Multiphase Laminar Flow with More Than Two Phases

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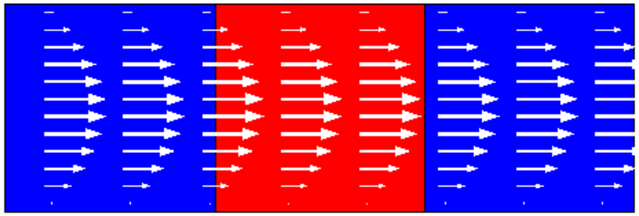
> Multiphase Flow

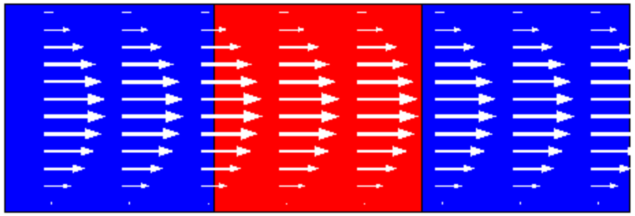
- In COMSOL MP, CFD fluid properties (density and viscosity) are associated with the mesh and does not travel with the fluid

- Thus, the properties of a fluid may change as it moves from place to place

1 kg/m^3 2 kg/m^3 1 kg/m^3



t = 0[s]



t = 1[s]

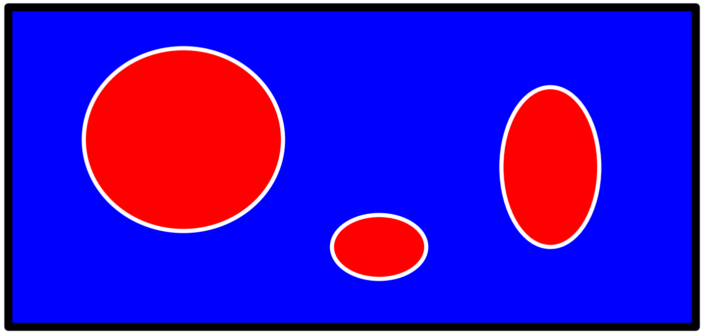
In this example, there are three regions. Each has a length of 2 cm and he fluid is moving at 1 cm/sec. The color is a surface map of fluid density. After 1 secpmd, the fluid has moved 1 cm to the right, but the density has not moved!

- In multiphase flow problems, we generally want each fluid to carry its properties as it moves

> Level Set Method for Two Fluids

- **COMSOL MP provides several interfaces for dealing with this:**
  - **Moving Mesh**: mesh deforms with the fluid boundaries
  - **Level Set Method and Phase Field Method**: an **auxiliary function** travels with the fluid

Level Set Interface



- **Anticipates two fluids**
- **Provide solver with**
  - Location of the t=0 fluid boundaries
  - Fluid properties for the two fluids
  - Identity of the fluid in each region
  - For inlets: The identity of the entering fluid

