Mathematical Model for Prediction of Transmission Loss for Clay Brick Walls

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Abstract: Standardized acoustic measurements are both expensive and time consuming. For porous materials the rigid matrix properties as well as porosity and pore properties contribute to sound reduction index of the material. The composite structure alignment itself is important. When trying to improve a composite structures sound reduction index by varying material properties or the structure alignment, a new measurement is necessary. Therefore a need for a mathematical model that can predict sound reduction index is necessary for the optimization to be carried out with computer and one final test to be made for validation.
In this work a standardized test room that fit ISO EN 10140 -1 to 10140-5 is used to predict a sound reduction index value for several composite structures. The results achieved are validated with real standardized measurements carried out in a certified laboratory.

Keywords: Transmission loss, acoustics.

1. Introduction

The problem of sound reduction of building structures is highly topical nowadays and acquires ever increasing importance in the construction engineering as the A class building constructions must have good sound insulating properties besides thermal insulation and microclimate requirements. To estimate the level of sound absorption by these structures a specific parameter − the sound reduction (SR) index − is employed[1,2]. The aim of this work is to set up a mathematical model for determination of SR index of building construction. To achieve the goal a finite element method (FEM) is used. This work will concentrate on evaluation of sound pressure level (SPL) assuming that noise is being generated as superposition of several monofrequencies. The results are later validated with measurements carried out at certified laboratory.

Previous works concerning this problem have been done by Papadupoulis[3,4]. In his papers the standardized 3D test rooms are considered and improvements are made to optimize the test facilities for better modal distribution. Virtual experiments are carried out and compared to real life measurement data. The low frequency range is considered as the computational cost increase with frequency. Del Coz Diaz [5] is using a 2D FEM model to analyse wall of building blocks with macroscopic enclosures similar as in this work, however the SR index calculation is different.

The second chapter „Experimental requirements" deals with ISO standard requirements for measurements of airborne SR index. Standard description with equations for SR index calculations and real life experiment setup is given.

The third chapter „Theory background" deals with mathematical description of sound propagation in air, solid media and porous materials. Boundary conditions for the problem are also discussed here.

The fourth chapter „Model setup" deals with assumptions made to set up a virtual experiment and to better fit it with real experiment. Two different model geometries are proposed here.

The fifth chapter „Noise and its measurements" deals with SPL evaluation for one-third octave bands and several possibilities are discussed to calculate SPL.

2. Experimental requirements

Experimental setup is given at EN ISO 10140-1 to 5 standards. The standards state that the test rooms must be at least 50 m^3 each. There must be 5 microphones or one rotating microphone in each room to register sound pressure [1,2]. The rooms must have diffuse sound field that means that form all directions at the given point all the frequencies must have same intensity [3]. To achieve such properties the reflection coefficient of the wall should be high.

The sound transmission class for a building structure is the shifted reference curve value in dB at 500 Hz frequency. This is called the weighted SR index for airborne sound and is calculated by using the EN ISO 717-1 standard. First the SR in each one-third octave band is calculated using

$$R = -10lg\left(10^{-\frac{R'}{10}} - 10^{-\frac{R'_T}{10}}\right) \quad (1)$$

Where R' – apparent SR index for one-third octave band, dB

R'_T - sideway noise, measured before the test, dB

The SR index for one-third octave bands is calculated as the SPL difference

$$R' = SPL_1 - SPL_2 \quad (2)$$

Where SPL_1 and SPL_2 is sound pressure levels in source room and receiving room respectively.

For the mathematical model the sideway noise cannot be taken into account and the $R = R'$

The experiment for the particular wall was made in certified laboratory in Latvia, the results are given in chapter 6 „Results and discussion".

