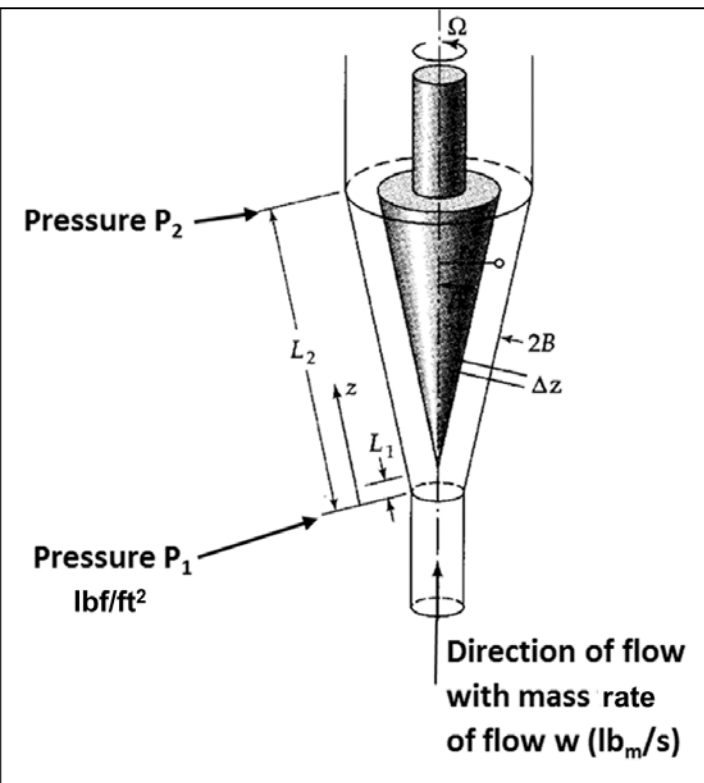


Session Chair: William Vetterling, Zink Imaging, Inc.

October 8, 2015

1:00 PM – 2:30 PM

# Hydrodynamic Modeling of a Rotating Cone Pump Using COMSOL Multiphysics™



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# Comparison of Several Different Pump Types

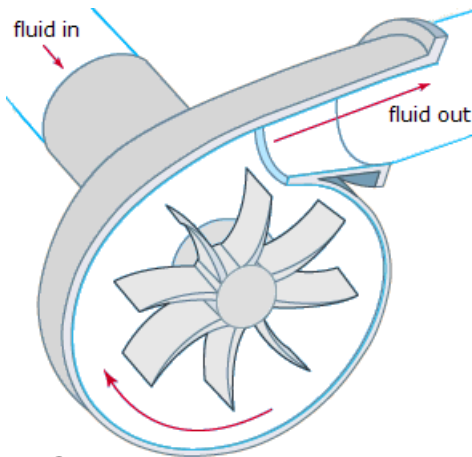
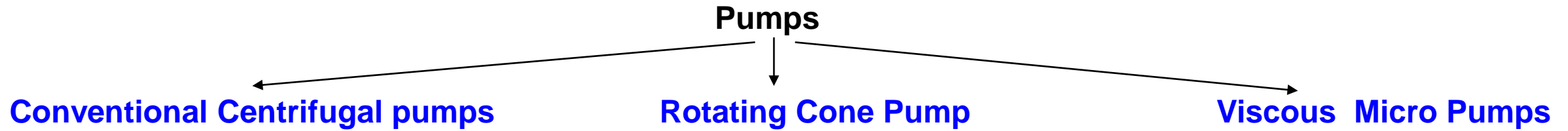
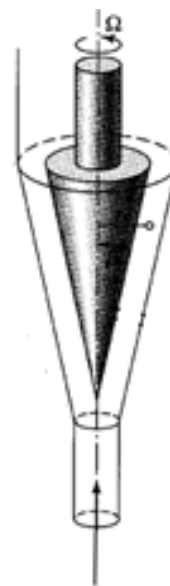


Fig. 1 Common centrifugal pump design[1]

- Reynolds number  $> 1000$
- High fluid throughput
- High pump head (meters)



- Reynolds number 10-100
- High fluid throughput
- Low pump head (millimeters)

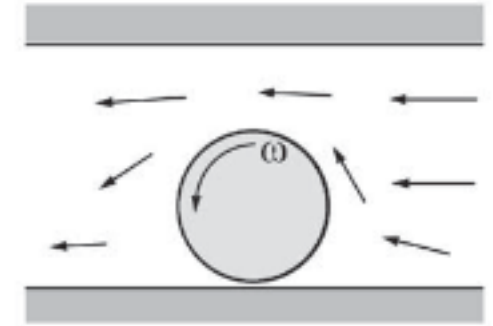


Fig.2 Eccentric cylinder pump[2]

- Reynolds number  $< 10$
- Moderate fluid throughput
- Low pump head (millimeters)

# Model Equations: Continuity, Momentum Transport and Turbulence Model

## Turbulence Model k-ε Model Equations

- Turbulent kinetic energy,  $k$ , is given by:

$$\rho \frac{\partial k}{\partial t} + \rho(\mathbf{u} \cdot \nabla)k = \nabla \cdot \left[ \left( \mu + \frac{\mu_T}{\sigma_k} \right) \nabla k \right] + P_k - \rho \epsilon$$