> Two-step solution

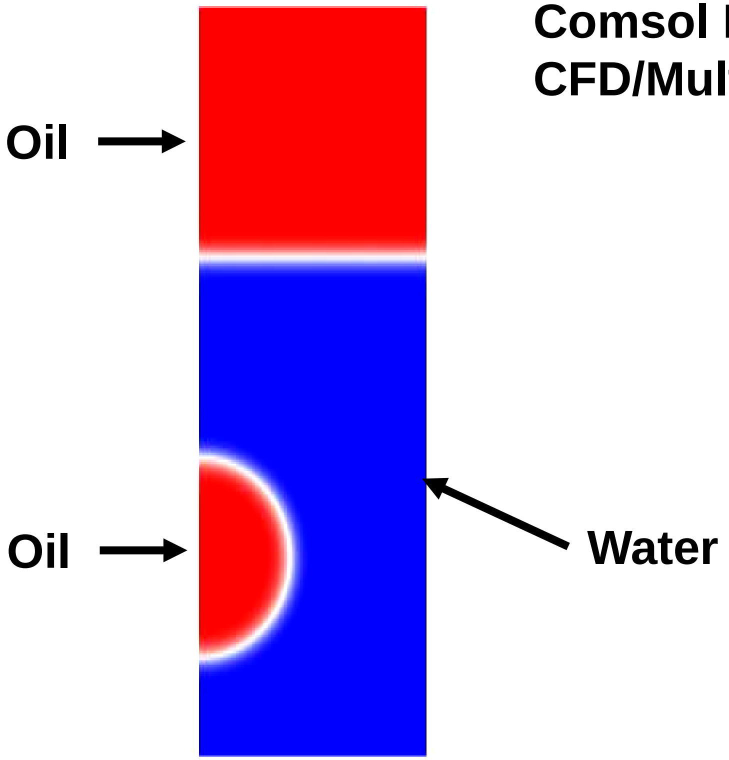
- **Step 1: Initialization. Create the auxiliary "Level Set" function**
  - First create a signed distance function $\phi(\vec{r},0)$ that represents the distance of each point in the fluid from the nearest interfacial boundary.
  - The sign of the function indicates which fluid
  - Then create a transformed version of this function:

$$\xi(\vec{r},0)=\frac{1}{2}\left[1+\tanh\left(\frac{\phi(\vec{r},0)}{2\epsilon}\right)\right]$$

  - Value = 0 in one fluid and 1 in the other fluid
  - Value = 0.5 at the interface

- **Step 2: Navier Stokes solution**
  - Move the function $\xi(\vec{r},t)$ with the fluid
  - Use its values to identify the fluid at each point in the domain and use the appropriate density and viscosity.

> Example from COMSOL library (movie)



"Rising Drop 2daxi" - From Comsol MP Application Library, CFD/Multiphase Tutorials

The fluids are tracked with an Auxiliary Function, called the "Level Set" Function

