Numerical evaluation of the tuning, pressure sensitivity and Lorentz force detuning of RF superconducting crab cavities

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1. Introduction

The Large Hadron Collider (LHC) is the largest and most powerful particle accelerator in the world. The 27 km circumference LHC machine accelerates and collides proton beams and heavy ions. The High Luminosity LHC (HL-LHC) is a project to upgrade the LHC by achieving instantaneous luminosities a factor of five larger than the nominal LHC luminosity. One of the key devices of the HL-LHC project are Superconducting Radio-Frequency (SRF) Crab Cavities. The crab cavities are RF components designed in such a way that the electromagnetic force is perpendicular to the motion of the particle beam crossing them. Therefore, when operated at the adequate frequency, the cavities will tilt the proton bunches, maximizing their overlap at the collision points. Two crab cavity concepts have been developed: the Double Quarter Wave (DQW) and the RF Dipole (RFD). Currently, two DQW crab cavity prototypes have been successfully fabricated and tested with beam in the Super Proton Synchrotron (SPS) at CERN in May 2018, whereas the fabrication stage of the RFD cavities is currently ongoing.

For an adequate operation, the crab cavities are designed to deflect the particle bunch at a specific resonant frequency. A tuning system, consisting of slightly modifying the shape of the cavity is used to correct possible deviations from the fundamental frequency of the cavity. In addition, Pressure Sensitivity (PS) and Lorentz Force Detuning (LFD) are two paramount parameters in the performance of RF cavities. PS analyses the variation of the fundamental frequency of the cavity when subjected to pressure fluctuations of the 2 K helium bath at which the cavity is subjected whereas LFD studies the shift on the cavity frequency due to the RF radiation forces acting on its walls.

The present contribution numerically evaluates the tunability, PS and LFD using COMSOL. On the one hand, experimental results of the tuning of the DQW cavity are used to study the DQW cavity tunability and to confirm the applicability of COMSOL as a powerful tool for RF-structural coupled calculations in RF superconducting cavities. On the other hand, the procedure to evaluate PS and LFD of the RFD crab cavity is described and discussed. The calculations were carried out integrally using COMSOL Multiphysics, which allows the integration of all the needed numerical models in a single software.

2. Theory

2.1. Cavity tunability

RF superconducting crab cavities are designed to operate at a specific frequency of 400.79 MHz. However, due to diverse effects during their fabrication, the fundamental frequency of the cavities may differ from its nominal value [1]. To correct these variations, a tuning system is installed on the cavity external part. This tuning system locally deforms the cavity walls in the elastic domain, promoting a variation of its fundamental frequency. From a structural point of view, one of the main parameters defining the cavity tuning is its tunability (in kHz/mm), that is, the ability of the cavity to change its fundamental frequency as a function of a controlled deformation applied to the cavity body.

2.2. Pressure sensitivity

Pressure sensitivity analysis in RF cavities studies the frequency shift due to pressure fluctuations around the cavity. These pressure changes are commonly caused by variations of the pressure of the cold liquid helium bath and are typically of the order of ±1 mbar. Changes in the He pressure promote small deformations in the cavity shape that affect the fundamental frequency of the cavity. Consequently, PS is regularly measured in Hz/mbar, directly relating the frequency shift to the magnitude of the pressure fluctuations.

Due to its implications in the fundamental frequency of the cavity and the cavity performance, one of the main goals in the RF-structural design stage of the cavity is focused in the reduction of the PS value (i.e. a stiffer structure).

2.3. Lorentz Force detuning

RF power produces forces in the form of a radiation pressure that acts on the cavity walls and is proportional to the square of the electromagnetic fields [2]:

$$P = \frac{1}{4}(\mu_0 H^2 - \varepsilon_0 E^2) \qquad (1)$$

Where E is the electric field amplitude (V/m), H is the magnetic field amplitude (A/m) and μ_0 and ε_0 are the vacuum permeability and permittivity, respectively. The deformation of the cavity due to this radiation pressure produces a shift on the fundamental frequency of the cavity. The relation between the frequency shift Δf and the deflecting voltage V_T is given by Equation (2) [2]:

$$\Delta f = K_{LFD} V_T^2 \qquad (2)$$

where K_{LFD} is a coefficient known as Lorentz force detuning and has units of Hz/MV^2. As in the case of PS, the design of the cavity is focused on obtaining LFD values as reduced as possible.

