Presented at the COMSOL Conference 2008 Boston

Coupled Electromagnetics-Multiphase Porous Media Model for Microwave Combination Heating

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- Fully coupled electromagnetics-multiphase porous media model
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Goals

Convection-Radiant Heating

- Used for cooking
- Slow

Microwave Heating

- Fast and convenient
- Non-uniform heating
- Mostly used for reheating

Combination Heating

- Fast and convenient
- Can be used for cooking
- Can provide custom cooking ability

Processing Variables
- Heating Modes
- Power levels
- Cycling

→

Food Factors
- Physical properties
  - Thermal
  - Dielectric
- Size

→

Desired Product
- Uniform heating
- Crust formation
- Appropriate texture

Modeling Methods

Primary Variables: microwave energy deposition, temperature, moisture, pressure

Microwave heating (Electromagnetics)

Model: **Exponential decay(empirical)- 1D, 2D**
Variables: *energy deposition- not valid in general*

Maxwell's equations- 3D
energy deposition

Increasing complexity (physics + geometry)

+

Transport

Model:

Heat Transfer only	Heat and mass transfer (no pressure driven flow)	Heat and mass transfer with pressure driven flow
temperature	*temperature and moisture*	*temperature, moisture, pressure, phase change*

Variables: see row above

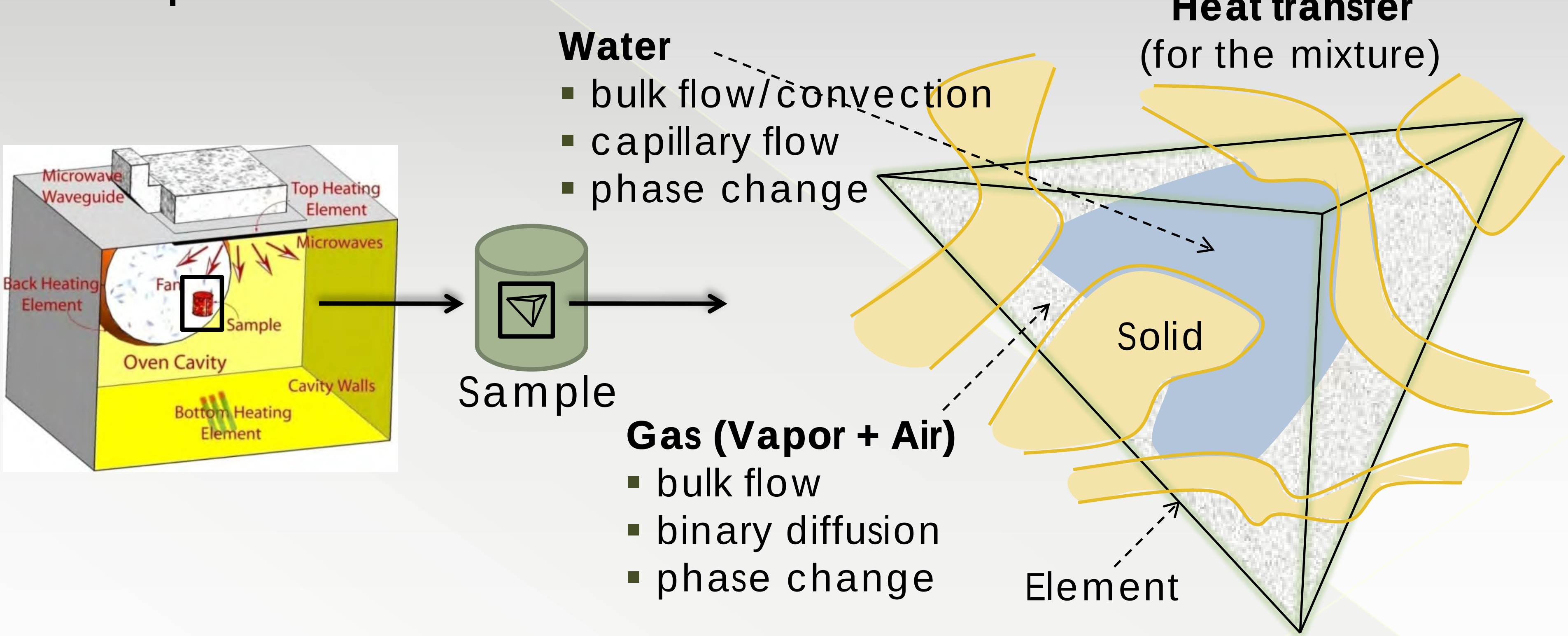
Increasing complexity (physics)