- Turbulent dissipation,  $\epsilon$ , is given by:

$$\rho \frac{\partial \epsilon}{\partial t} + \rho(\mathbf{u} \cdot \nabla)\epsilon = \nabla \cdot \left[ \left( \mu + \frac{\mu_T}{\sigma_\epsilon} \right) \nabla \epsilon \right] + C_{c1} \frac{\epsilon}{k} P_k - C_{c2} \rho \frac{\epsilon^2}{k}$$

- Turbulent viscosity is modelled by:

$$\mu_T = \rho C_\mu \frac{k^2}{\epsilon}$$

- Production of turbulent kinetic energy defined as:

$$P_k = \mu_T [\nabla \mathbf{u} : (\nabla \mathbf{u} + (\nabla \mathbf{u})^T)] - \frac{2}{3} \rho k \nabla \cdot \mathbf{u}$$

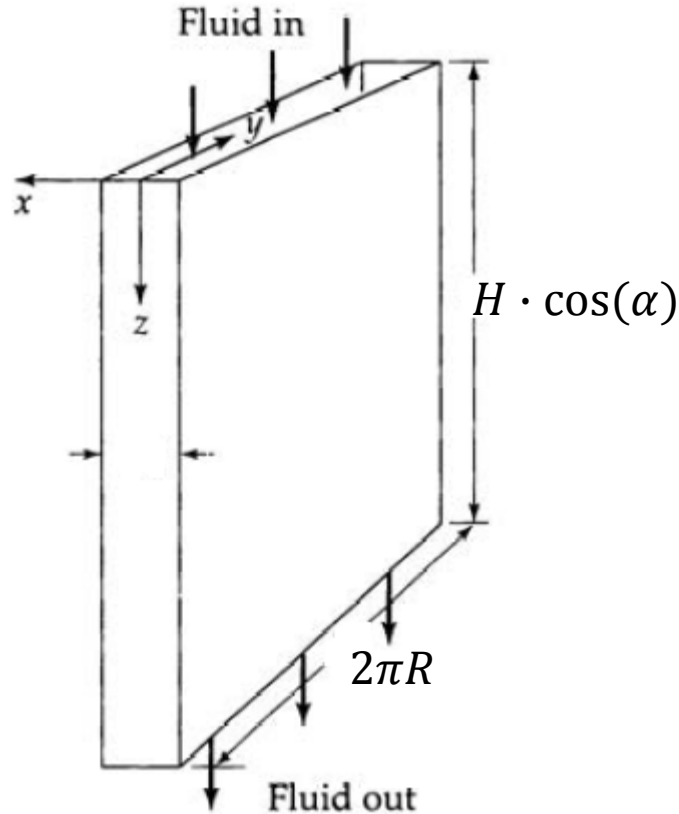
## Equation of Continuity

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$

## Momentum Transport Equation

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho(\mathbf{u} \cdot \nabla)\mathbf{u} = \nabla \cdot \left[ -p\mathbf{I} + (\mu + \mu_T)(\nabla \mathbf{u} + (\nabla \mathbf{u})^T) - \frac{2}{3} \rho k \mathbf{I} \right] + \mathbf{F}$$

# Equations for 1-D Momentum Transport Model



Approximation of an annular region flow as a narrow slit flow.

$$\rho \left[ \frac{\partial u_z}{\partial t} \right] - \mu \left[ \frac{\partial^2 u_z}{\partial x^2} + \frac{\partial^2 u_z}{\partial y^2} + \frac{\partial^2 u_z}{\partial z^2} \right] + \rho \left[ u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_y}{\partial y} + u_z \frac{\partial u_z}{\partial z} \right] + \frac{\partial p}{\partial z} = 0$$

- Steady-state;
- $u_x, u_y = 0$ ;
- $u_z = f(x)$ ;

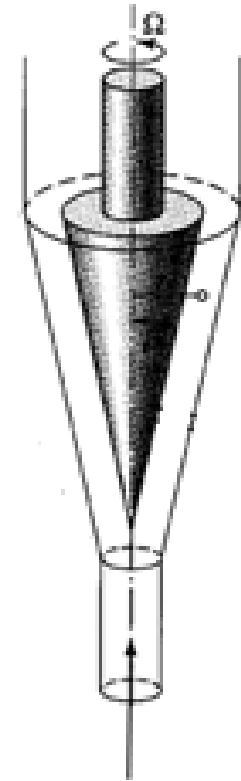
$$\mu \left[ \frac{\partial^2 u_z}{\partial x^2} \right] + \frac{\partial p}{\partial z} = 0$$

Separation of variables gives:

$$u_z = \frac{\Delta P \cdot B^2}{2\mu L} \left[ 1 - \left( \frac{x}{B} \right)^2 \right]$$

Integration over cross section gives:

$$w = \frac{4\pi B^3 \rho \sin \beta}{3\mu \ln(L_2/L_1)} \left[ \Delta P + \left( \frac{1}{8} \rho \Omega^2 \sin^2 \beta \right) \cdot (L_2^2 - L_1^2) \right]$$



