COMSOL CONFERENCE 2015 BOSTON

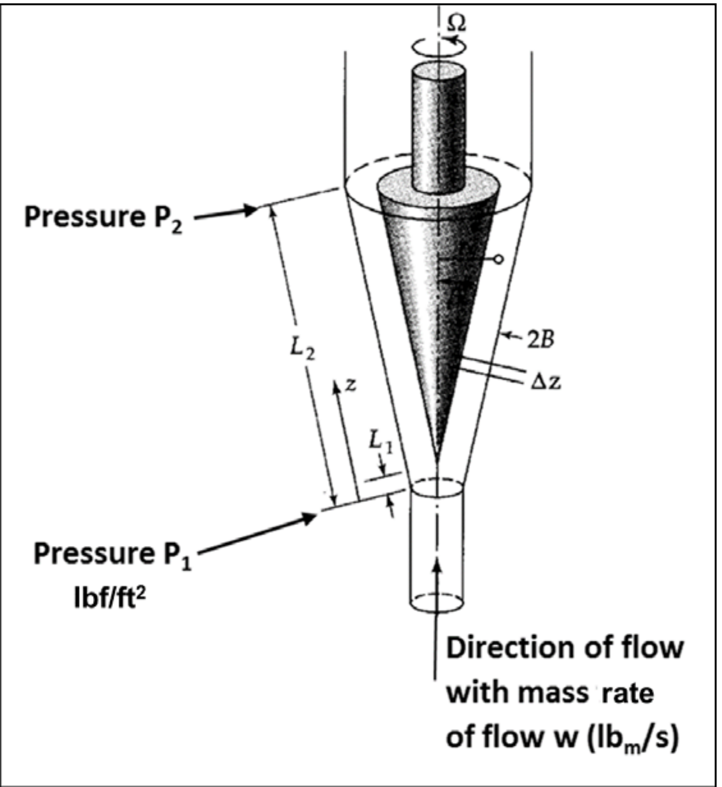
COMSOL Conference 2015 Boston

Session: Computational Fluid Dynamics

Session Chair: William Vetterling, Zink Imaging, Inc.

October 8, 2015 1:00 PM – 2:30 PM

Hydrodynamic Modeling of a Rotating Cone Pump Using COMSOL Multiphysics™



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Comparison of Several Different Pump Types

Pumps

Conventional Centrifugal pumps

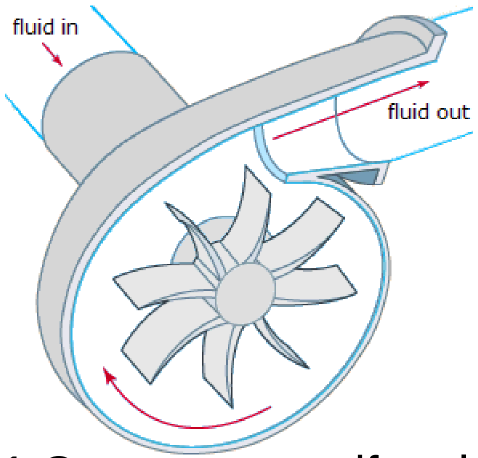
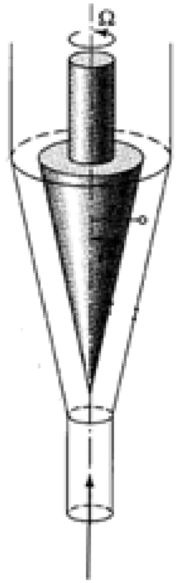


Fig. 1 Common centrifugal pump design[1]

- Reynolds number > 1000
- High fluid throughput
- High pump head (meters)

Rotating Cone Pump



- Reynolds number 10-100
- High fluid throughput
- Low pump head (millimeters)

Viscous Micro Pumps

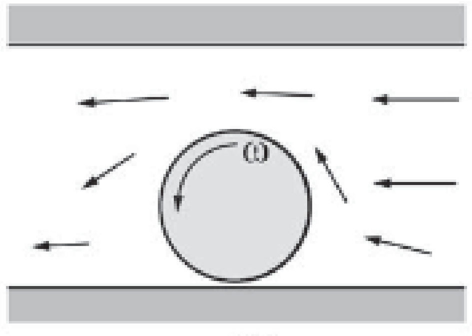


Fig.2 Eccentric cylinder pump[2]

- Reynolds number < 10
- Moderate fluid throughput
- Low pump head (millimeters)

Model Equations: Continuity, Momentum Transport and Turbulence Model

Turbulence Model
k-ε Model Equations

- Turbulent kinetic energy, k, is given by:

$$\rho\frac{\partial k}{\partial t}+\rho(\boldsymbol{u}\cdot\nabla)k=\nabla\cdot\left[\left(\mu+\frac{\mu_T}{\sigma_k}\right)\nabla k\right]+P_k-\rho\epsilon$$

- Turbulent dissipation, ε, is given by:

$$\rho\frac{\partial \epsilon}{\partial t}+\rho(\boldsymbol{u}\cdot\nabla)\epsilon=\nabla\cdot\left[\left(\mu+\frac{\mu_T}{\sigma_\epsilon}\right)\nabla \epsilon\right]+C_{c1}\frac{\epsilon}{k}P_k-C_{c2}\rho\frac{\epsilon^2}{k}$$