Present work

Fully Coupled Electromagnetics- Heat and Mass Transfer with pressure driven flow (multiphase porous media model) : in 3D

Process Description

- Microwave, hot air, radiant heating
- Multiphase transport in sample treated as a porous medium[1,2,3]



[1]Whitaker S. , *Advanced Heat Transfer*, **13**, 119-203 (1997)

[2]Ni H., Datta, A. K. and Torrance, K. E., *International Journal of Heat and Mass Transfer*, **42**, 1501-12 (1999)

[3]Halder A., Dhall A., and Datta A. K. , Food & Bioproducts Processing, 85, 209-19 (2007)

Governing equations: Electromagnetics

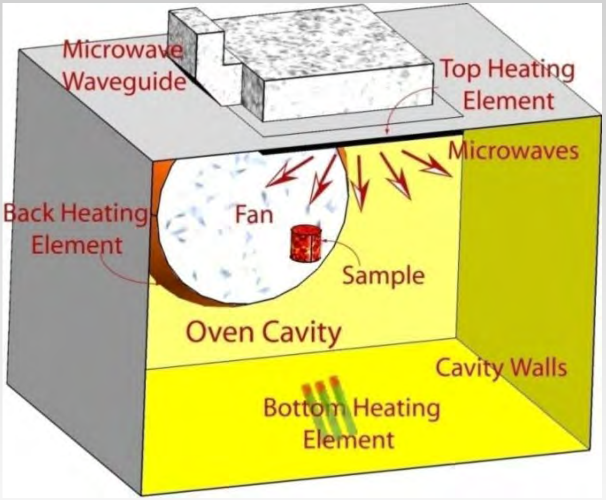
- Maxwell's equations:

$$\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H}$$
$$\nabla \times \mathbf{H} = j\omega\varepsilon\varepsilon_0\mathbf{E}$$
$$\nabla \cdot \left(\varepsilon\mathbf{E}\right) = 0$$
$$\nabla \cdot \mathbf{H} = 0$$

$\mathbf{E}$ = Electric field intensity

$\mathbf{H}$ = Magnetic field intensity

ω = microwave angular frequency



Relative permittivity: $\varepsilon = \underbrace{\varepsilon'}_{\text{dielectric constant}} - j\overbrace{\varepsilon''}^{\text{dielectric loss}}$

- Boundary condition (oven walls):

$$E_{tangential,\,oven\,wall} = 0$$

- Power absorbed (by the sample): $Q_{mic} = \dfrac{1}{2}\omega\varepsilon_0\varepsilon''\left|\mathbf{E}\right|^2$

Governing Equations: Multiphase Porous Media Model

- Momentum balance
  - Darcy's Law:

$$\mathbf{v_i} = -\frac{k_i k_{r,i}}{\mu_i} \nabla P$$

for water and gas (vapor/ air)

