Homogenization Approaches for Laminated Magnetic Cores using the Example of Transient 3D Transformer Modeling

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Abstract: A specific issue in transformer modeling using the finite element method is the consideration of electric sheets or other laminated core materials which are used to reduce eddy currents. It would be impractical to explicitly model a large number of sheets as this would lead to a large number of elements and hence to unacceptable computational costs. Homogenization procedures overcome this problem. They substitute the laminated structure by a solid having nearly the same electromagnetical behavior. In our study, we have implemented several of them in a transformer model using COMSOL Multiphysics. Simulation results obtained with the different homogenization approaches are compared to experimental test results. This reveals the accuracy and the application limits of the investigated homogenization approaches.

Keywords: transformer model, laminated core, electric sheets, nonlinear ferromagnetic material, transient analysis.

1. Introduction

Proper modeling and simulation of transformers with finite element methods is a challenge due to the involved nonlinearities and coupling effects between different physical domains. Primarily, there are the highly nonlinear magnetic characteristics of the core materials including ferromagnetic hysteresis, eddy currents in the windings and in the core, and temperature-dependent material properties like the electrical conductivity. With nonlinear magnetic material or a non-harmonic excitation, simulations need to be performed in transient rather than in time-harmonic mode.

The transformer models presented in former publications solve most of the mentioned problems in a satisfactory manner. This was demonstrated for the example of a current transformer [1, 2] which has a primary winding of only one turn and is short-circuited on the secondary side.

A specific issue in transformer modeling is the consideration of magnetic cores based on stacked electric sheets or strip wound or other laminated core materials which are used to reduce eddy currents and thus to minimize the dynamic hysteresis losses and to improve the dynamic behavior of magnetic actuators.

Normally, the thickness of the individual sheets is small as compared to the core thickness. Therefore, it would be impractical to explicitly model a large number of sheets with a high dimensional aspect ratio, as this would lead to a large number of elements and hence to unacceptably high computational costs. This is even more the case as the transition boundary condition is not applicable between domains in time-dependent studies. It would be therefore also necessary to model the insulation layers between the sheets.

To overcome this problem, several homogenization approaches for laminated cores have been proposed in the past [3-7]. They replace the laminated structure by a single domain of an electrically orthotropic material which exhibits the same macroscopic behavior in a certain range of conditions. Thus, the computational effort can be significantly reduced.

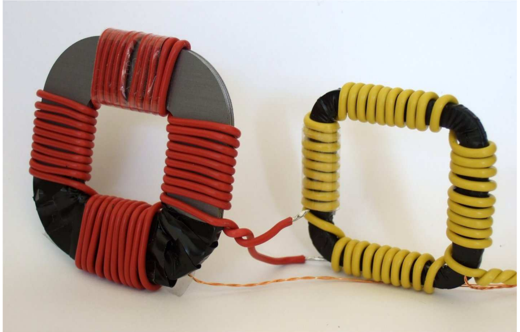

Figure 1. Transformers with laminated soft magnetic ring core for static and dynamic hysteresis measurements.

In our study, we have implemented these approaches in a 3D-transformer model. The obained solutions are compared to experimental test results. This comparison is made both for ring core arrangements similar to those which are used for magnetic material characterization according to DIN EN 60404-6 and for configurations when the core is operated as a current transformer.

2. Transient Electromagnetic Transformer Model

2.1 Principle of Modeling

Figure 1 depicts the transformer samples which which were both experimentally investigated (s. Sec. 4) and simulated. The core consists of stacked electric sheets which wear a closely wound secondary coil with N_2 turns at one leg and a primary coil with N_1 turns equally distributed over all legs of the closed ring core. In order to evaluate modeling approaches for laminated magnetic cores, measured and simulated hysteresis loops $B(H)$ of the core material have to be compared. They can be measured and simulated as follows.

A primary current i_1 fed by the current source generates a magnetic field $H(t)$ in the core material according to Ampère’s circuital law. For a closed core with a constant cross section area A_c and a mean length of the ferromagnetic path L_c a simplified relation can be derived:

$$H(t) = \frac{i_1(t) \cdot N_1}{L_c} \tag{1}$$

According to the law of induction a voltage occurs in the secondary winding. The change of the mean flux density ΔB in the core in the time interval $t_2 - t_1$ can be calculated by integrating the secondary voltage u_2:

$$\Delta B = \frac{1}{N_2 A_c} \int_{t_1}^{t_2} u_2 \, \mathrm{d}t \tag{2}$$

Equations (1) and (2) are used to measure the static and dynamic magnetic hysteresis [8, 9].