3. Theoretical background

3.1. Air domains

The sound propagation in a lossless air is described with Helmholtz equation

$$\frac{1}{\rho c^2}\frac{\partial^2 p}{\partial^2 t} + \nabla\left(-\frac{1}{\rho}(\nabla \mathrm{p} - \mathrm{q})\right) = Q \; (3)$$

Where p – acoustic pressure, Pa

ρ – density, kg/m^3

c – speed of sound m/s

q – dipole source N/m^3

Q – monopole source $1/s^2$

For time harmonic case when $p(x,t)=p(x)e^{i\omega t}$ the equation (1) become

$$\nabla\left(-\frac{1}{\rho}(\nabla \mathrm{p} - \mathrm{q})\right) - \frac{\omega^2 p}{\rho c^2} = Q \quad (4)$$

Where ω – angular frequency, rad/s

The dipole source q=0 and the monopole source Q is expressed as

$$Q = 2\sqrt{\frac{P\omega}{\rho}} \quad (5)$$

Where P – sound power W/m

3.2. Solid domains

Two methods have been chosen to model the sound transmission through construction. First is the solid mechanics approach described in this subsection and the second is by using equations derived by *Biot* for porous materials.

The sound propagation in solids is described by stress-strain relationships

$$\sigma_{ij} = \sigma_0 + C_{ijkl}\varepsilon_{kl} \quad (6)$$

Where σ_{ij} – stress, Pa

σ_0 – initial stress, Pa

C_{ijkl} – elasticity tensor

ε_{kl} – strain

The wave equation in solids is

$$\rho\frac{\partial^2 u}{\partial t^2} - \eta\frac{\partial u}{\partial t} + \nabla(\nabla \mathrm{u} - \mathrm{f_s}) = F \quad (7)$$

Where u – displacement, m

F – volume source

f_s – dipole source

As there is no sound sources inside the wall f=0 and F=0

Similar to air domains the time harmonic case when is assumed and eq.(7) reduce to

$$-\rho\omega^2 u - i\eta\omega u + \nabla(\nabla \mathrm{u} - \mathrm{f_s}) = F \quad (8)$$

3.3. Biot equivalent

For porous material domain a set of equations derived by *Biot* are used [8-10]. In this work they are not derived once more, but basic principles and equations are given. *Biot* model assume a cube for which a pore size is small compared to its side length and pores. The main idea is that fluid and solid parts move independent of each other and the energy dissipation occurs because of Poisuelle type flow between two media. These equations must be solved for two displacement fields and pressure field. To make the equations numerically optimal the u-p formulation suggested by Allard[11] is used.

$$\begin{cases} -\omega^2\left(\rho_{av} - \dfrac{\rho_f^2}{\rho_c}\right)u - \nabla(\sigma_{dr} - s_0) = F \\ -\dfrac{\omega^2}{M}p_f + \nabla\left(-\dfrac{1}{\rho_c}(\nabla p_f - q)\right) = Q \end{cases} \quad (9)$$

3.4. Boundary conditions

For test room boundaries there are several possibilities the two opposite cases are sound hard boundary that is completely reflecting

$$\frac{\partial p}{\partial n} = 0 \quad (10)$$

The other case is sound soft boundary that is completely absorbing

$$p = 0 \quad (11)$$

The rest of cases can be modelled using impedance boundary condition that is partly reflective depending on impedance.

$$-n\left(-\frac{1}{\rho}(\nabla p)\right) = -\frac{i\omega p}{Z} \quad (12)$$

Where Z – acoustic impedance

The problem with sound soft boundaries is that there is no diffuse field and therefore the experimental requirements are not fulfilled. The problem with sound hard boundary is the unrealistically high sound pressure values at the resonance frequencies, so these boundary conditions should be avoided. Therefore the impedance boundary condition was chosen. The reflection at the boundary is determined by characteristic impedances of materials.

For the test wall ends there are also several possible boundary conditions. One of the options is absolutely fixed boundary when

$$u = 0 \quad (13)$$

Another option is to allow the wall to move freely. The average between both can be done by adding mass to the test wall ends.