G.E.- Mass balance

- Liquid phase (water):

$$\frac{\partial c_w}{\partial t} + \nabla \cdot \underbrace{(\mathbf{n_w})}_{\text{flux}} = \underbrace{-\dot{I}}_{\text{phase change}}$$

$$\mathbf{n_w} = -\rho_w \frac{kk_r}{\mu} \nabla (P - p_{cap}) = -\rho_w \frac{kk_r}{\mu} \nabla P + \rho_w \frac{kk_r}{\mu} \frac{\partial p_{cap}}{\partial S_w} \nabla S_w$$

$$D_w = -\frac{kk_r}{\varphi \mu} \frac{\partial p_{cap}}{\partial S_w} \qquad c_w = \rho_w \varphi S_w$$

$$\frac{\partial c_w}{\partial t} + \nabla \cdot \underbrace{(\rho_w \mathbf{v_w})}_{\text{bulk flow}} = \nabla \cdot \underbrace{(D_w \nabla c_w)}_{\text{capillary flow}} \underbrace{- \dot{I}}_{\text{phase change}}$$

- Gas phase (vapor and air):

$$\frac{\partial c_g}{\partial t} + \nabla \cdot \underbrace{(\rho_g \mathbf{v_g})}_{\text{bulk flow}} = \underbrace{\dot{I}}_{\text{phase change}}$$

G.E.- Mass balance (contd..)

- Vapor:

$$\frac{\partial c_v}{\partial t} + \nabla \cdot \underbrace{\left(\rho_g \omega_v \mathbf{v_g}\right)}_{\text{bulk flow}} = \nabla \cdot \underbrace{\left(\varphi S_g \frac{C^2}{\rho_g} M_a M_v D_{eff,g} \nabla x_v\right)}_{\text{binary diffusion (vapor and air)}} + \dot{I}$$

$$\underbrace{\omega_v + \omega_a}_{\text{mass fractions}} = 1$$

- Phase change# (evaporation/ condensation):

$$\dot{I} = K \frac{M_v}{RT}\left(p_{v,eq} - p_v\right)$$

#Halder A., Dhall A., and Datta A. K. , *Food & Bioproducts Processing*, **85**, 209-19 (2007)

G.E.- Energy balance

- For the mixture:

$$\frac{\partial}{\partial t}\left[\sum_{i=s,w,v,a}\left(c_i c_{p,i} T\right)\right] + \nabla \cdot \left[\sum_{i=w,v,a}\underbrace{\left(c_{p,i}\mathbf{n_i} T\right)}_{\text{bulk flow}}\right] = \underbrace{\nabla\left(k_{eff}\nabla T\right)}_{\text{conduction}} - \overbrace{\lambda \dot{I}}^{\text{phase change}} + \underbrace{Q_{mic}}_{\text{microwave source}}$$

  - Fluxes:

$$\mathbf{n_v} = \rho_g \omega_v \mathbf{v_g}$$

$$\mathbf{n_a} = \rho_g \omega_a \mathbf{v_g}$$

$$\mathbf{n_w} = \underbrace{\left[\rho_w \mathbf{v_w} - D_{w,cap}\nabla c_w\right]}_{\text{water flux due to liquid pressure, } P - p_{cap}}$$

  - Average thermal conductivity:

$$k_{eff} = \left(1-\varphi\right)k_s + \varphi\left\{S_w k_w + S_g\left(\omega_v k_v + \omega_a k_a\right)\right\}$$