# COMSOL Model Setup

ConePumpForCOMSOLturb.mph - COMSOL Multiphysics (Trial version)

File Home Definitions Geometry Materials Physics Mesh Study Results

Application Builder Model Data Access Record a New Method Test Application Application

Component 1 (comp1) Add Component Model

Parameters a= Variables f(x) Functions Definitions

Build All Import LiveLink Geometry

Add Material Materials

Turbulent Flow, k-ε (spf) Add Physics Physics

Build Mesh Mesh Mesh 1

Compute Angle45\_H\_10\_A\_07 Add Study Study

Pressure (spf) Add Plot Group Results

Windows Reset Desktop Layout

**Model Builder**

- ConePumpForCOMSOLturb.mph (root)
  - Global Definitions
    - Parameters
    - Materials
  - Component 1 (comp1)
    - Definitions
    - Geometry 1
      - OuterCone (cone1)
      - InnerCone (cone2)
      - Extrude 1 (ext1)
      - Extrude 2 (ext2)
      - Difference 1 (dif1)
      - Form Union (fin)
    - Materials
    - Turbulent Flow, k-ε (spf)
      - Fluid Properties 1
      - Initial Values 1
      - Wall 1
      - Inlet 1
      - Rotating domain
      - Outlet
    - 3D Laminar (spf2)
    - Mesh 1
    - Component 2 (comp2)
    - 3D
    - 2D
    - Different Mass flows
    - Transient
    - Angle45\_H\_10\_A\_07

**Settings**

Inlet

Label: Inlet 1

Boundary Selection

Selection: Manual

11

Active

Override and Contribution

Equation

Boundary Condition

Pressure

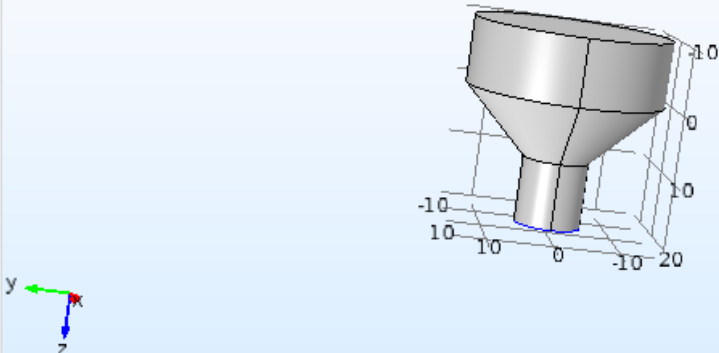
Pressure Conditions

Pressure:  $p_0$  Pin Pa

Suppress backflow

Flow direction: Normal flow

**Graphics**



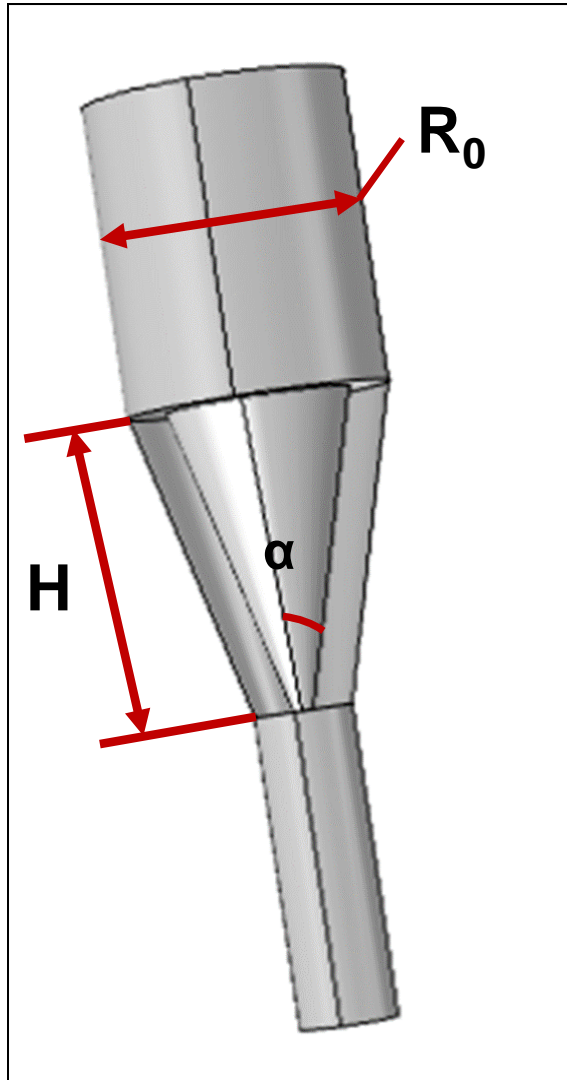
**Messages Progress Log Table 1**

Q (kg/s)	Omega (1/s)	Pressure (Pa)
1.0000E-4	50.000	1.0133E5
5.0000E-4	50.000	1.0133E5
7.5000E-4	50.000	1.0133E5
0.0050000	50.000	1.0132E5
0.0010000	50.000	1.0133E5
0.010000	50.000	1.0132E5