Figure 2 shows the parametric transformer model. It allows to compute the interactions between the electrical and the magnetic subsystems. An external circuitry (a) including a current source and an external resistor feeds the finite element transformer model (b). In contrast to the measurement, B can directly be derived from the solution in the transformer model. Therefore, the secondary winding is not modeled in this case.

2.2 Multi-Turn Coil Modeling and Coupling to External Circuitry

We did not use the multi-turn coil domains provided by the AC/DC Module due to their strict requirements concerning the geometry of the electrically conductive cross section area.

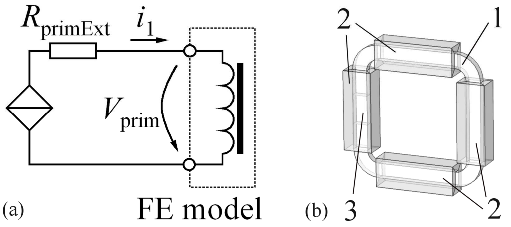

Figure 2. Model of the transformer with laminated core; (a) external circuitry, (b) finite element model; 1 – magnetic core, 2 – primary windings, 3 – measuring volume (surrounding air domain is not shown).

Instead, we modeled the coils as homogenized, electrically poorly conductive domains. A current source as part of the external circuit generates i_1 and feeds the coil domains with an external current density. The primary voltage V_{prim} is computed by integrating the induced electrical field over the coil domains and considering the ohmic voltage drop across the winding resistance [2]. Eddy currents in the windings can be neglected in the investigated frequency range.

Finite element transformer model and primary external circuit are bi-directionally coupled with an *IvsU* element which is an inherent component of the *cir* application mode.

If it is necessary to build a complete transformer model, a secondary external circuit can be coupled to the finite element transformer model in the same way.

The transformer model can also be fed by a voltage source in the primary circuit instead of a current source. In this case, the correct current value must be iteratively found in each time step for which the sum of the ohmic and induced voltage of the primary winding equals the source voltage. This increases computing time and may cause convergence problems if nonlinear magnetic material is applied.

3. Laminated Magnetic Core with Nonlinear Magnetization Curve

3.1 Approximation of the Nonlinear Magnetization Curve

The magnetic core material in the model is described by magnetic and electrical properties. The nonlinear magnetic behavior of the ferromagnetic transformer core needs to be considered. In order to avoid circular variable definitions in the constitutive relations, the $\mathbf{H} = f(|\mathbf{B}|)\mathbf{e}_\mathbf{B}$ form is used. The used approach does not account for the static magnetic hysteresis. However, the resulting error is acceptable in many practical cases, especially if high quality magnetic material is used.

At least two types of implementations are possible. Firstly, the commutation curve can be implemented in the form of an interpolation table. Piecewise cubic interpolation, additional data points at higher field strengths, and linear extrapolation are the optimum settings for fast computation and improved convergence [2]. A monotonously increasing sequence of the data points must be ensured by a good quality of the measurement data.

Secondly, a global approximation function valid for $B = [0; \infty]$ and derived from measurements by fitting, can also be used.

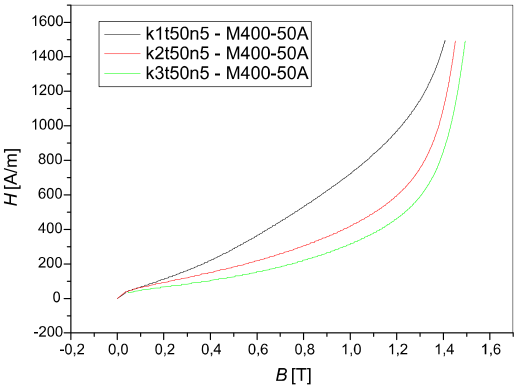

Figure 3. Implemented commutation curves from measurements of the investigated core materials.

By using appropriate setup functions for fitting $H(B)$, strictly increasing values, differentiability and high determination are achieved even from qualitatively poor data [2].

Currently, both approaches are implemented only for isotropic magnetic material. Anisotropic material, e. g. grain-oriented sheets cannot yet be adequately considered.