For all of the above, the design of the cavity focuses on providing a locally flexible (tunable) cavity while granting sufficient stiffness to minimize PS and LFD.

3. Numerical model

Two different numerical models were used along the present contribution. The first model corresponds to the DQW cavity, which has been already tested at CERN and of which experimental measurements of the tunability are available. Figure 1 shows a three dimensional (3D) view of the numerical model. For the sake of simplicity, a symmetry condition is applied in the yz plane of the cavity, as shown in Figure 4.

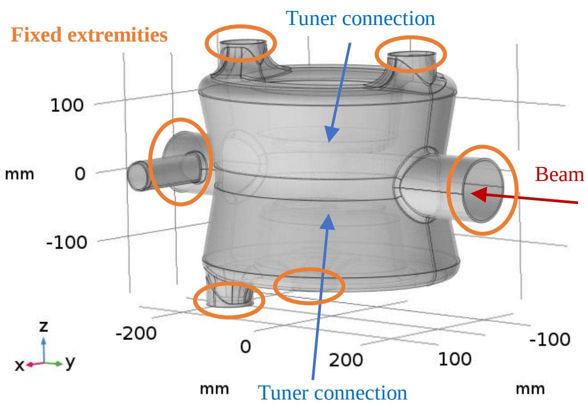


Figure 1. Three-dimensional model of the DQW crab cavity.

The pressure sensitivity and Lorentz force detuning were calculated using the numerical model of the RFD cavity shown in Figure 2, the results of these analyses provided valuable information for the final design of the cavity body. Note that the numerical model includes also details such as thickness reductions in the cavity body shape, which were done for welding purposes and have demonstrated a non-negligible impact on the cavity PS and LFD values.

Tuning, PS and LFD analyses imply performing both RF and structural calculations. Therefore, both models consider at least two domains: the cavity body (for structural purposes) and the vacuum volume inside the cavity body (for RF calculations).

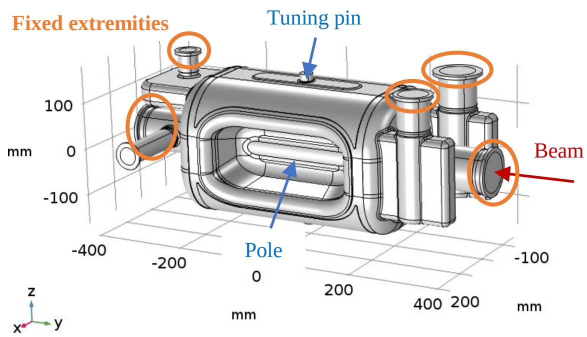


Figure 2. Three-dimensional model of the RFD crab cavity.

3.1. Materials

The cavity body is in niobium RRR300. The mechanical properties in Table 1 were used for the RFD analyses and correspond to the properties of Nb at 2 K. For DQW tunability analysis, properties similar to those of previous analyses [3] were used for the sake of comparison.

Table 1. Mechanical properties used for Niobium [4, 5].

Density [kg/m^3]	Young's modulus [GPa]	Poisson's ratio [-]
8600	118	0.38

The material chosen for the volume inside the cavity body was air with relative permittivity and permeability equal to 1 and zero electrical conductivity.

3.2. Simulation procedure and boundary conditions

Figure 3 presents the simulation procedure for the coupled RF and structural analyses. This procedure is coincident for the three types of analysis performed (i.e. tunability, pressure sensitivity and Lorentz force detuning), as they only differ in the loads applied on the cavity body.

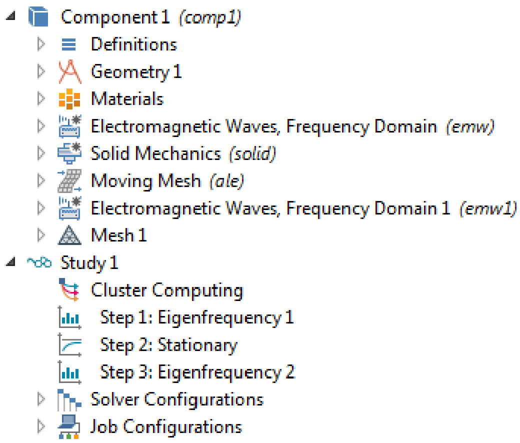


Figure 3. List of components and studies used in COMSOL for both the RF-structural studies.