- Turbulent viscosity is modelled by:

$$\mu_T=\rho C_\mu\frac{k^2}{\epsilon}$$

- Production of turbulent kinetic energy defined as:

$$P_k=\mu_T[\nabla\boldsymbol{u}:(\nabla\boldsymbol{u}+(\nabla\boldsymbol{u})^T]-\frac{2}{3}\rho k\nabla\cdot\boldsymbol{u}$$

Equation of Continuity

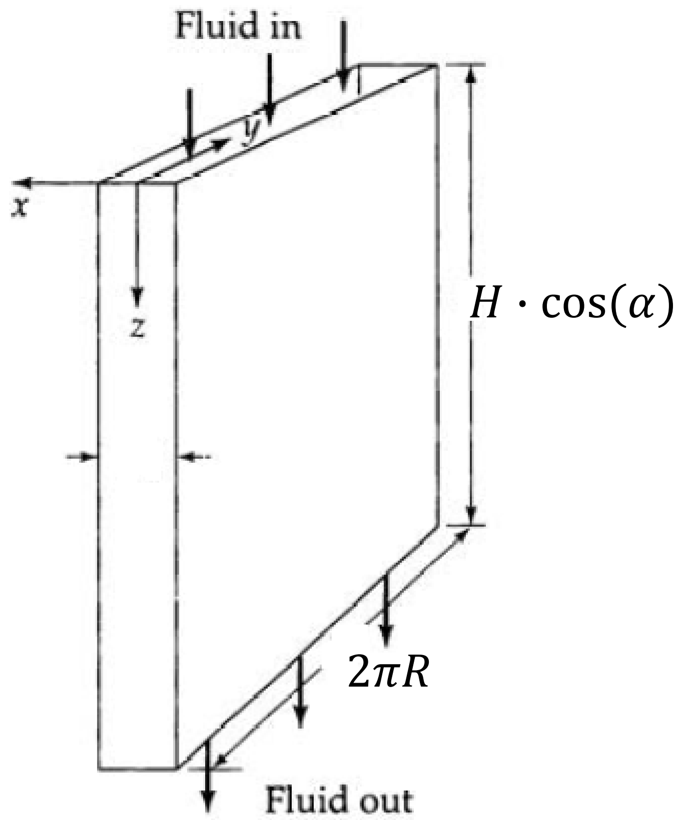
$$\frac{\partial\rho}{\partial t}+\nabla\cdot(\rho\boldsymbol{u})=0$$

Momentum Transport Equation

$$\rho\frac{\partial u}{\partial t}+\rho(u\cdot\nabla)u=$$

$$\nabla\cdot\left[-pI+(\mu+\mu_T)(\nabla u+(\nabla u)^T)-\frac{2}{3}\rho kI\right]+F$$

Equations for 1-D Momentum Transport Model



Approximation of an annular region flow as a narrow slit flow.

$$\rho\left[\frac{\partial u_z}{\partial t}\right] - \mu\left[\frac{\partial^2 u_z}{\partial x^2} + \frac{\partial^2 u_z}{\partial y^2} + \frac{\partial^2 u_z}{\partial z^2}\right] + \rho\left[u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_y}{\partial y} + u_z \frac{\partial u_z}{\partial z}\right] + \frac{\partial p}{\partial z} = 0$$

- Steady-state;
- $u_x, u_y = 0$;
- $u_z = f(x)$;

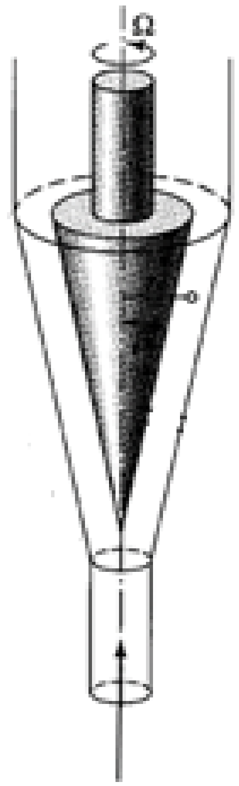
$$\mu\left[\frac{\partial^2 u_z}{\partial x^2}\right] + \frac{\partial p}{\partial z} = 0$$

Separation of variables gives:

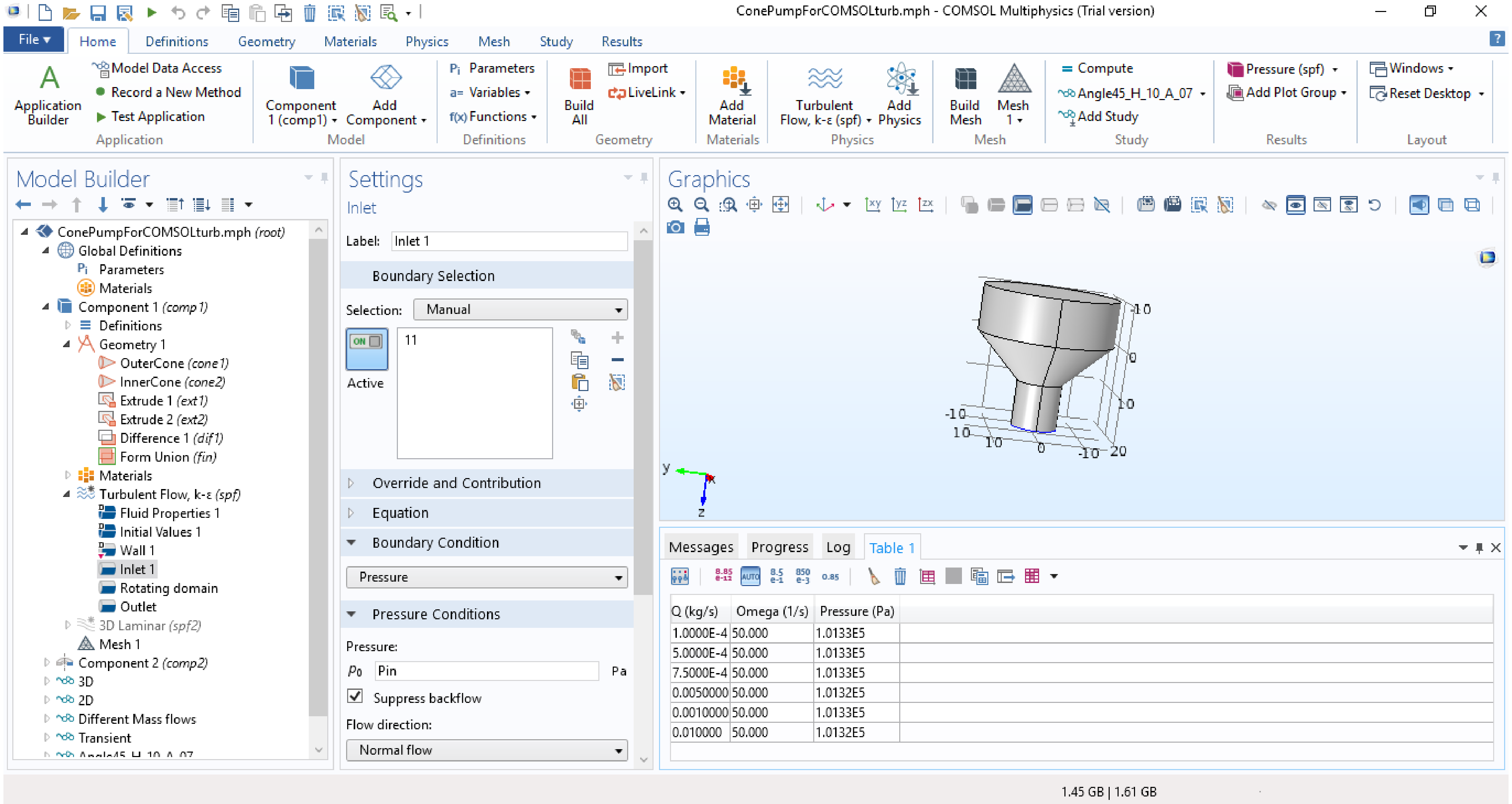
$$u_z = \frac{\Delta P \cdot B^2}{2\mu L}\left[1 - \left(\frac{x}{B}\right)^2\right]$$

Integration over cross section gives:

$$w = \frac{4\pi B^3 \rho \sin\beta}{3\mu \ln\left({L_2}/{L_1}\right)}\left[\Delta P + \left(\frac{1}{8}\rho\Omega^2 sin^2\beta\right) \cdot (L_2^2 - L_1^2)\right]$$

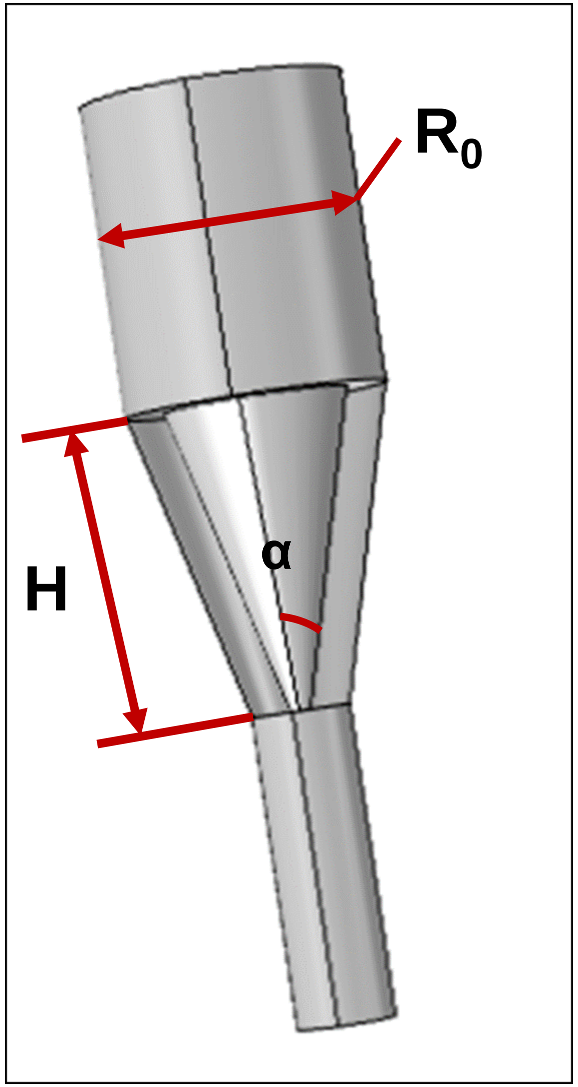


COMSOL Model Setup



COMSOL Model Description

Assumed Geometry



Parameter	Value		Units
	Min	Max	
H	10	35	mm
$\kappa = R_i/R_0$	0.7	0.9	-
R_0	15	15	mm
P_{in}	1	1	atm
Ω	3	10	rpm·10^{-3}
α	10	45	deg