$$\xi(\vec{r}, t)$$

$\xi(\vec{r}, t) < 0.5$ **=> Fluid 1**

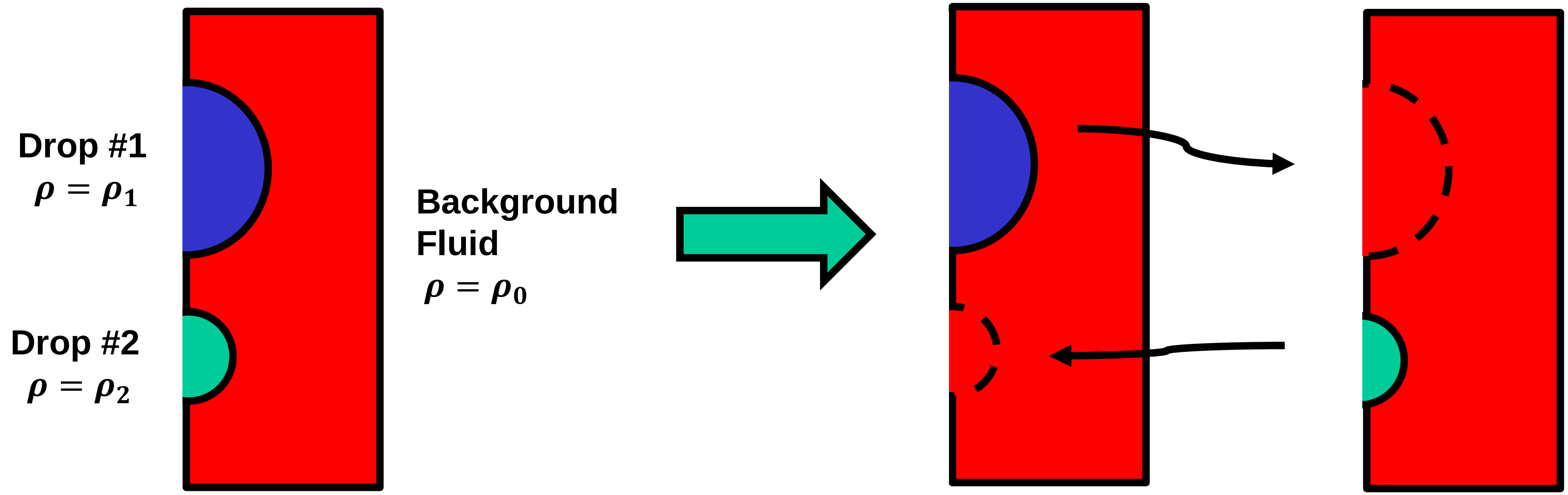
$\xi(\vec{r}, t) > 0.5$ **=> Fluid 2**

> More than two fluids

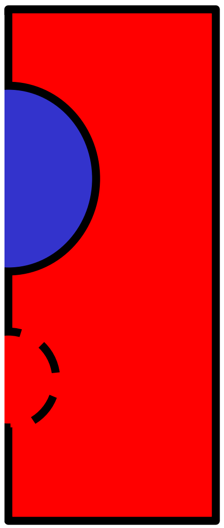
When there are more than 2 fluids, each fluid-fluid interface needs to know which two fluids it separates.

We can achieve this by turning the problem into multiple 2-fluid Level Set problems.

In general, if there are N different fluids, we will use N-1 Level Set interfaces.



> Example with three fluids

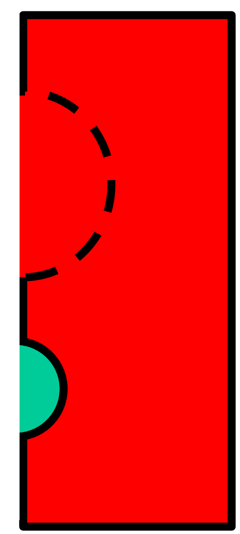


Level Set interface #1

Calculates $\xi_1(\vec{r},t)$, **… using** $\xi_2(\vec{r},\mathrm{t})$
from Level Set interface #2.

$$\rho = \rho_0 + (\rho_2 - \rho_0) \cdot (\xi_2(\vec{r},t) > 0.5)$$

$$\nu = \nu_0 + (\nu_2 - \nu_0) \cdot (\xi_2(\vec{r},t) > 0.5)$$



Level Set interface #2

Calculates $\xi_2(\vec{r},t)$, **… using** $\xi_1(\vec{r},\mathrm{t})$
from Level Set interface #1.

$$\rho = \rho_0 + (\rho_1 - \rho_0) \cdot (\xi_1(\vec{r},t) > 0.5)$$

$$\nu = \nu_0 + (\nu_1 - \nu_0) \cdot (\xi_1(\vec{r},t) > 0.5)$$

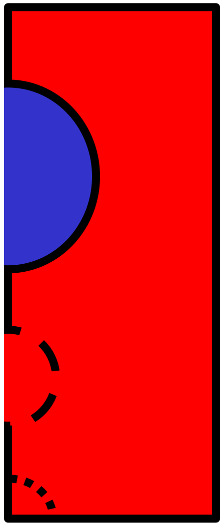
> General case: N Fluids

Create N-1 Level Set interfaces. In each one we specify two fluids

1. the droplet being tracked by that interface
2. a composite fluid, composed of the background fluid and all the other droplets

The density and viscosity of the composite fluid is computed by using the Level Set functions of the other droplets.

In interface j, we have a droplet of density ρ_j and viscosity ν_j and we have a composite fluid with



$$\rho = \rho_0 + \sum_{k \neq j} (\rho_k - \rho_0) \cdot (\xi_k(\vec{r}, t) > 0.5)$$

$$\nu = \nu_0 + \sum_{k \neq j} (\nu_k - \nu_0) \cdot (\xi_k(\vec{r}, t) > 0.5)$$