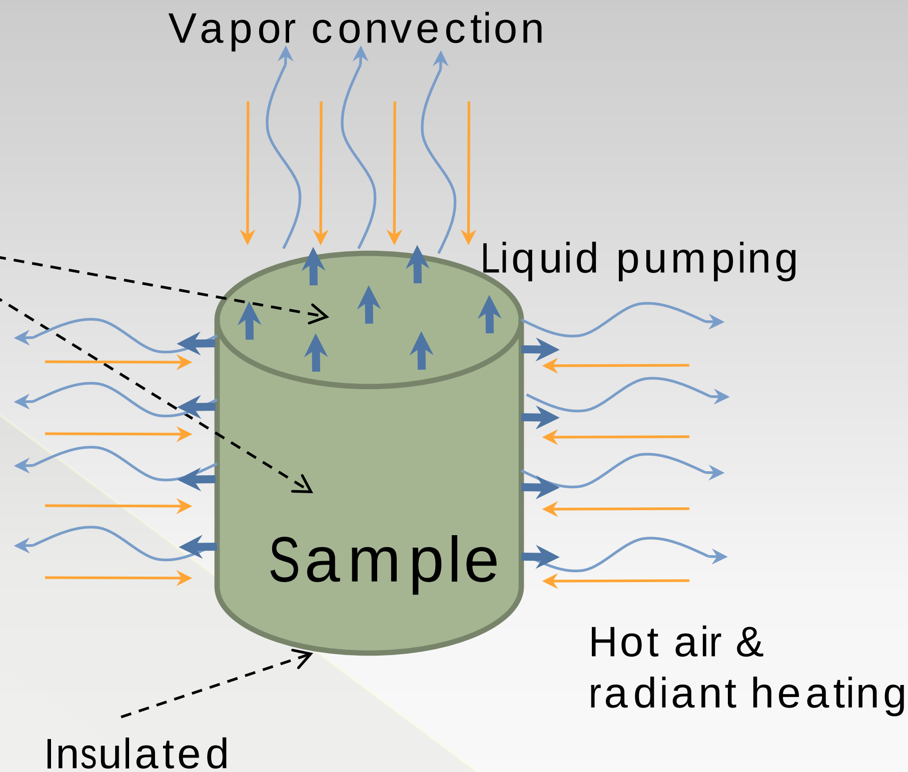
Boundary Conditions

$$P = P_{amb}$$

$$n_v = h_m c_v$$

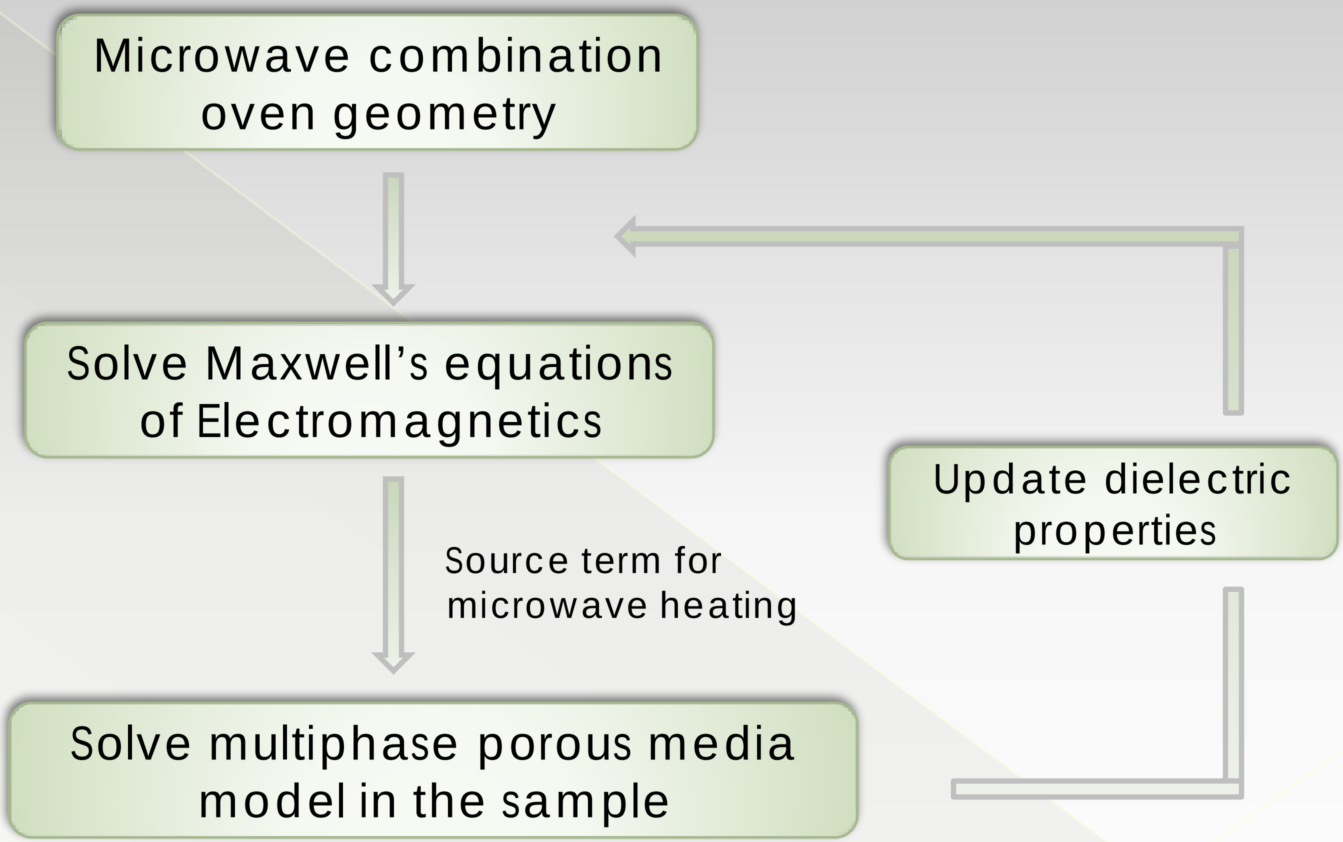
$$n_w = \rho_w u_w, \quad \text{when } S_w = 1$$