H - Height of the cone

κ - Ratio of inner to outer cylinder radii

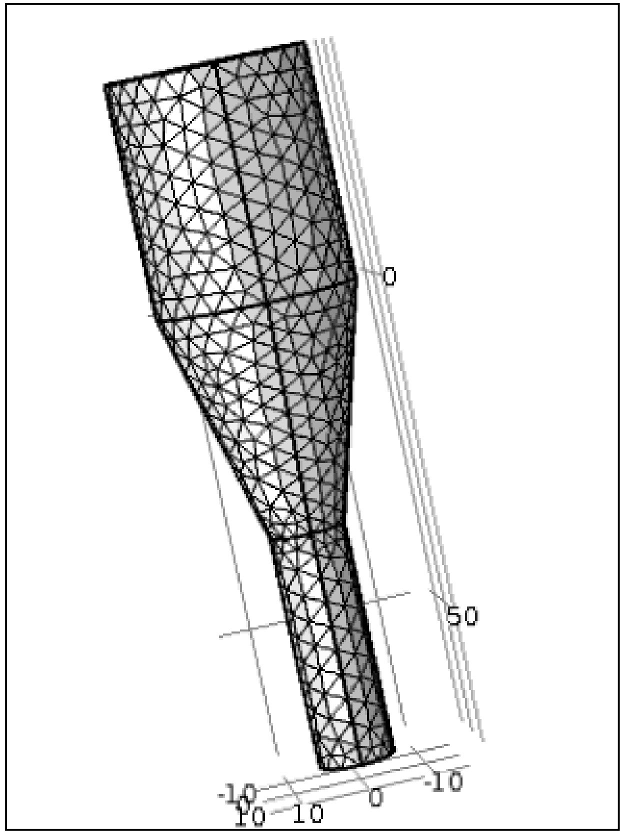
P_{in} – Inlet pressure

Ω - Angular velocity

α - Semi-angle of the cone

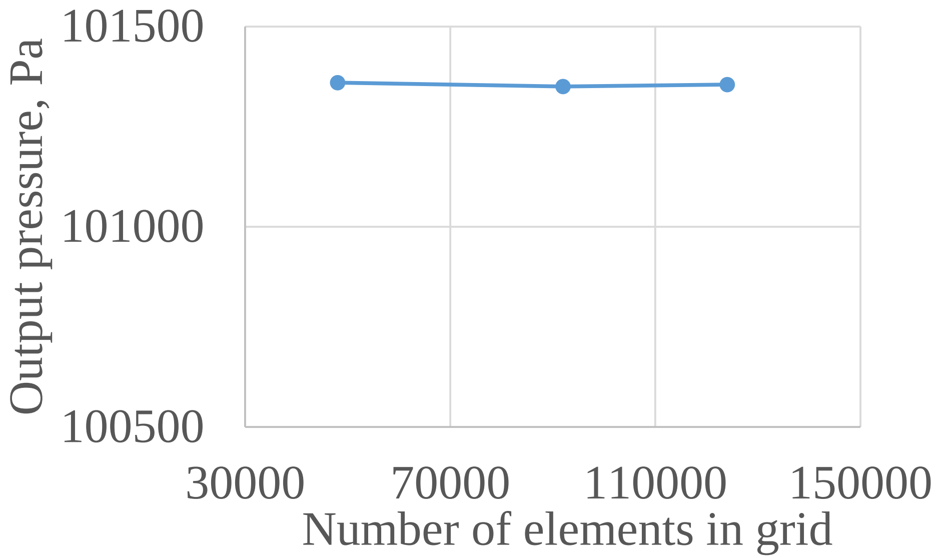
Meshing

Tetrahedral Mesh
(48,000 elements)