From a practical point of view, the first task to perform is to import the geometry of the system and to create the calculation mesh. Previous works done at CERN revealed that mesh refinement is needed in the high deformation regions, so that the contribution of the deformation to the frequency shift in the cavity is properly modelled. For that reason, a mesh with a maximum element size of 2 mm was used in the most deformed regions of the cavity body (i.e. the pole and the flat top and bottom regions of the cavity which correspond to the tuner connection region, see Figure 2), whereas a standard "finer" mesh was used for the rest of the geometry. In order to estimate the minimum mesh size needed to obtain reliable results, a mesh sensitivity study was performed for the tunability of the DQW cavity and it is presented in the results section of the present contribution.

The procedure followed to calculate the frequency shift of the cavity is the following:

(a) A fundamental RF frequency (eigenfrequency) problem was solved on the vacuum domain by using the electromagnetic waves module available in COMSOL. This allows to obtain the fundamental frequency of the cavity, namely f_0.

(b) The application of structural loads on the cavity walls promotes a deformation of the vacuum volume inside the cavity, which causes a variation of the cavity fundamental frequency. To capture this effect, the structural and moving mesh modules were fully coupled. Therefore, a deformation on the cavity body caused a displacement of the mesh in the vacuum volume, whose deformed state was subsequently used for the calculation of the new eigenfrequency.
In all the cases studied, the cavities were fixed at their extremities. Nevertheless, depending on the study being performed, different loads were applied in the system:

(b.1) Tunability – measures the frequency shift of the cavity when subjected to a specific displacement of the tuning system. Therefore, the boundary condition in that region consists in a vertical displacement in the top and bottom pins of the cavity tuner (see Figure 1).

(b.2) Pressure sensitivity – measures the frequency shift due to pressure variations in the external He cold bath at which it is subjected. The frequency shift is considered to have a linear relation with pressure. Therefore, a pressure P_{PS}=1 bar was applied to the exterior walls of the cavity.

When the cavity is subjected to the cold external He bath, the whole tuning system (not shown in Figure 1 and Figure 2 for simplicity) is connected. The tuning system components provide an additional stiffness to the cavity top and bottom plates. Considering the configuration of the tuning system, it can be modelled as if the top and bottom tuner pins were connected by a spring with a spring constant k_s that is, in a first approximation, assumed to be equal to the spring constant of the tuning frame. This is because the tuning frame corresponds to the less stiff component of the tuning system [6]. Thus, a force boundary condition was applied to each of the tuning surfaces as follows:

$$F_{s1} = k_s(v_{s2} - v_{s1}) \quad F_{s2} = k_s(v_{s1} - v_{s2}) \qquad (3)$$

Where F_{s1} and F_{s2} are the vertical forces on the tuner pins and v_{s1} and v_{s2} are the vertical displacement of the tuning interfaces at the top and bottom surfaces of the cavity, respectively, and are monitored by means of boundary probes in COMSOL.

(b.3) Lorentz force detuning. As in the pressure sensitivity analysis, a pressure, P, was applied to the cavity walls to measure the frequency shift due to the deformation of the cavity as an impact from the cavity's own electromagnetic field, as indicated in Equation (1).

In the eigenvalue problem, COMSOL scales the electric and magnetic fields to some value of the stored energy in the cavity. Therefore, the value of the pressure in the eigenvalue problem was scaled up by a factor SF to account for the pressure on the cavity walls at the nominal deflecting voltage (3.4 MV in this specific case). The value of SF accounts for the energy stored in the cavity at the nominal deflecting voltage (3.4 MV) with regard to the energy stored in the eigenvalue problem:

$$SF = \frac{U\ (3.4\ MV)}{U\ (COMSOL)} \quad (4)$$

The procedure followed to calculate the scaling factor *SF* and, in consequence, the pressure on the cavity interior walls was also calculated using COMSOL:

Firstly, making use of the eigenfrequency problem solved in (a), the total energy stored in the cavity volume, $U\ (COMSOL)$ was integrated.

Secondly, the corresponding deflecting voltage at that energy was calculated by integrating the deflecting voltage along the axis, as described in Equations (5) and (6) [7]. Due to the design of the cavity, only the *x* component of the electric field and the z component of the magnetic field will promote a transversal kick on the particle bunch (see Figure 7 and Figure 10):

$$V_T = \left| \int [E_x + \mu_0 H_z] e^{j\frac{\omega y}{c}} dy \right| \quad (5)$$

$$V_T = \left| \int E_x \cos\left(\frac{\omega y}{c}\right) dy + \int \mu_0 c \cdot H_z \cdot \sin\left(\frac{\omega y}{c}\right) dy \right| \quad (6)$$

Where *x*, y and z represent the system of coordinates as shown in Figure 2, c is the speed of light, ω is the angular frequency and μ_0 is the vacuum permeability.