1.45 GB | 1.61 GB

# COMSOL Model Description

## Assumed Geometry



Parameter	Value		Units
	Min	Max	
H	10	35	mm
$\kappa = R_i/R_0$	0.7	0.9	-
$R_0$	15	15	mm
$P_{in}$	1	1	atm
$\Omega$	3	10	$\text{rpm} \cdot 10^{-3}$
$\alpha$	10	45	deg

H - Height of the cone

$\kappa$  - Ratio of inner to outer cylinder radii

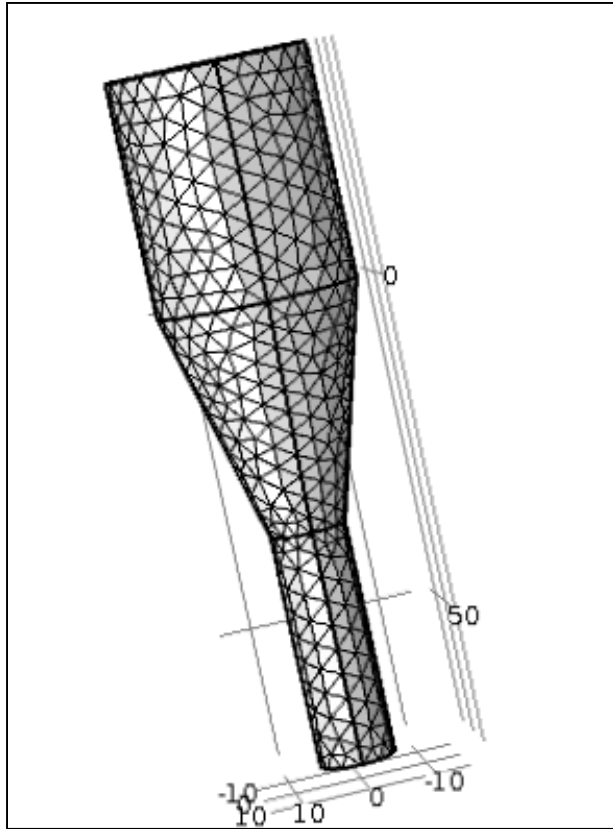
$P_{in}$  - Inlet pressure

$\Omega$  - Angular velocity

$\alpha$  - Semi-angle of the cone

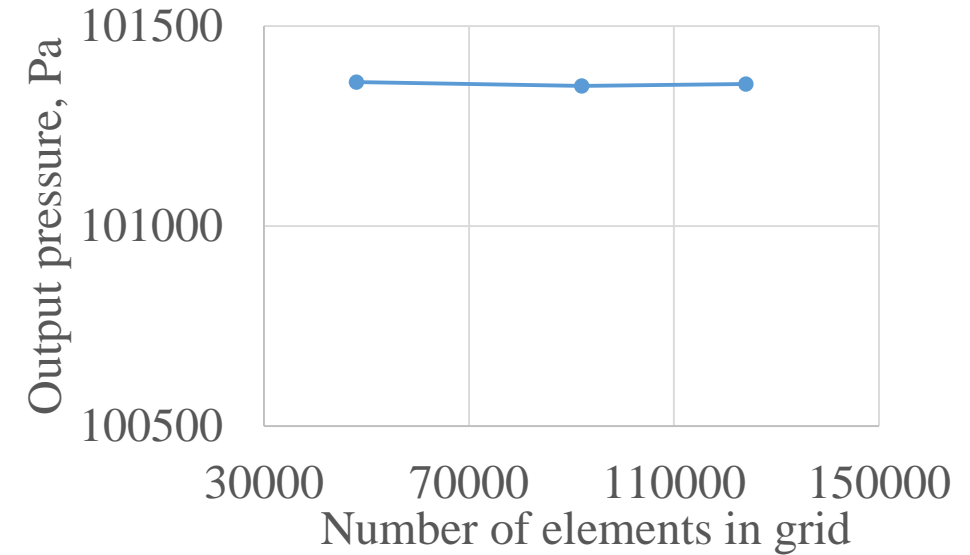
# Meshing

## Tetrahedral Mesh (48,000 elements)

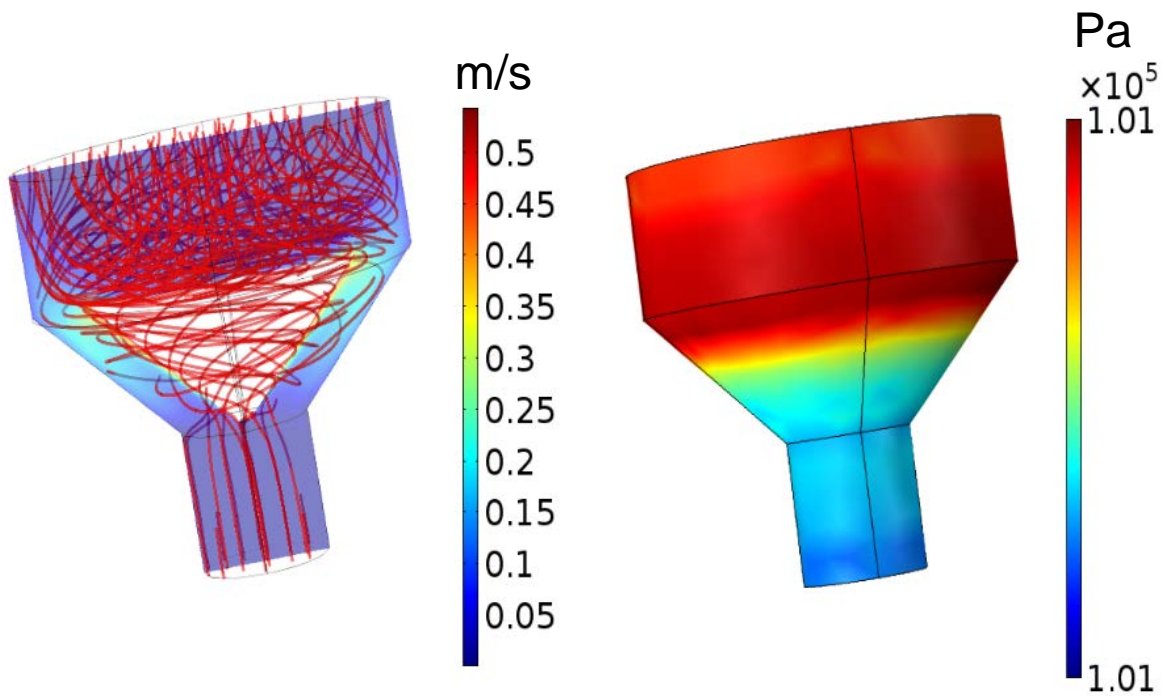


- Tetrahedral mesh with boundary layers has been used
- Number of elements ~48000
- Average quality of the mesh - 0.75