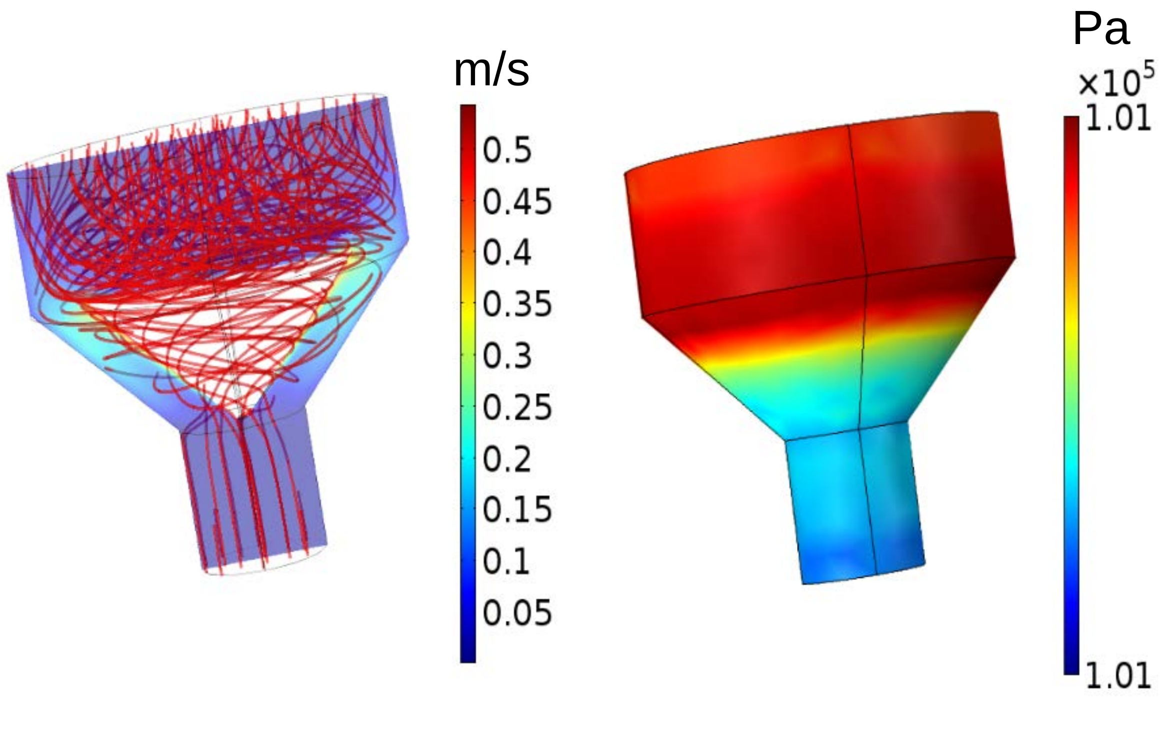


- Tetrahedral mesh with boundary layers has been used
- Number of elements ~48000
- Average quality of the mesh - 0.75