The energy stored in the cavity at the nominal voltage was scaled by using the value of V_T previously calculated, considering the fact that the energy varies quadratically with the voltage:

$$U\left(V_{T,nominal}\right) = U\left(V_{T,COMSOL}\right)\left(\frac{V_{T,nominal}}{V_{T,COMSOL}}\right)^2 \quad (7)$$

From which the scaling factor, *SF*, for the radiation pressure was explicitly deduced.

Note that, similarly to the pressure sensitivity analysis, the tuning system is at all times connected to the cavity top and bottom plates and, therefore, the spring forces indicated in Equation (3) were also applied.

(c) Moving mesh - The original mesh created for the vacuum volume was deformed according to the deformation of the solid body of the cavity. To do so, the moving mesh module was fully coupled with the solid mechanics module. To correctly capture the mesh displacement, free deformation conditions were set in all the domains. The mesh was deformed by a prescribed mesh displacement imposed (resorting from the structural analysis results) in the boundary between the solid domain and the vacuum volume. Due to the configuration of the cavity, zero normal mesh displacements were imposed in the ports end surfaces, which prevents the mesh of the cavity to deform in the axial direction (see Figure 4). In the particular case of the DQW tunability calculation, a perfectly magnetic boundary condition was set in the symmetry plane.

(d) Finally, a new eigenfrequency problem was solved using the same guidelines as in point (a). This eigenfrequency problem provides the fundamental frequency of cavity, f_1, in the deformed state of each of the previously described cases. Therefore, the cavity tunability, pressure sensitivity and Lorentz force detuning can be calculated as indicated in Table 2.

Table 2. Equations to calculate tunability, PS and LFD.

Tunability [kHz/mm]	PS [Hz/mbar]	LFD [Hz/MV^2]
$\frac{f_1 - f_0}{\lvert v_{s2}\rvert + \lvert v_{s1}\rvert}$	$\frac{f_1 - f_0}{P_{PS}}$	$\frac{f_1 - f_0}{V_{T,nominal}^{\ 2}}$

4. Results and discussion

4.1. Tunability – DQW cavity

Figure 4 shows the deformation of the DQW cavity body when a pulling displacement of 1 mm was applied on the top and bottom regions of the cavity. Note that the deformation is concentrated in the tuning region. The frequency shift of the cavity after this deformation was calculated following the guidelines presented in Section 3.2 and resulted in 315.5 kHz/mm, which is consistent with the 318 kHz/mm encountered in previous works for the same numerical conditions and a different software [3].

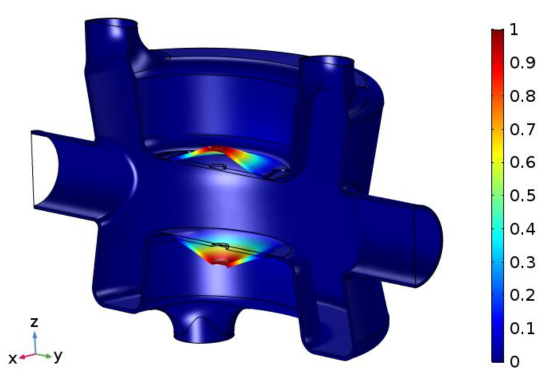


Figure 4. Contour plot of the displacement in the DQW cavity, in mm. Tuner displacement of 1 mm.

Before proceeding to a further analysis of the results, a mesh sensitivity analysis was performed. This mesh sensitivity considers a refinement of the mesh in the regions in which the mesh is deformed. The reason why only this area is relevant is because the RF frequency will change only due to the deformation of the cavity which, in this model, only takes place in the tuner region. The resulting values of RF frequency against the maximum local mesh size can be seen in Figure 5. It can be concluded that beyond a mesh dimension of 4 mm, the results start to diverge. At 5 mm the difference is already in the order of 2 kHz.

From the study, it is also concluded that a safe compromise between accuracy and computational cost for the mesh dimension in the region of deformation is 2 mm. It must be stated that this is so in a case where the maximum deformation is 1 mm and the sheet thickness is 4 mm. In the case of larger deformations and/or thinner sheets, it is likely that a smaller mesh dimension would be more suitable.