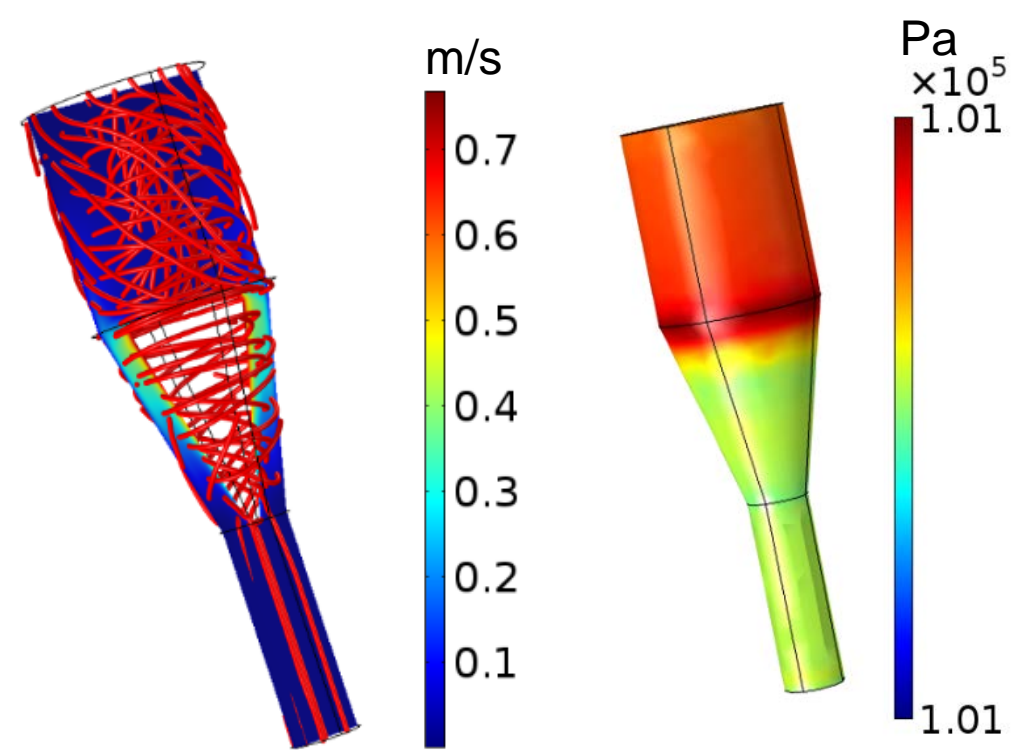
## Mesh Independent Solution



# Velocity and Pressure Profiles: Effect of Semi angle



Velocity and pressure profiles for a semi angle of 45°,  
 $Q = 1 \text{ ml/s}$



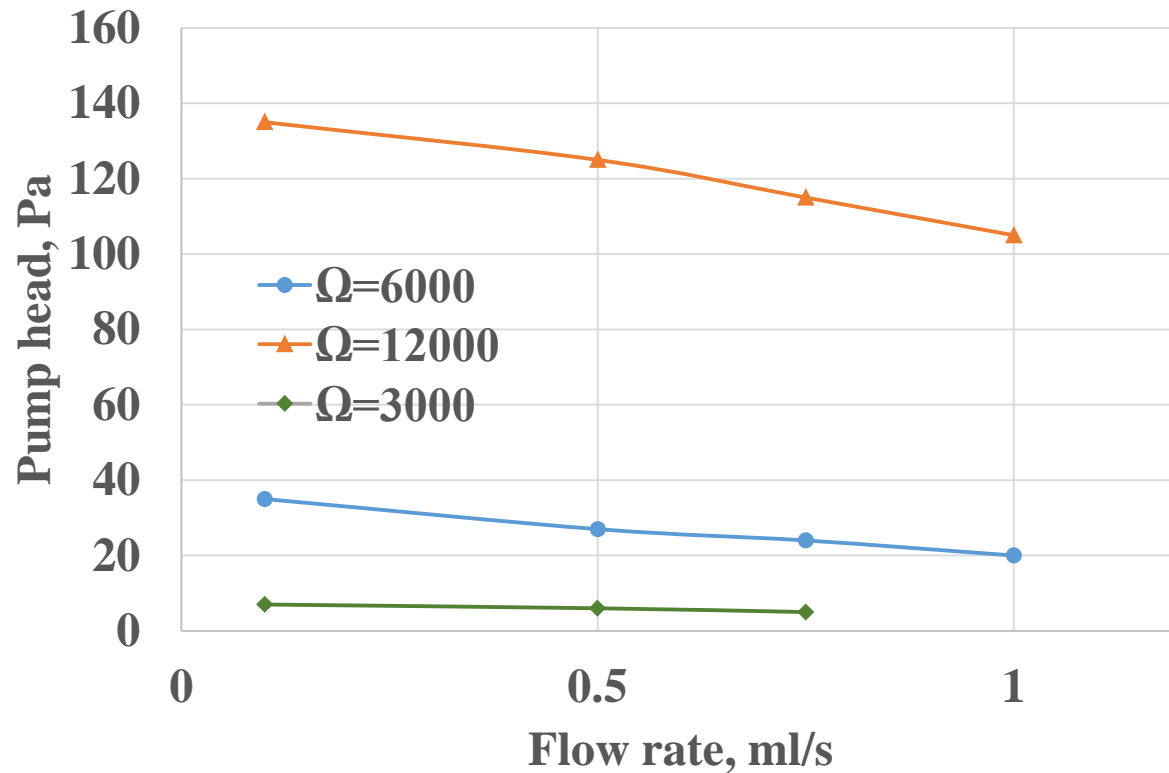
Velocity and pressure profiles for a semi angle of 12°,  
 $Q = 1 \text{ ml/s}$

The pressure profiles show that the hydraulic head of the pump is a weak function of the cone semi angle and height of the rotating cone.