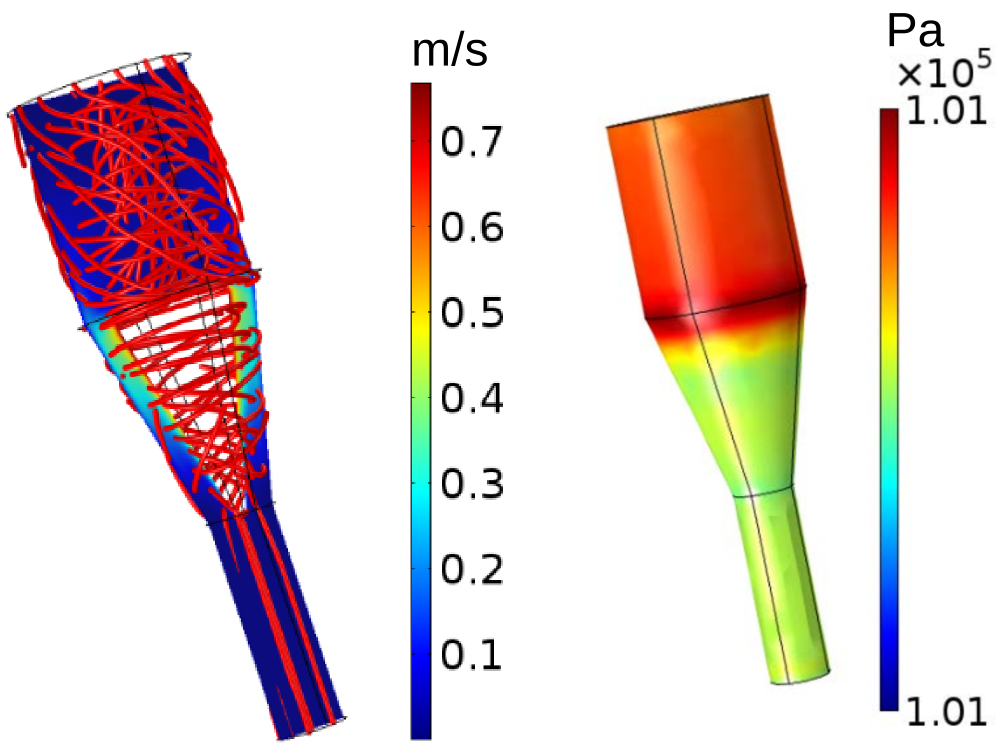
Mesh Independent Solution



Velocity and Pressure Profiles: Effect of Semi angle



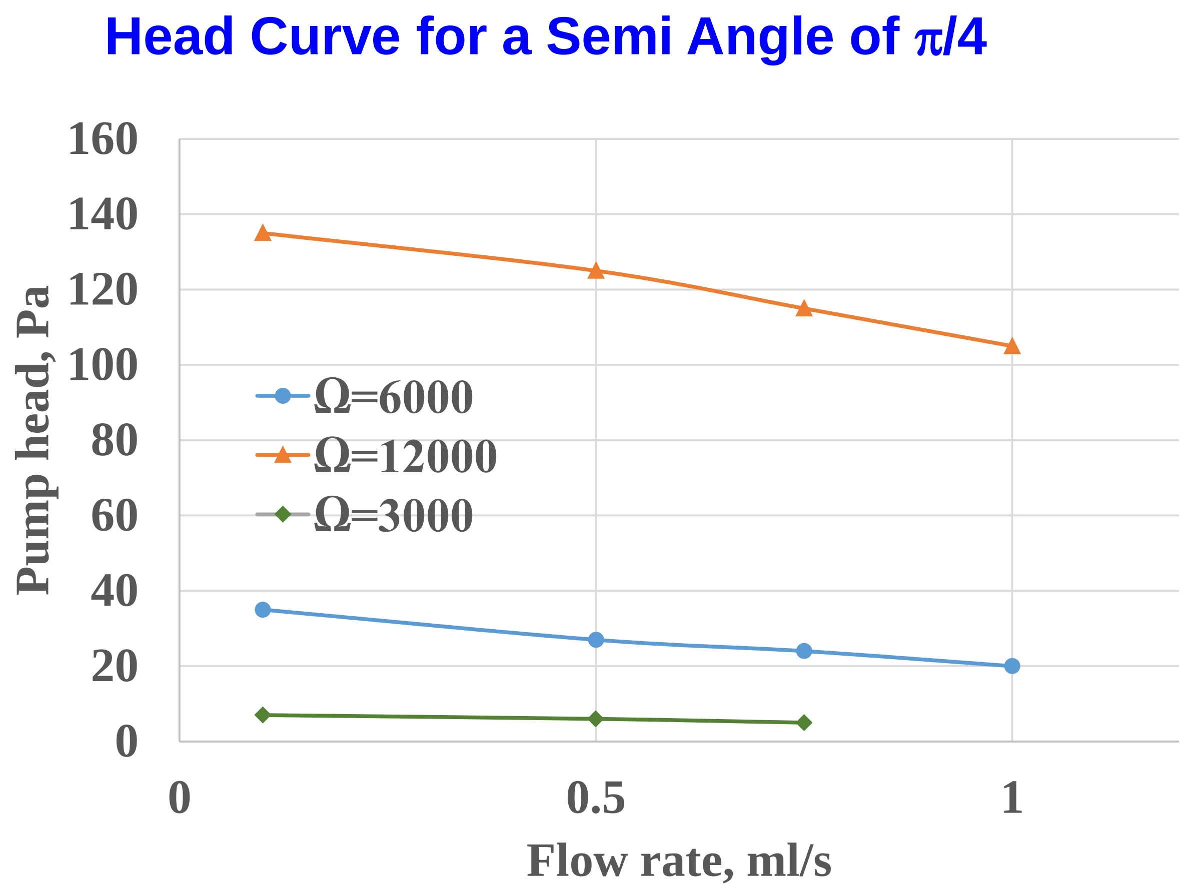
Velocity and pressure profiles for a semi angle of 45°, Q = 1 ml/s



Velocity and pressure profiles for a semi angle of 12°, Q = 1 ml/s

The pressure profiles show that the hydraulic head of the pump is a weak function of the cone semi angle and height of the rotating cone.

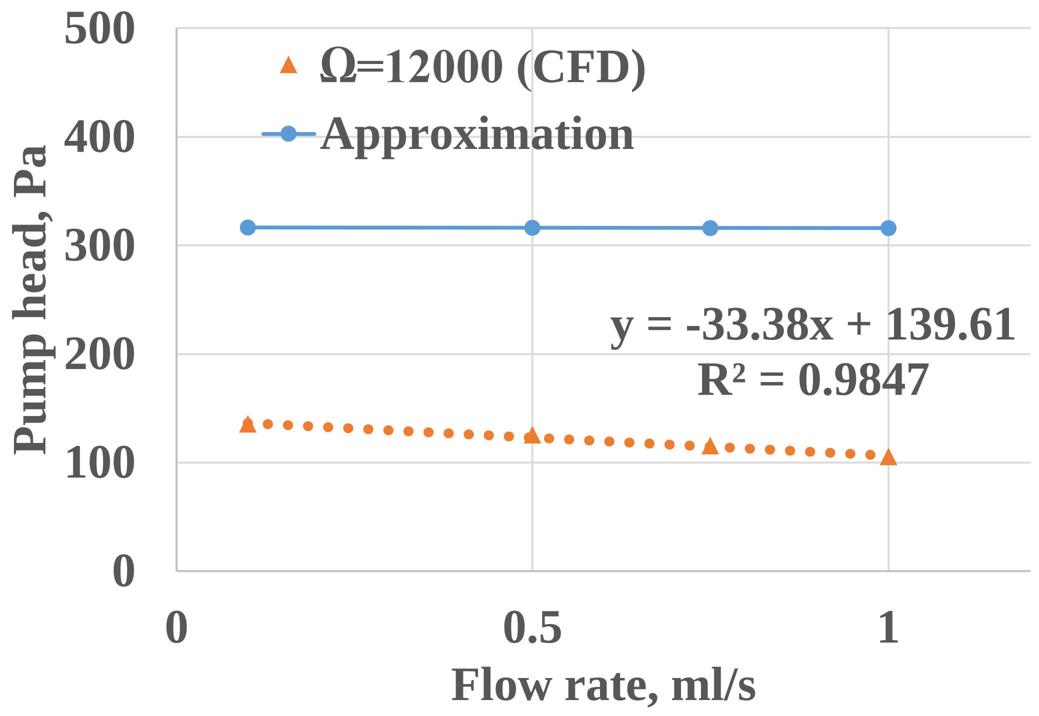
Pump Head *vs* Flow Rate: Effect of Rotational Speed



- Outlet pressure decreases almost linearly with increasing volumetric flowrate.
- Pump head is proportional to the square of rotational speed.
- Pump outlet pressure does not exceed 135 Pa even for highest values of angular velocity.