3.2 Homogenization Approaches for Laminated Magnetic Cores

Eddy currents in laminated structures would be correctly represented in a transient simulation model when the electrical and magnetic material properties and the geometry of the magnetic core are explicitly modeled (Figure 4.a). However, in most cases, this is practically impossible. As mentioned above, homogenization approaches replace the laminated structure by a single solid domain of an electrically anisotropic material which exhibits nearly the same macroscopic behavior in a certain range of conditions (Figure 4.b). This is achieved by adapting the conductivity such that the Ohmic resistance in the homogenized core is equal to that of the current path in the laminated structure. .

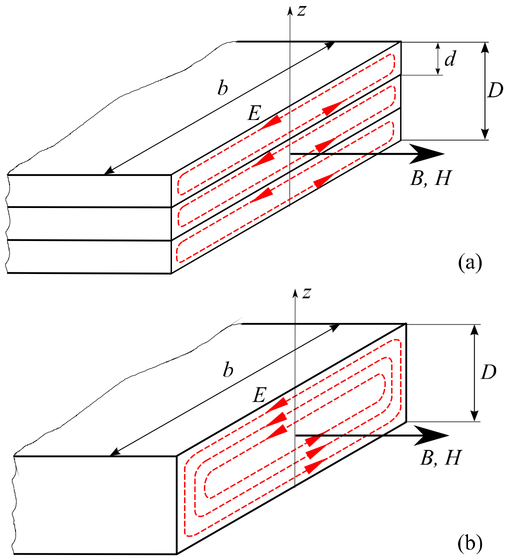

Figure 4. Eddy currents in a transformer core with laminated electric sheets (a) and homogenized core (b).

In each of the approaches, an orthogonal electrical conductivity σ is proposed to adapt the behavior as desired. With the direction z normal to the sheet plane, σ is

$$\sigma = \begin{bmatrix} \sigma_x & & \\ & \sigma_y & \\ & & \sigma_z \end{bmatrix} \qquad (3)$$

Based on different assumptions and applying different simplifications, the homogenization approaches listed in Table 1 have been derived. σ_b is the isotropic conductivity of the basic material, n the number and b the width of the stacked sheets, and D the overall thickness of the laminated core (Figure 4).

Table 1. Homogenization approaches for laminated magnetic core materials

SILVA [3]	$\sigma_x = \sigma_y = \sigma_b$ $\sigma_z = 0$
HAHNE [4]	$\sigma_x = \sigma_y = \sigma_b$ $\sigma_z = \sigma_b \left[\frac{D - 2\delta_L}{n(b + d - 2\delta_L) - b} \right]^2$ $\delta_L = 1 / \sqrt{\pi \mu \sigma_b f}$
KIWITT [5]	$\sigma_x = \sigma_y = \frac{1}{n^2} \sigma_b$ $\sigma_z = \sigma_b$
KÜHNER [6]	$\sigma_x = \sigma_y = \sigma_b$ $\sigma_z = \frac{1}{n^2} \sigma_b$
WANG [7]	$\sigma_x = \sigma_y = \sigma_b$ $\sigma_z = \left(\frac{d}{b} \right)^2 \sigma_b$
Solid core	$\sigma_x = \sigma_y = \sigma_z = \sigma_b$

In a strict sense, the HAHNE model [4] cannot be used with non-harmonic excitations or nonlinear magnetic materials due to its explicit dependence on the signal frequency and the core permeability. We simplified this model by using the maximum permeability and investigated only harmonic excitations.

All these homogenization procedures do not adjust the core permeability , apart from a filling factor which considers the thickness of the insulation layers between the sheets if necessary.

With the simplification of [4], the conductivity is depending only on geometric parameters in all of the approaches..

4. Simulation Results and Measurements

We investigated laminated ring cores of different sizes both with simulations and tests (Table 2). Depending on the core size (Figure 3), significant differences in the static magnetic behavior of the core material was found in the tests. They originate from edge-effects caused by the laser cutting process performed on the electric sheets. Therefore, the influences of the geometry and the material cannot be strictly separated in the tests.