$$-k \frac{\partial T}{\partial n} = h(T - T_{oven}) - n_v c_{pv} T - \underbrace{n_w c_{pw} T}_{\text{when } S_w = 1}$$



Oven at temp, T_{oven}

Computational Scheme



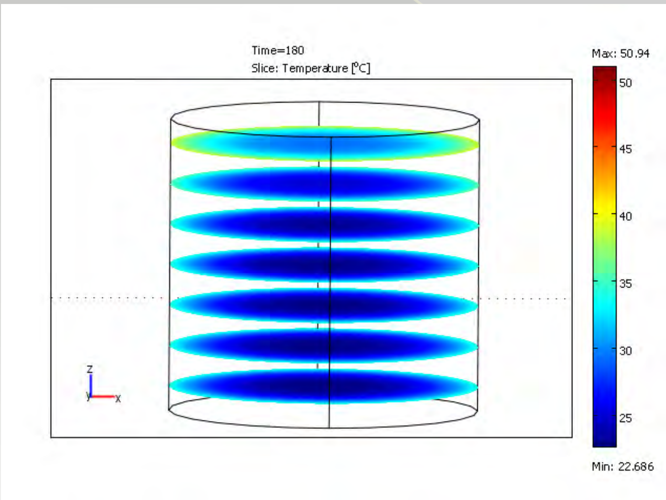
Implementation

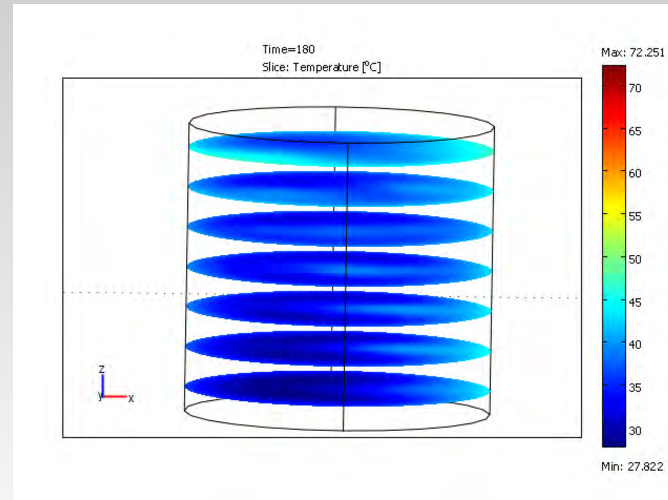
- Equations coupled using scripting in COMSOL
- Problems
  - 3D first time- EM (~2.4M DOFs) and Porous media (~200k DOFs): 42 hrs for 10 min heating: 16 Gb RAM
  - Different mesh and solver (stationary, transient, iterative/ direct) needed for the two physics
  - To make equations implementable in COMSOL additions terms were added- convergence problems