4. Model Setup

Solving a complete 3D acoustic problem is difficult task. For example a 3150Hz 3D model with dimensions approximately 9.7m x 4.3m x 3.3m can be taken. As the wavelength must be resolved with mesh by 5 nodes per wavelength at worst, the degrees of freedom (DOF) for a problem can be evaluated as 5^3 times model volume measured in wavelengths. As the wavelength for 3150Hz is 10.9cm, for this particular assumption it would be 13.3 million DOF. Therefore a 2D model (figs. 1,2) is chosen. This eliminates some resonance frequencies of the room and of the test wall, but the physics remain the same.

The time harmonic case allows simulating monofrequencies so the noise is made up from superposition of them with step 1 Hz (fig.3).

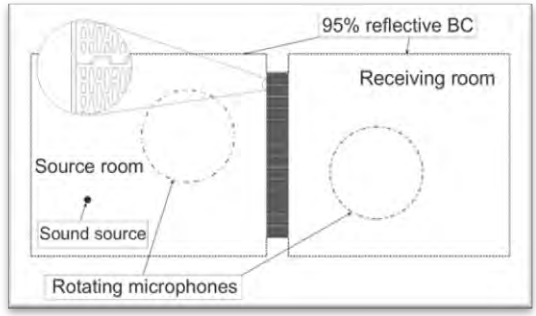

Figure 1. Geometry of mathematical model. Simple case with parallel test room walls.

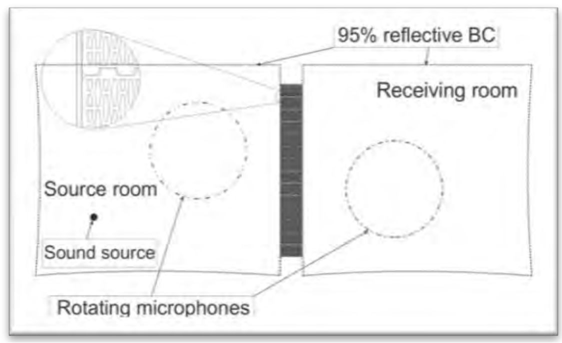

Figure 2. Arc type geometry with radius 20 m

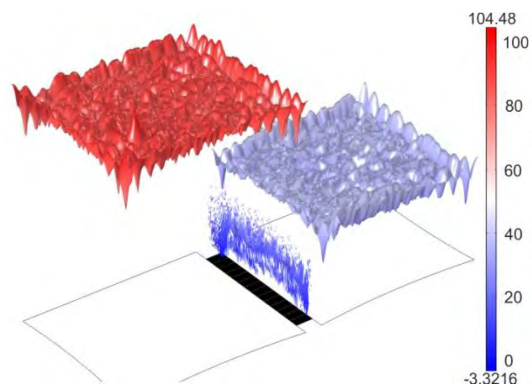

Figure 3. Noise made of monofrequencies for one-third octave band 500Hz middle frequency.

The modal density which here must be understood as the number of eigenfrequencies for the given test room per one-third octave band is small at low frequencies so the result mostly depends on few frequencies around the resonant one. The modal density can be improved by choosing 3D model that is possible for low

frequency range; however this is a future task and is not performed here.
For the sound pressure measurements at the room the rotating microphone is modelled as a circle where sound pressure level is integrated

5. Noise and its Measurements

The noise used in real measurements is pink noise that means over every octave band the total sound intensity is constant. To model this effect the input sound power was made frequency dependent.

The time harmonic case finds stationary solution to problem. The superposition principle implies that the resulting SR index for one-third octave band should be calculated using

$$R = 10lg\frac{\sum_1^n p_s^2}{\sum_1^n p_r^2} \quad (14)$$

Where p_s – sound pressure in source room
p_r – sound pressure in receiving room

The other way the SR index for octave band was calculated is a by averaging over frequency range. This approach assumes that all the frequencies have equal impact on final result and eliminate the impact on resonance frequencies. This approach is used in [5]. The results were calculated using both methods and the results are compared. The former method is physically correct for real test environment while the latter assume that the sound field is perfectly diffuse and the impact is