Pump Head *vs* Flow Rate: Approximate Solution *vs* CFD Solution

Comparison of Head Curve Acquired from Approximation and CFD Results



Approximate solution for rotating cone pump problem is available in open literature. (Bird et al., 2007)

Key assumptions

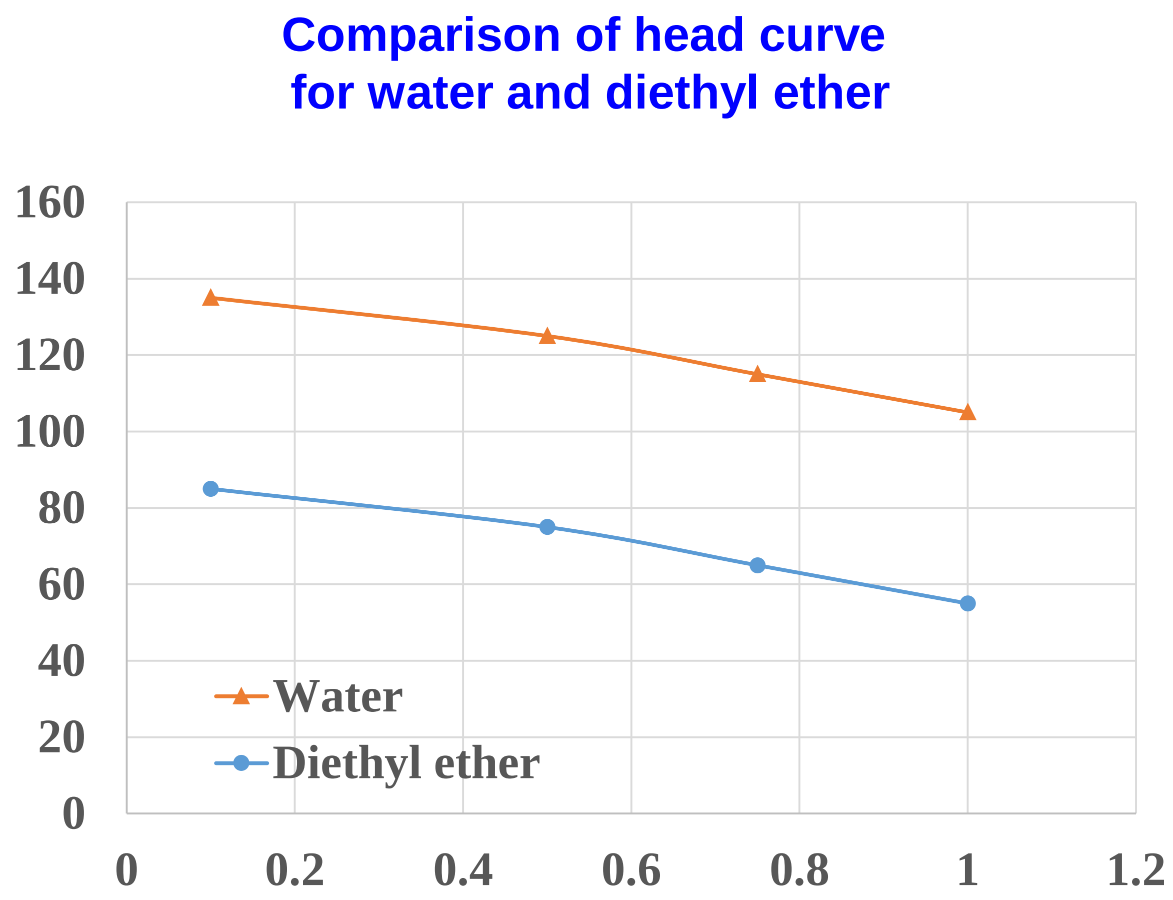
- Laminar flow
- Curvature and entrance effects are neglected.

$$w = \frac{4\pi B^3 \rho \sin\beta}{3\mu \ln\left(L_2/L_1\right)}\left[(p_1 - p_2) + \left(\frac{1}{8}\rho\Omega^2 sin^2\beta\right)\cdot\left(L_2^2 - L_1^2\right)\right]$$

where

L_1, L_2 are heights corresponding to pressures p_1 & p_2 respectively.

