Conclusions

Comsol software is used to solve the dynamic of a chain where the dependent variables are displacements and not deformations.

Where the cable goes slack or when there is a rapid lifting and lowering of the cable to and from the sea bottom, the convergence is slow, of order 30 mins and the default accuracy of the variables involves some tampering (and tempering!). Anyway the solution fully agrees to the experimental measurements.

The drag coefficient was calibrated and a value of 1.2-1.4 was found appropriate, the cross section being determined by the total widths of the links.

References

Aamo O.M., Fossen T.I. (2000). Finite element modelling of mooring lines. *Math Comput Simul* 53(4-6), 415–22.

Brown DT, Mavrakos S. (1999). Comparative study on mooring line dynamic loading. *Marine Struct*; 12, 131–51.

Fitzgerald, J. (2008). Position mooring of wave energy converters, phd thesis, Water environment technology, Chalmers University of Technology, Göteborg, Sweden, 93 pp.

Gobat and Grosenbaugh (2006). Time-domain numerical simulation of ocean cable structures. *Ocean Eng* 33 1373–1384

Goodman TR, Breslin JP (1976). Statics and dynamics of anchoring cables in waves. *J Hydronaut* 1976;10(4):113–20.

Harris R., L. Johanning, J. Wolfram (2004). Mooring systems for wave energy converters: A review of design issues and choices. Marec2004.

Johanning L., G. Smith (2006). Comparison of simulation and test results for a generic moored WEC using a catenary mooring arrangement. *Proc. Int Conf Ocean Energy*, Bremerhaven, 2006.

Martinelli L., A. Lamberti, P. Ruol, P. Ricci, P. Kirrane, C. Fenton, L. Johanning (2010). Power Umbilical for Ocean Renewable Energy Systems - Feasibility and Dynamic Response Analysis, *Proc. Int Conf Ocean Energy*, Bilbao, 2010.

Acknowledgements

The support of the CORES project, funded by the European Commission under grant agreement n°213633, is gratefully acknowledged.