Table 2. Investigated soft magnetic ring cores

Core width b	5, 10, 15 mm
Mean length of the flux path	150 mm
Core material	M400-50A
Sheet thickness d	0,5 mm
Number of sheets n	5
Number of primary turns	60
Number of secondary turns	22

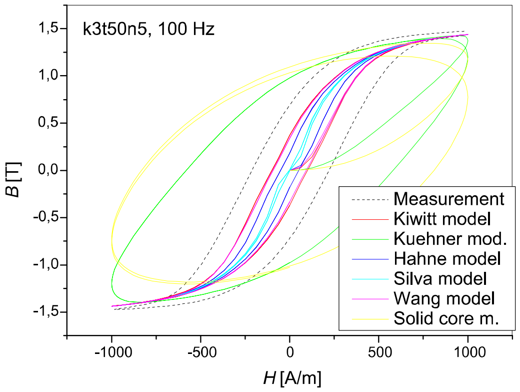

Figure 5. Simulated dynamic hysteresis curves calculated with different homogenization models; core width b = 15 mm

We used harmonic current excitation with a peak field of 1000 A/m in a frequency range from 1 Hz up to 2 kHz both in the tests and in the simulations. Additionally, static hysteresis measurements were performed to derive commutation curves in $H(B)$ form for implementation in the models (s. Sec. 3.1).

Figure 5 depicts examples of simulated dynamic hysteresis curves for a core width of b = 15 mm and a frequency of 100 Hz in comparison to measured data. Figure 6, Figure 7 and Figure 8 compare the measured and simulated coercivity for all core sizes and frequencies.

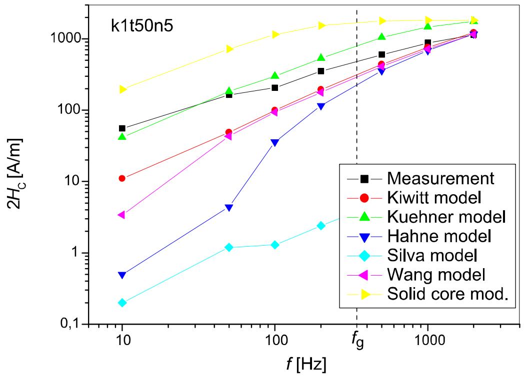

Figure 6. Simulated and measured coercivity of the dynamic hysteresis, core width b = 5 mm

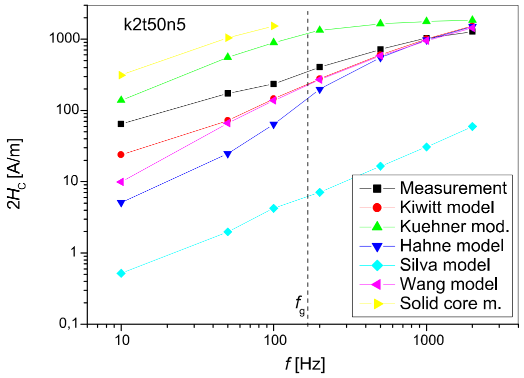

Figure 7. Simulated and measured coercivity of the dynamic hysteresis, core width b = 10 mm

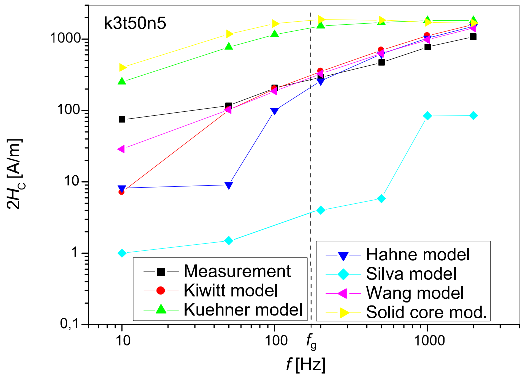

Figure 8. Simulated and measured coercivity of the dynamic hysteresis, core width b = 15 mm

The coercivity may be used to compare the approximation quality of the different homogenization procedures. For better comparability, the measured dynamic coercivity was reduced by the measured static one. Obviously, the approaches according to Table 1 calculate quite different dynamic hysteresises framed by the Silva and the solid core model. Even in the validity range of the models below f_g considerable deviations occur.

5. Conclusions and Outlook

It has been shown that homogenization procedures known from literature do not precisely predict the form of dynamic hysteresis in the investigated parameter range. The comparison of simulation results and measurements based on the coercivity of the hysteresis curves for the same excitation reveal significant deviations. These results do not encourage an uncritical use of the homogenization procedures according to Table 1 if precise modeling of laminated magnetic cores is required. Especially a reliable prediction of dynamic losses is not to be expected.

Focus of future work will be on models with explicitly modeled core sheets to compare their results with those from homogenized models and measurements. Additional parameters should be included to improve comparability of measured and simulation results.

Further, homogenization procedures which base on additional terms in the magnetic material equation depending on the flux density rate should be included and compared to the linear approaches we have involved up to now.

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