6. Results and Discussion

You The computation took approximately 9 hours for the every frequency range (89 Hz – 3548 Hz) on the 3.4 GHz intel i7 processor, so the whole calculation took approximately 36 hours. The following figures show SR values of one-third octave bands. As it can be seen in all cases the there is major differences from experimental data in low frequency range when using superposition. The averaging gives much closer predictions in the low frequency range. In the middle and high frequency range the averaging approach gives slightly higher SR values then superposition. This is due to resonance frequencies, that have high peak values that mostly affect the SPL and the rest of frequencies have much smaller impact. This can be seen in figs. 7 and 8 where the superposition of one-third octave band frequencies with the middle frequency of 100 Hz can be seen.

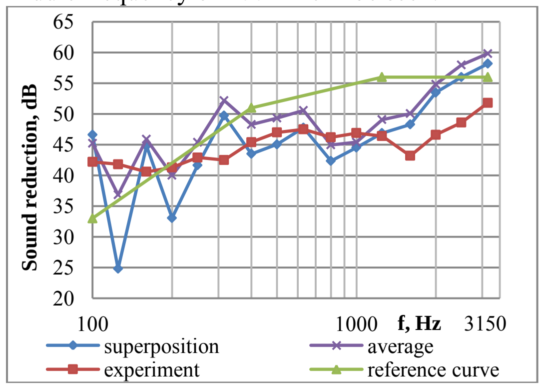

Figure 4. SR index curve for one-third octave bands for simple geometry and structural mechanic equations, compared to experimental data.

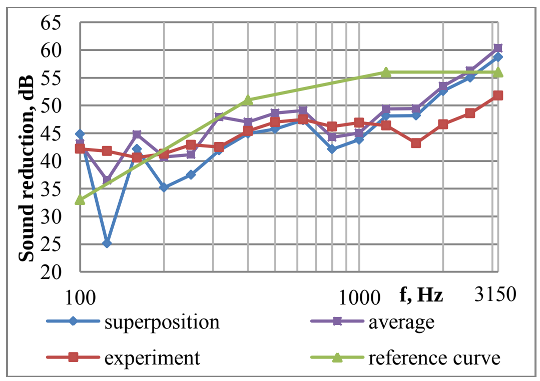

Figure 5. SR index curve for one-third octave bands for curved geometry and structural mechanic equations, compared to experimental data.

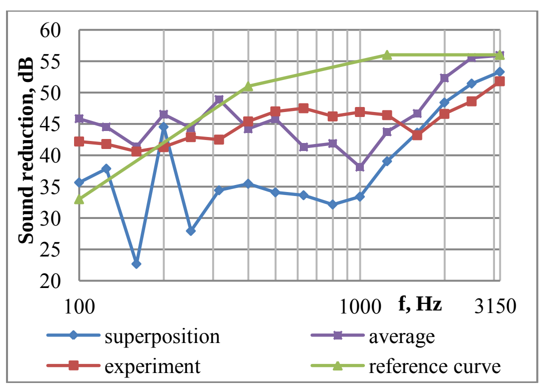

Figure 6. SR index curve for one-third octave bands for simple geometry and *Biot* equations, compared to experimental data.

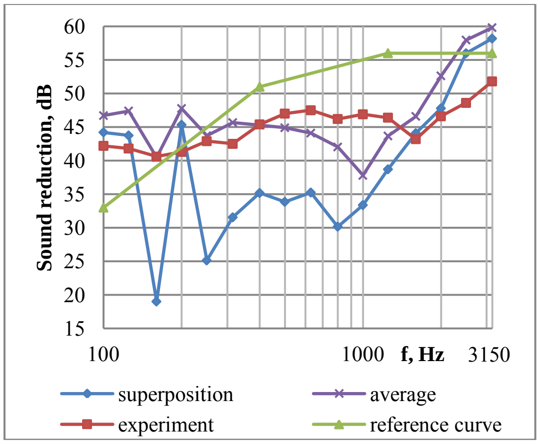

Figure 7. SR index curve for one-third octave bands for curved geometry and *Biot* equations, compared to experimental data.