# Pump Head vs Flow Rate: Effect of Rotational Speed

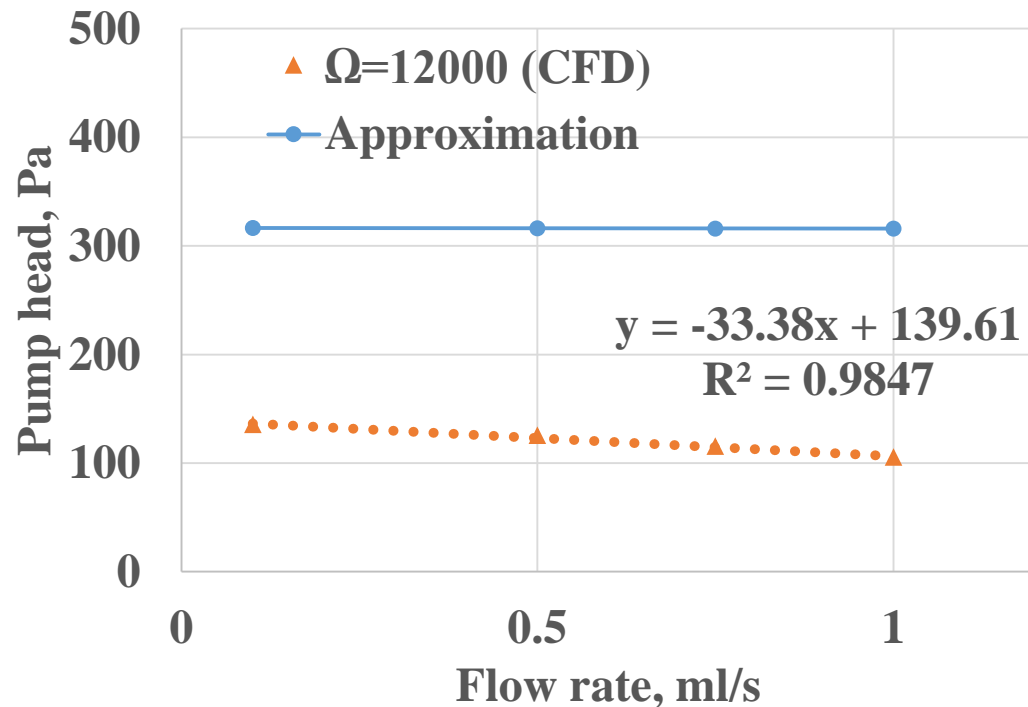
## Head Curve for a Semi Angle of $\pi/4$



- Outlet pressure decreases almost linearly with increasing volumetric flowrate.
- Pump head is proportional to the square of rotational speed.
- Pump outlet pressure does not exceed 135 Pa even for highest values of angular velocity.