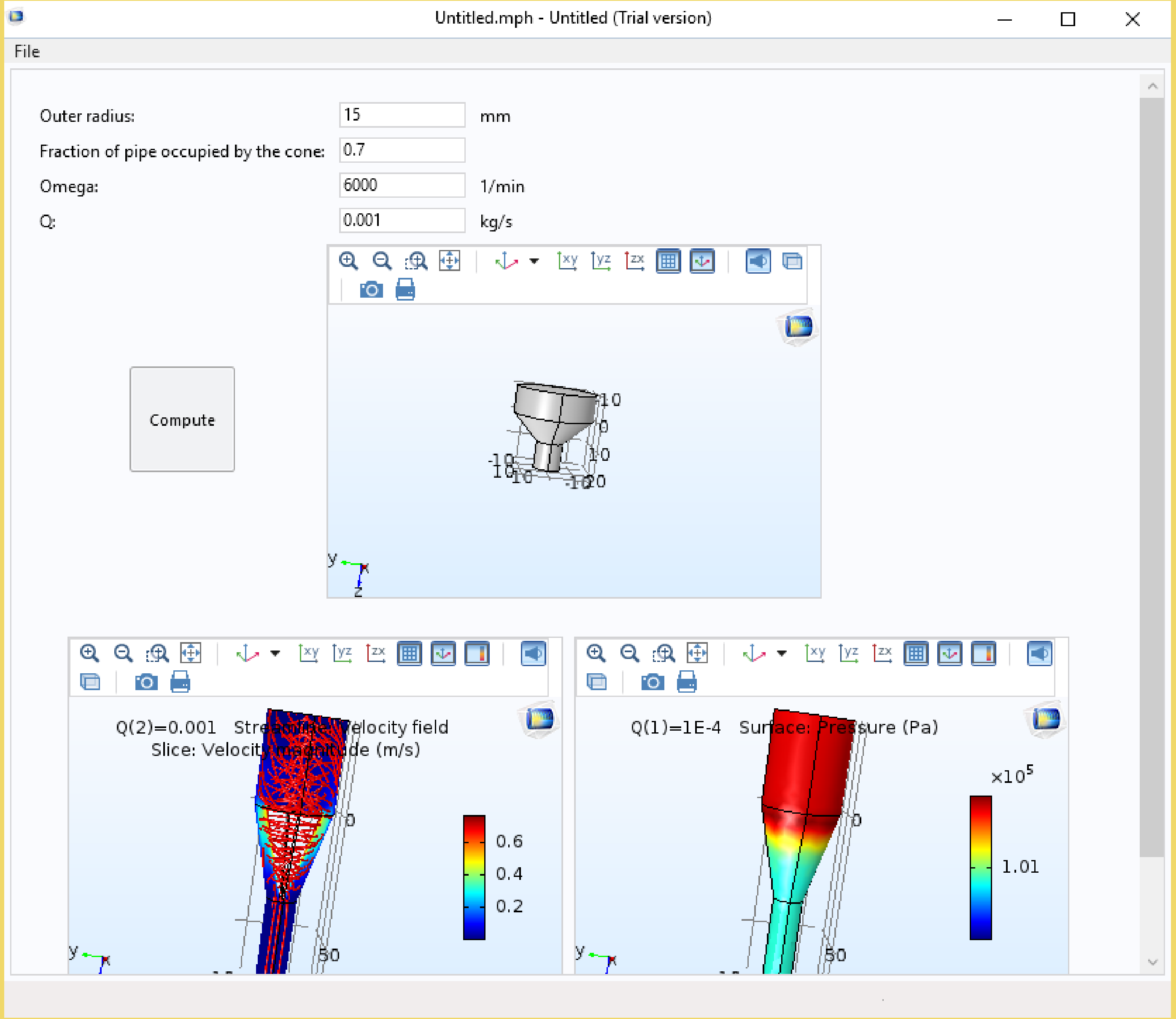
Pump Head *vs* Flow Rate: Effect of Fluid Density and Viscosity



Fluid	Density, kg/m^3	Viscosity, cP
Water	999.66	1.01
Diethyl ether	713.58	0.24

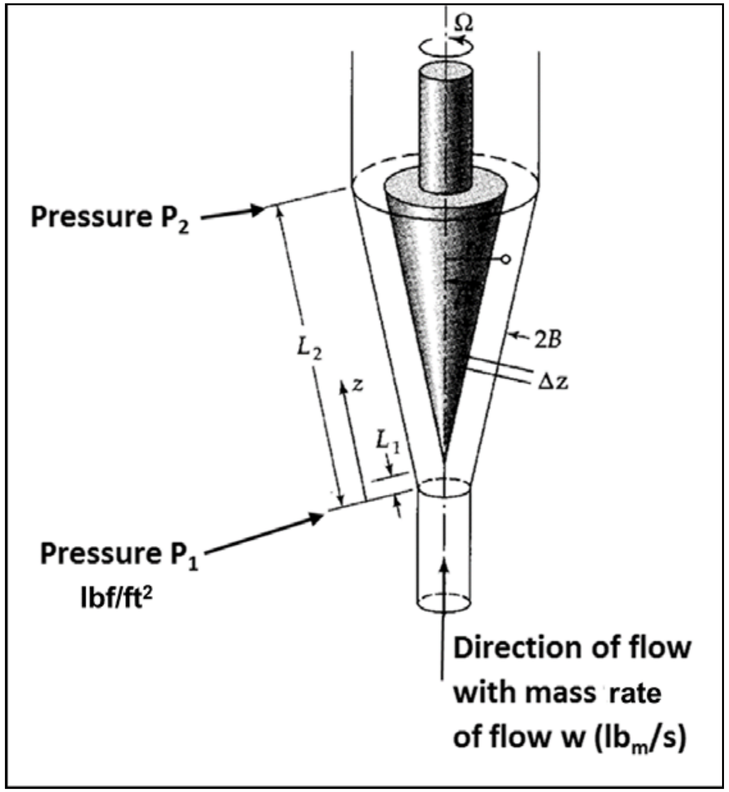
- Rotating pump head increases with increasing viscosity and density.
- Rotating pump head behavior follows the same trend as that for a centrifugal pump.

COMSOL Application for Rotating Cone Pump



Conclusions

- The rotating cone pump is a simple design compared to other pump configurations that has applications where a low pump head is sufficient, such as in microprocess systems.
- Analysis of the rotating cone pump performance can be facilitated by using the COMSOL Multiphysics CFD Module.
- The CFD model predicts the correct trends in pump head performance for various model parameters.
- The approximate formula, which is based upon a 1-D fluid mechanics model, over predicts the pump head performance by about a factor of 2.
- Optimization of the cone pump design, which might include modification to the cone head surface, *e.g.*, addition of spiral fins, would be facilitated by COMSOL CFD module simulations *vs* using empirical approaches.



Thank you for your attention