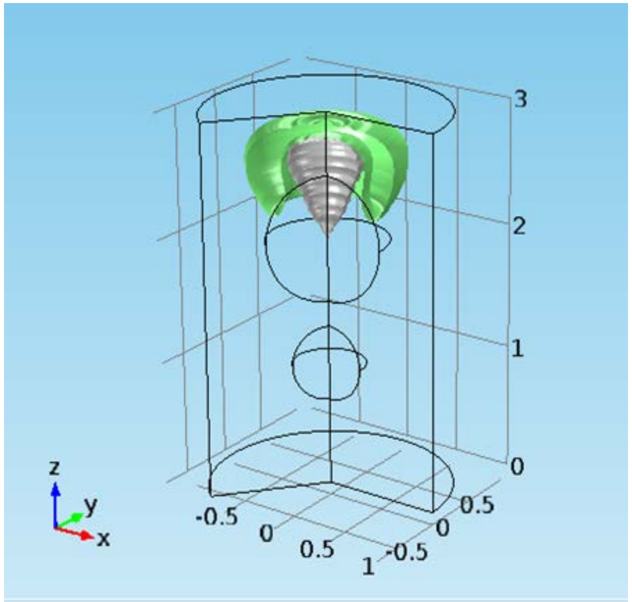
Background fluid properties includes all of the other droplets through their respective level-set functions

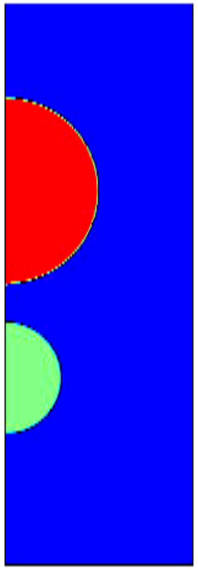
> Navier Stokes variables

- **Along with the Level Set function, each Level Set interface also has variables p, u, v, w describing the pressure and velocity of the fluids**
- **Normally:**
  - Level Set interface #1 has variables p, u, v, w
  - Level Set interface #2 has variables p2, u2, v2, w2
  - etc …
- **In the present solution, we need to change the names so that there is only one set of Navier Stokes variable:**
  - Level Set interface #1 has variables p, u, v, w
  - Level Set interface #2 has variables p, u, v, w
  - etc …
- **COMSOL MP does not complain about duplicated variable names when you do this**

> Example Result (movie)

	Drop #1	Drop #2	Background
Viscosity (mPa-s)	100	100	100
Density (kg/m^3)	800	600	1000





> Dilute Species method

As a check on results, we can do the same problem by a different method.

Rather than using a Level Set function, use a Dilute Species interface to put a non-diffusing tracer in each droplet.

Then solve with Navier-Stokes physics. The tracer travels with the fluid.

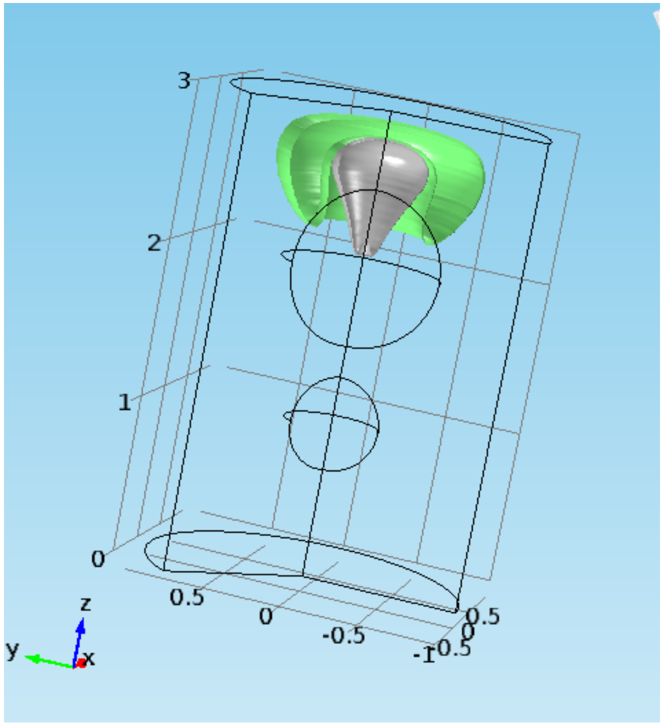
With N fluids, use N-1 tracers. Use starting concentrations of $c_k = 1\left[\frac{M}{m^3}\right]$**.**