Computed Results

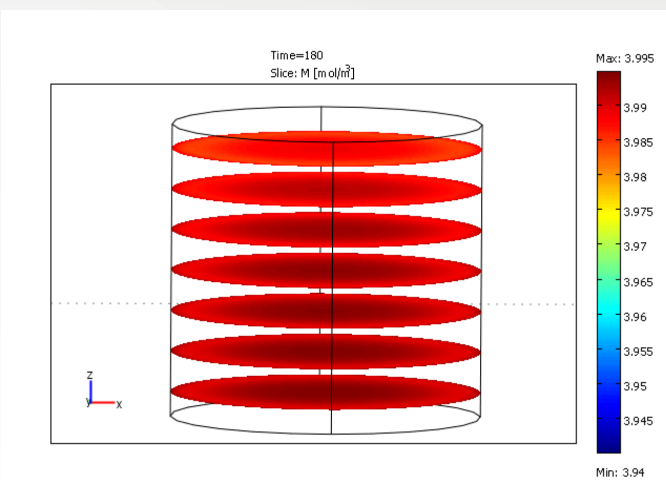
Radiant & hot air

Microwave(10s/50 on), radiant & hot air

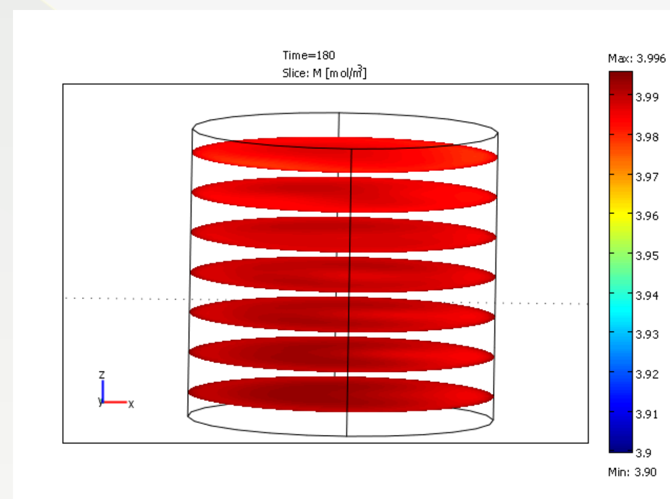




Temperature



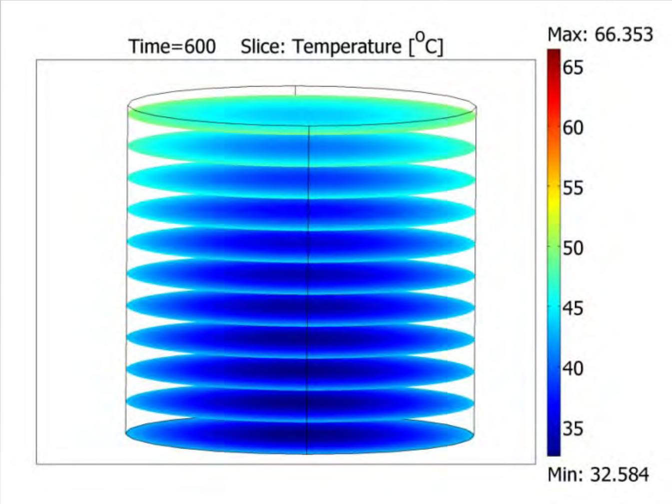
Moisture



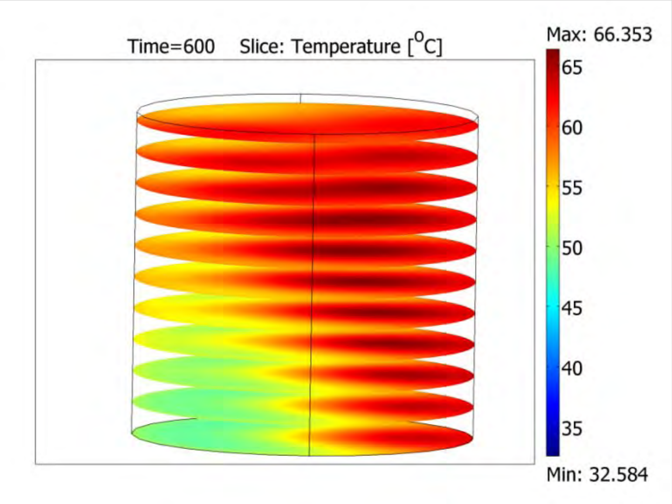
Temperatures & moisture distributions after 10 min of heating