The *Biot* equations and structural mechanic ones give completely different results. The first gives much lower SR values with superposition evaluation then with the averaging. The latter gives only slightly lower values for superposition calculation then for averaging.
The sound transmission class for all cases is also computed (table 1) the resulting values are close to experimental ones.

	Simple geometry		Arc type geometry	
	str. mech.	*Biot*	str. mech.	*Biot*
superposition	46	37	46	36
average	50	45	49	46

Table 1. SR index values for calculations using porous material and solid approximations. Superposition method and averaging are compared.

The various boundary conditions was assumed and the resulting SR index (table 2) was calculated.

Boundary conditions	free	fixed	added mass	
			1000	10000
R value, dB	47	47	47	48

Table 2. SR index for different boundary conditions.

7. Conclusions

The results show that the in low frequency range the results fluctuate far from experimental ones and that model must be improved. This is mainly because of resonance frequencies that have great impact on the SR index in low frequencies where modal density is low. This means that the results will greatly differ from one laboratory to another. The possible improvements would be 3D test rooms that are easy to compute in low frequency range because of lower computation costs. Another possibility is making the test room boundaries curved that would enlarge the modal density. The artificial sound diffusers could also help to increase modal density.

The *Biot* equations undervalue the damping and therefore if used for this kind of materials, additional damping term should be introduced.

Another important thing as already pointed out by Papadopoulos [3,4] is the need for boundary conditions at the ends of test wall. Various boundary conditions give different results over one-third octave bands; however the sound reduction index does not change. An extensive laboratory work is needed to determine correct boundary conditions; however they could change from wall to wall that leads to laboratory tests once more. This method con not determin the correct values at octave bands jet, but can serve as a tool to approximately determine the SR index.

The drawback of the model is the lack of third dimension because additional reflections happen at the ceilings and floor. For this an additional term for evaluation of SR index must be introduced.

8. References

1. EN ISO 10140 – 1,2,3,4,5 *Acoustics – Laboratory measurements of sound insulation of building elements* (2010)
2. EN ISO 717 -1 *Acoustics – Rating of sound insulation in buildings and of building element,* (1996)
3. Papadopoulos, C. I., Development of an optimized standard-compliant procedure to calculate sound transmission loss: Design of transmission rooms" *Applied Acoustics 63*, 1003-1029, (2002)
4. Papadopoulos, C. I. Development of an optimized standard-compliant procedure to calculate sound transmission loss: Numerical measurements, *Applied Acoustics 64*, 1069-1085, (2003)
5. del Coz Diaz, J.J., et.al. Sound transmission through a multilayer lightweight concrete hollow

brick wall by FEM and experimental validation *Building and Environment 45,* 2373-2386, (2010)

6. Rossing, T.D. et. al. Springer Handbook of Acoustics, *Springer* ISBN: 978-0-387-30446-5, (2007)

7. Comsol Inc., Acoustic module User's guide, Comsol Multiphysics, (2011)

8. Biot, M. A. Theory of Propagation of Elastic Waves in a Fluid-Saturated Porous Solid. I. Low Frequency Range., *The Journal of the Acoustical Society of America*, *28* (2), 168-178, (1956)

9. Biot, M. A. Theory of Propagation of Elastic Waves in a Fluid-Saturated Porous Solid. II. High Frequency Range, *The Journal of the Acoustical Society of America*, *28* (2), 179-191, (1956)

10. Biot, M. A. „Generalized theory of Acoustic Propagation in Porous Dissipative Media, *The Journal of the Acoustical Society of America, 34* (5), 1254-1264, (1962)

11. Allard, J.F., Atalla, N. Propagation of sound in porous media: modelling sound absorbing materials, second edition, *John Wiley and Sons Ltd*., ISBN 978-0-470-74661-5, (2009)

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EIROPAS REĢIONĀLĀS ATTĪSTĪBAS FONDS

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