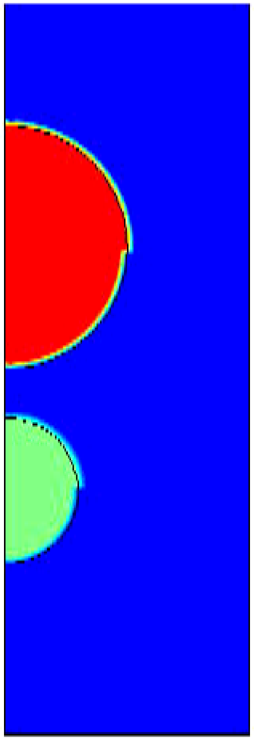
Consider this whole system as a single fluid with density and viscosity given by:

$$\rho(\vec{r},t) = \rho_0 + \sum_{k=1}^{N-1} (\rho_k - \rho_0) \cdot \left(c_k > 0.5\left[\frac{M}{m^3}\right]\right)$$

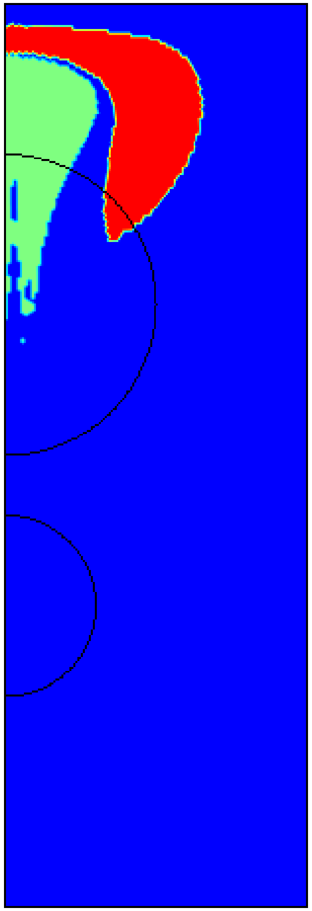
$$\nu(\vec{r},t) = \nu_0 + \sum_{k=1}^{N-1} (\nu_k - \nu_0) \cdot \left(c_k > 0.5\left[\frac{M}{m^3}\right]\right)$$

> Dilute Species Example (movie)

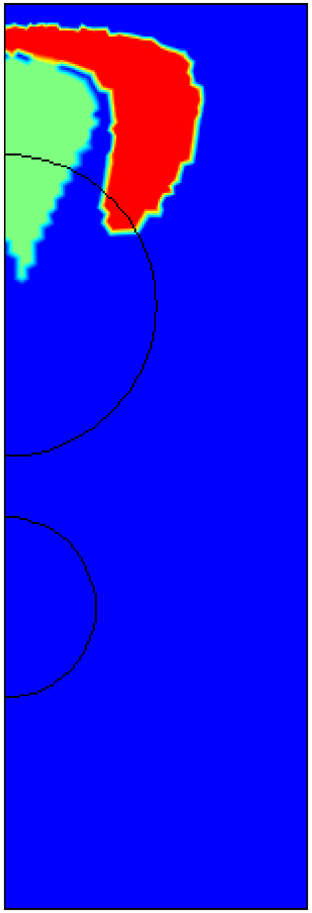




> Comparison at t=0.3 s



Level Set



Dilute Species

> Conclusions

- **The Dilute Species method can be easier to set up**

Dilute Species Method	1 Navier Stokes Interface 1 Dilute Species Interface
Level Set Method	N-1 Coupled Level Set Interfaces

- **However, no method currently exists for calculating surface tension or contact angles from the concentration functions.**
  - If concentration functions are smoothed to allow the calculations of gradients, then the identification of fluids becomes blurred

> Result with Surface Tension (movie)

- Using the Surface Tension of Olive Oil/Water, the results of the level-set calculation are a quite different.