# Pump Head vs Flow Rate: Approximate Solution vs CFD Solution

## Comparison of Head Curve Acquired from Approximation and CFD Results



Approximate solution for rotating cone pump problem is available in open literature. (Bird et al., 2007)

## Key assumptions

- Laminar flow
- Curvature and entrance effects are neglected.

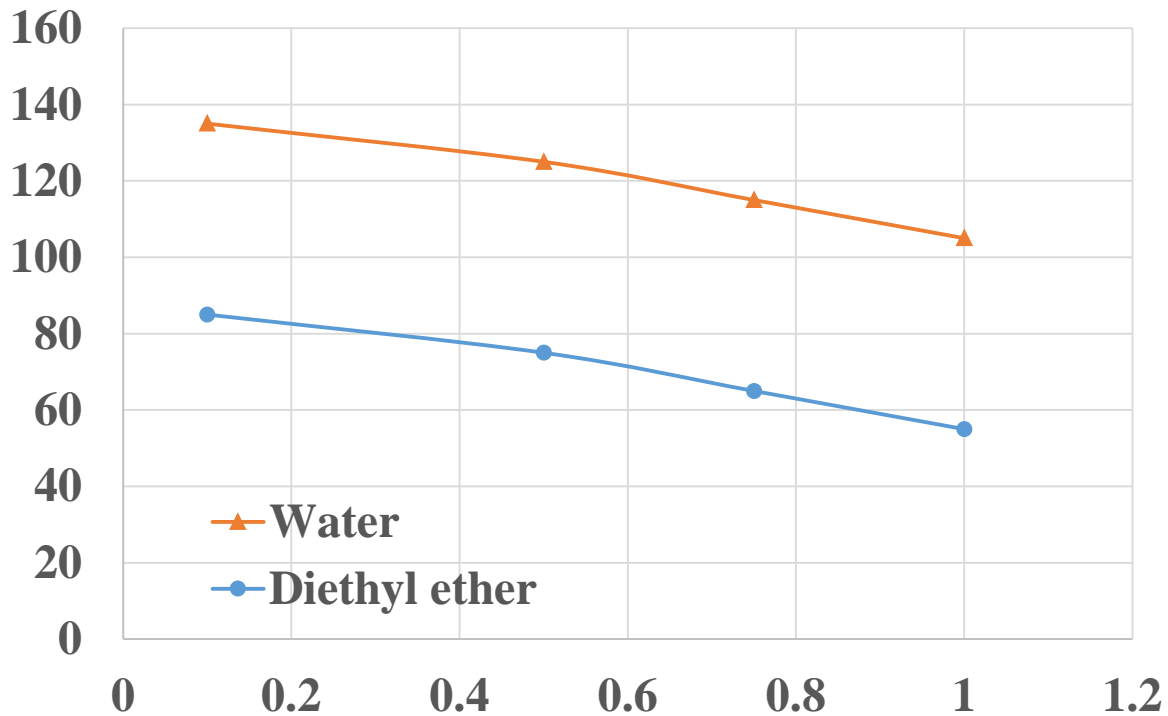
$$w = \frac{4\pi B^3 \rho \sin\beta}{3\mu \ln(L_2/L_1)} \left[ (p_1 - p_2) + \left(\frac{1}{8} \rho \Omega^2 \sin^2\beta\right) \cdot (L_2^2 - L_1^2) \right]$$

where

$L_1, L_2$  are heights corresponding to pressures  $p_1$  &  $p_2$  respectively.