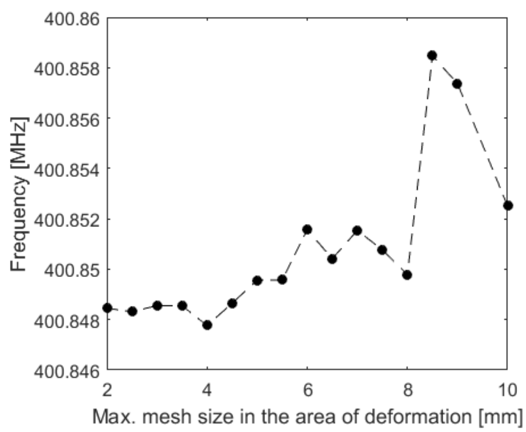


Figure 5. Fundamental frequency of the cavity as a function of the maximum mesh side in the area of deformation.

The numerical value of the cavity tunability was compared against experimental results obtained during the testing of the DQW cavities at CERN SM18 in 2018. Figure 6 shows a comparison of the experimental and numerical results of the fundamental frequency change of the cavity as a function of the force applied by the tuner. It can be observed that the numerical predictions provide a good estimation of the experimental measurements. For small frequency changes, the numerically predicted values match well the measured frequency change whereas the difference between the experimental and numerical results increases progressively due to the different slope of the curves. The force caused by the tuner mechanism results into the deformation of the cavity. The tuner system consists of many elements that form an equivalent system that transmits the force on the tuning system (measured) into displacement in the cavity body. Thus, the frequency change is estimated based on the calculation of the stiffness of the different components and on the tuning sensitivity derived from COMSOL. This may cause the slight difference between the experimental and numerical results. Nevertheless, the certainty, the simplicity and the closeness between the numerical and the experimental results indicates that COMSOL can be a useful tool not only during the design stage but also to improve and recalculate values of high incertitude once experimental outcome is available.

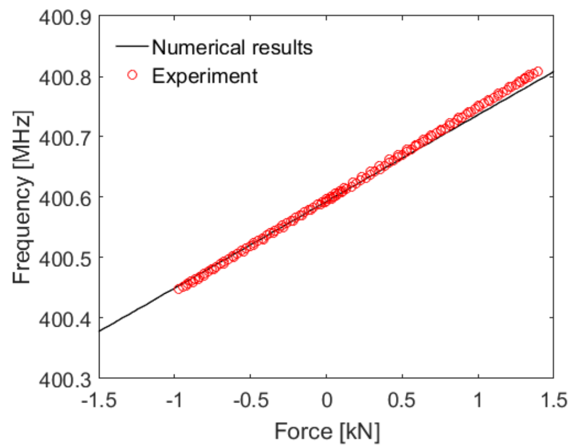
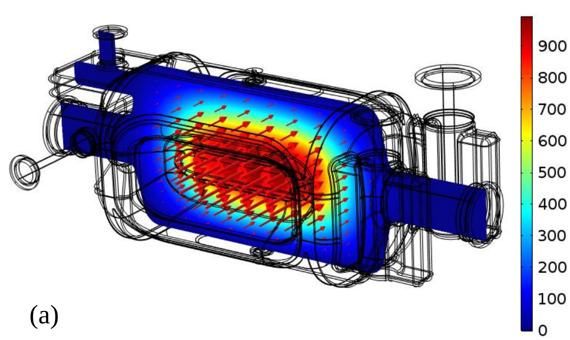


Figure 6. Frequency of the DQW cavity as a function of the force generated by the tuner system. Numerical results are derived from COMSOL and the tuning system stiffness.

4.2. Pressure sensitivity analysis and Lorentz force detuning – RFD cavity

4.2.1. Pressure sensitivity analysis

Figure 7 shows the electric and magnetic fields of the RFD cavity volume at the fundamental frequency. It can be observed that the electric field is maximum in the centre of the cavity and provides a deflecting kick that is perpendicular to the beam direction through the cavity. The magnetic field is perpendicular to the beam velocity so that it also provides a deflecting kick in the x direction.