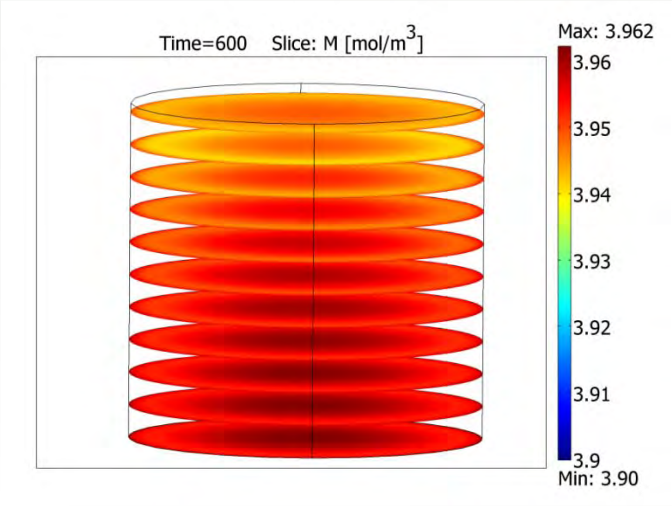
Radiant & hot air

Microwave(10s/50 on), radiant & hot air

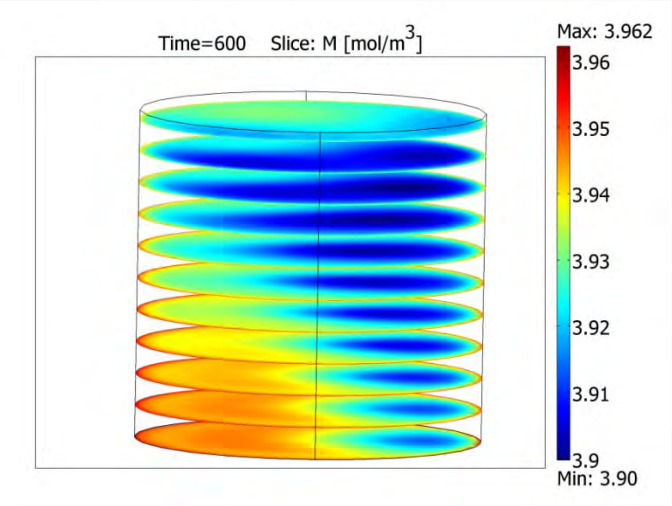


Temperature

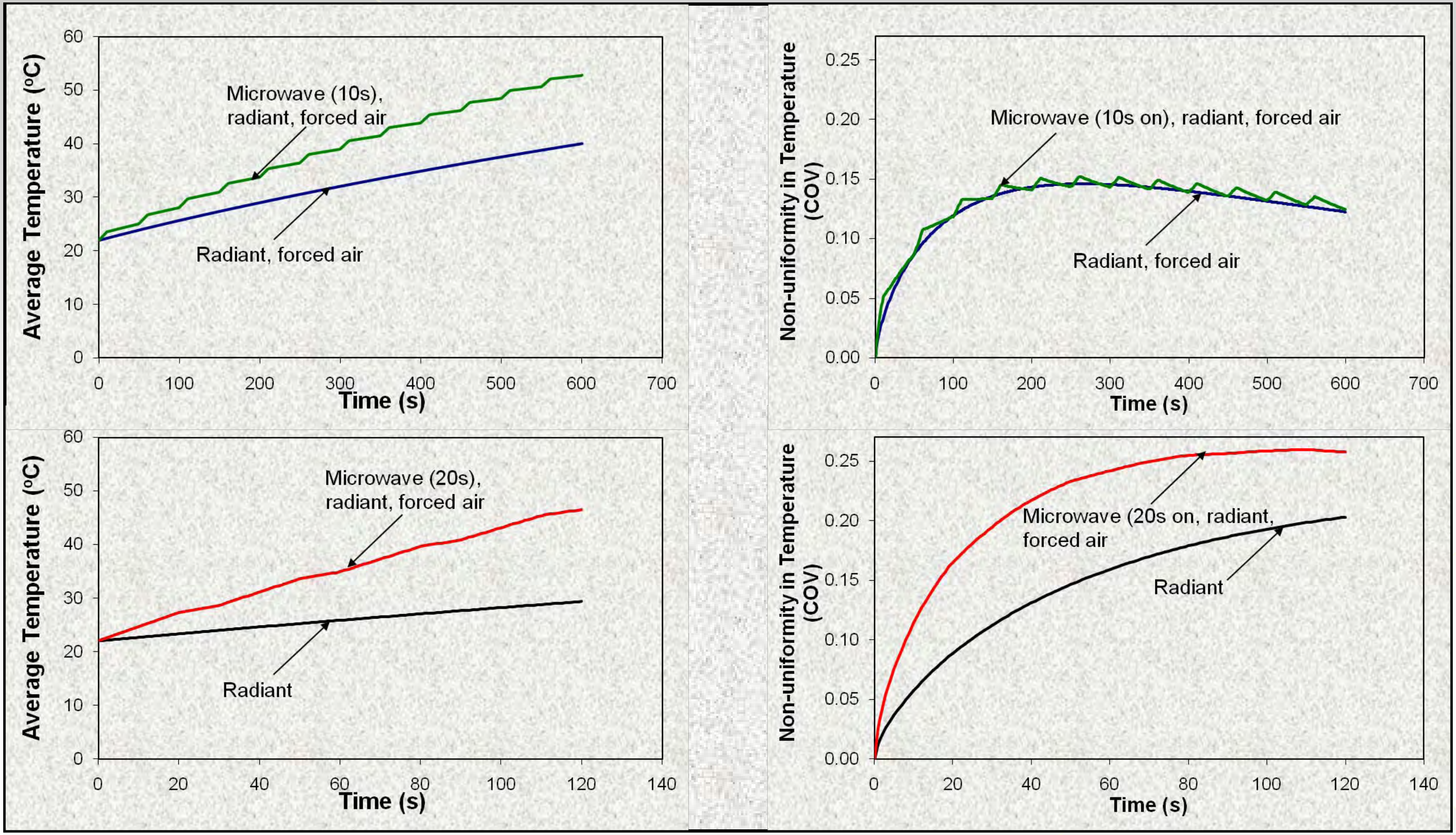




Moisture



Microwave combination heating increases the speed of heating while maintaing the uniformity



Conclusions

- First study - complex coupling of Maxwell's equations with a multiphase porous media model
- Optimum combination of heating parameters can be developed that can speed up the process and maintain heating uniformity at the same time
- Results can be used to develop design recommendations for combination heating for different thermal processes

Ongoing Work

- Experimental validation using Magnetic Resonance Imaging (MRI): UC Davis
- Coupled multiphase porous media-solid mechanics model to study processes with large volume change (e.g. microwave puffing)

Acknowledgements

- USDA Grant
- GE for the oven
- Research group members(BEE, Cornell University)
  - Amit Halder
  - Ashish Dhall

Questions?

Thank You!