# Pump Head vs Flow Rate: Effect of Fluid Density and Viscosity

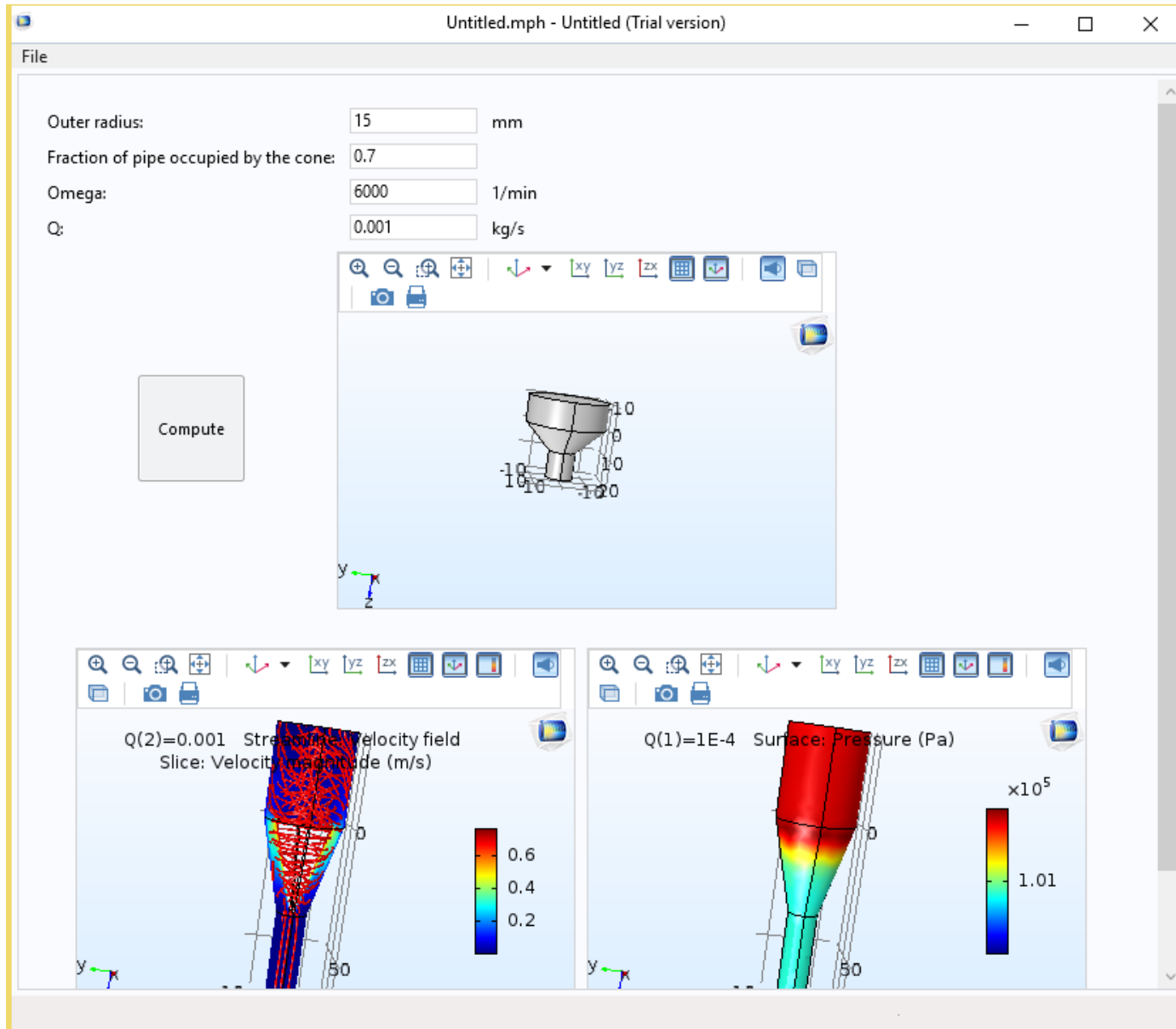
Comparison of head curve for water and diethyl ether



Fluid	Density, kg/m <sup>3</sup>	Viscosity, cP
Water	999.66	1.01
Diethyl ether	713.58	0.24

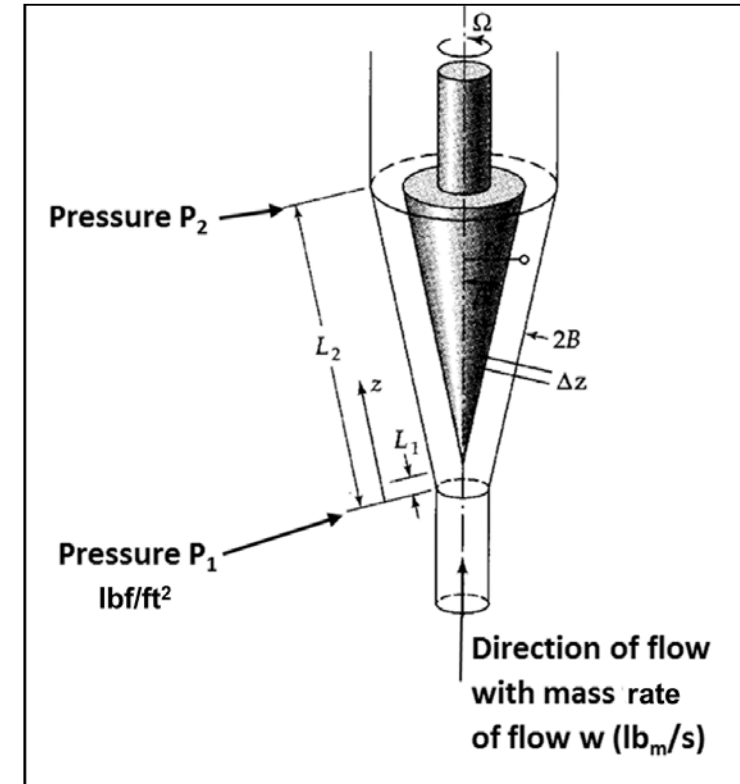
- Rotating pump head increases with increasing viscosity and density.
- Rotating pump head behavior follows the same trend as that for a centrifugal pump.

# COMSOL Application for Rotating Cone Pump



# Conclusions

- The rotating cone pump is a simple design compared to other pump configurations that has applications where a low pump head is sufficient, such as in microprocess systems.
- Analysis of the rotating cone pump performance can be facilitated by using the COMSOL Multiphysics CFD Module.
- The CFD model predicts the correct trends in pump head performance for various model parameters.
- The approximate formula, which is based upon a 1-D fluid mechanics model, over predicts the pump head performance by about a factor of 2.
- Optimization of the cone pump design, which might include modification to the cone head surface, *e.g.*, addition of spiral fins, would be facilitated by COMSOL CFD module simulations vs using empirical approaches.



Thank you for your attention