(a)

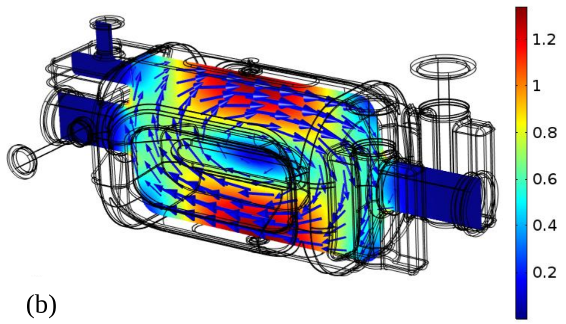


Figure 7. 2D slice and volume vectors of the (a) electric field in V/m and (b) magnetic field in (A/m) in the cavity vacuum volume for the eigenfrequency problem.

The calculated value of the fundamental frequency of the cavity in this situation is $f_0=400.805$ MHz. The volume integrated total energy in the cavity is $U=15\cdot10^{-9}$ J. Once the fundamental frequency of the cavity was calculated, a pressure of P_{PS} = 1 bar was applied to the cavity walls, which promoted a deformation of the cavity body, as shown in Figure 8. Subsequently, the new eigenfrequency was calculated for the deformed cavity, which results in f_1=400.562 MHz. Therefore, the pressure sensitivity of the cavity is 244 Hz/mbar (see Table 2).

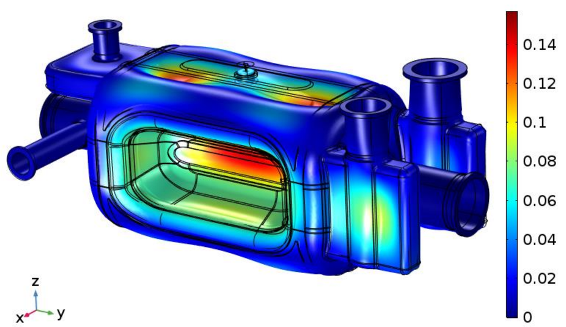


Figure 8. Total displacement of the cavity walls, in mm, when subjected to an external pressure of 1 bar. The 3D representation of the deformation has been scaled up for the sake of visual inspection of the results.

Parametric analysis of the tuning system stiffness

It was identified that a critical parameter of the pressure sensitivity of the cavity was the stiffness of the tuning system, k_s. In fact, a calculation performed assuming $k_s = 0$, revealed that PS reduced to nearly half its value to PS = 115 Hz/mbar. Therefore, in order to optimize the design of the cavity tuning system, a parametric study was performed, where the spring constant, k_s was varied around possible design values of the tuning system stiffness. Figure 9 shows the results from this analysis, which reveal a clear increase of the pressure sensitivity of the cavity with k_s. This behaviour is attributed to the counteracting effect that the pressure-induced deformation of the pole (negative frequency shift) and the tuning regions (positive frequency shift) have on the fundamental frequency of the cavity.

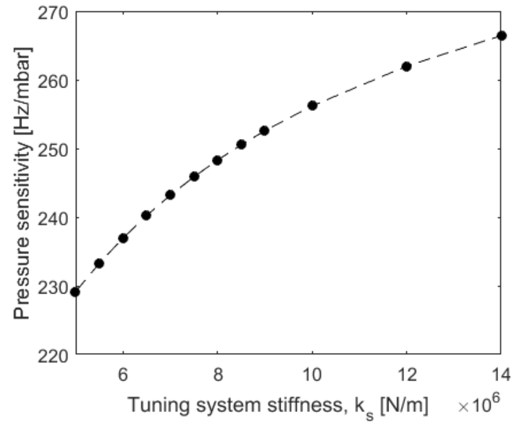


Figure 9. Evolution of the pressure sensitivity as a function of the tuning system stiffness.

4.2.2. Lorentz Force detuning

Figure 10 shows electric and magnetic field components along the cavity axis as a function to the distance to the cavity centre, which is directly related to the results shown in Figure 7. Note that only the x component of the electric field and the z component magnetic field are shown in the figure, which correspond to the only non-zero components of these fields along the axis. Using Equations (5) and (6) and integrating in COMSOL along the axis yields an integrated deflecting kick of 127.72 V. Therefore, the scaling factor to apply to the radiation pressure is $SF=7.086\cdot10^8$ and the total energy stored in the cavity for a deflecting kick of 3.4 MV is equal to 10.63 J, which is well in line with the expected value.

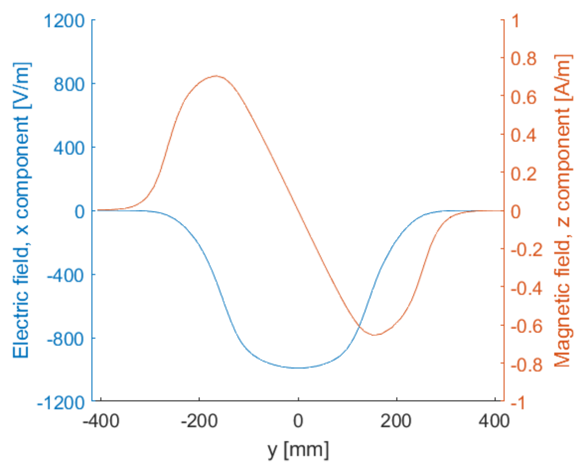


Figure 10. Electric field in the x direction and magnetic field in the y direction used for the calculation of the deflecting kick voltage (Equation (6)).

Figure 11 shows the pressure applied to the cavity walls resorting from the electromagnetic forces and scaling up Equation (1) by the factor *SF*. In this situation, the magnetic repulsive forces tend to push the cavity top and bottom plates to the exterior of the cavity, promoting a decrease of the cavity fundamental frequency, whereas the electric forces tend to join the poles together, which also promote a decrease of the cavity frequency.

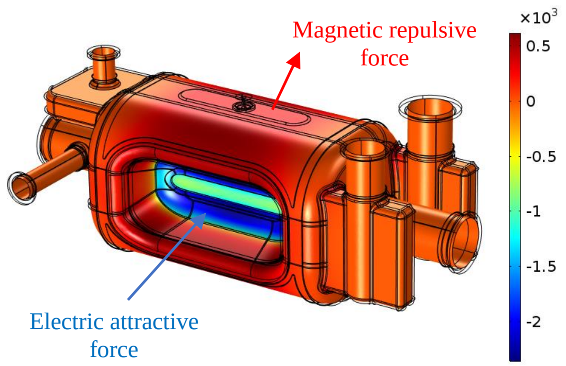


Figure 11. Radiation pressure, in Pa, applied to the cavity internal walls.

Figure 12 shows the total displacement of the cavity after the application of the radiation pressure. As in the previous section, the new eigenfrequency of the deformed cavity was calculated, resulting in a frequency of 400.798 MHz. Therefore, as indicated in Table 2, the value obtained for the Lorentz force detuning from the non-deformed to the electromagnetically deformed cavity results in 659 Hz/MV^2.

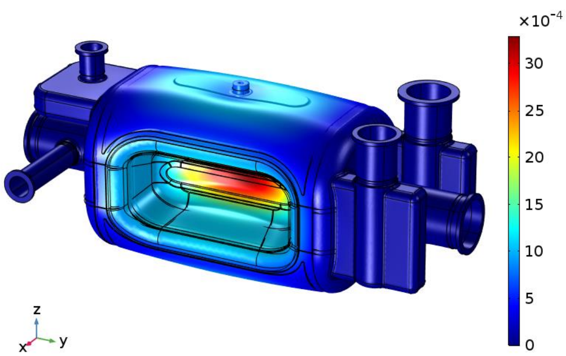


Figure 12. Total displacement of the cavity walls, in mm, when subjected to the radiation pressure described in Equation (1) and Figure 7. The 3D representation of the deformation has been scaled up for the sake of visual inspection of the results.

5. Conclusions

The present work summarizes the procedure followed to calculate the tuning, pressure sensitivity and Lorentz force detuning of superconducting RF crab cavities. The model used corresponds to the DQW (tunability) and RFD cavities (PS and LFD) of the HL-LHC project, but it could be applied to any RF cavity of similar characteristics.

The numerical calculations were integrally performed using COMSOL Multiphysics. The steps followed required the use of three particular modules: first, the RF module was used to calculate the fundamental frequency of the cavity. Then, the structural and moving mesh modules were coupled to deform the vacuum volume inside the cavity in accordance with the appropriate loads and boundary conditions (displacement in the case of tunability, pressure in the case of pressure sensitivity and Lorentz Force detuning analyses). Once the loads were applied, the mesh was deformed and the frequency shift in the cavity was calculated. Special mention requires the Lorentz force detuning calculations, in which the deflecting kick voltage and the energy stored in the cavity were required and also calculated using COMSOL.

The results demonstrate the capability of COMSOL to perform RF-structural coupled calculations and confirms it as a very powerful tool during the validation and design stages of superconducting RF cavities.

Acknowledgements

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