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Implementation of the Perfectly Matched Layer to Determine the Quality Factor of Axisymmetric Resonators in COMSOL

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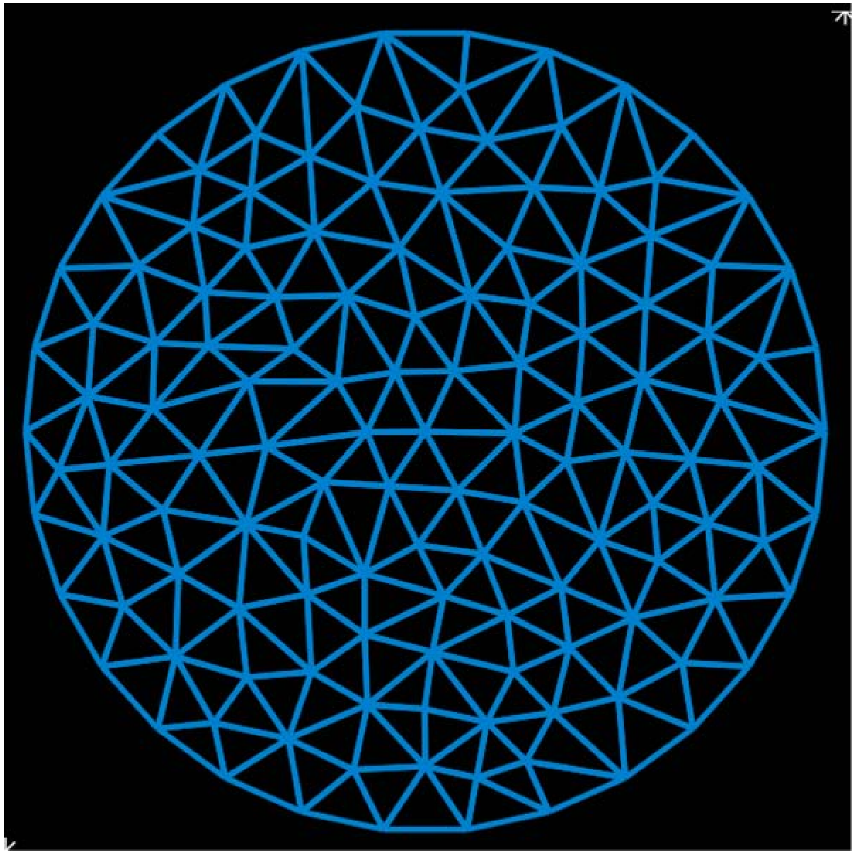
Outline

- Introduction
- Whispering Gallery modes
- Previous FEM Model
- Perfectly Matched Layer
- Our FEM Model
- Conclusions

Introduction

- Accurate electromagnetic model is needed for various axisymmetric optical resonators such as micro-discs and micro-toroids
- A COMSOL model for such resonators exists but without perfectly matched layer
- Unwanted reflections from the computation wall reduces the accuracy of the model
- Quality factor determination with high accuracy is important for certain applications



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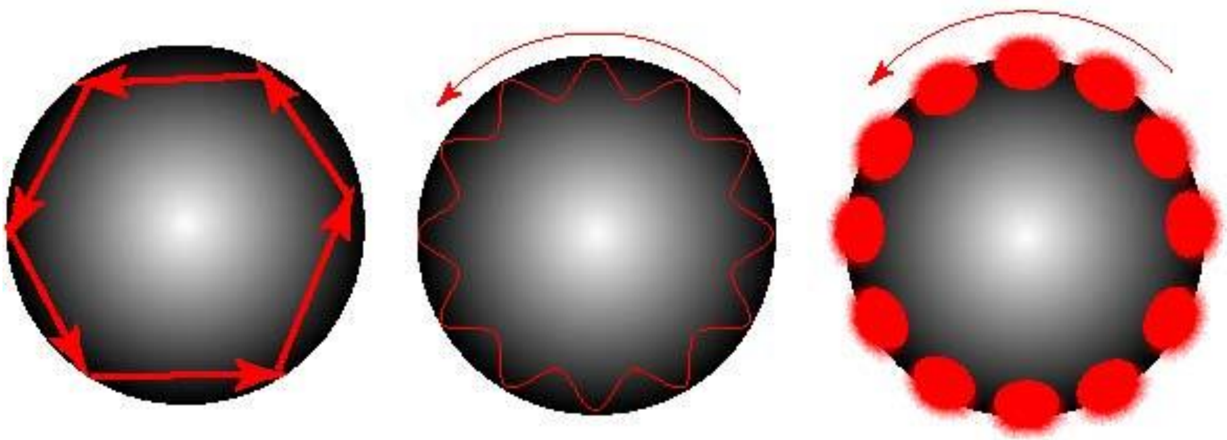
Whispering Gallery Modes (WGM)



- In open cavities light circulates in the form of WGM
- WGM field does not occupy the whole cavity
- Portion of a WGM field lies outside the cavity

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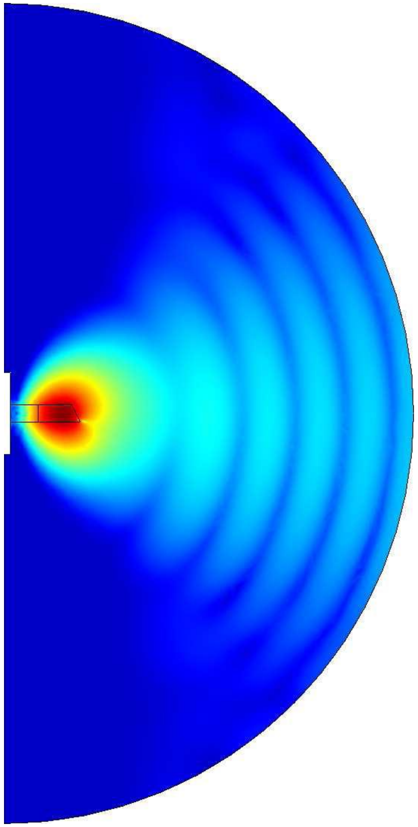
Previous Finite Element (FEM) Model

- Full vectorial model – No transverse approximation
- No PML or any other absorbing boundary condition
- Important parameters can be extracted for various cavity geometries:
  - Quality factors
  - Mode Volumes
  - Resonant Frequencies
  - Shapes of fundamental and higher order modes
- Estimated quality factor for the disc resonator is $1.31 \times 10^7 < Q < 3.82 \times 10^7$
  - Prior knowledge of the mode frequency is required
  - Quality factor for one mode at a time
  - Need to change the boundary conditions for each bound and recalculation of the model each time



Mark Oxborrow, IEEE Transactions on Microwave Theory and Techniques, 55, 1209-1218, 2007.

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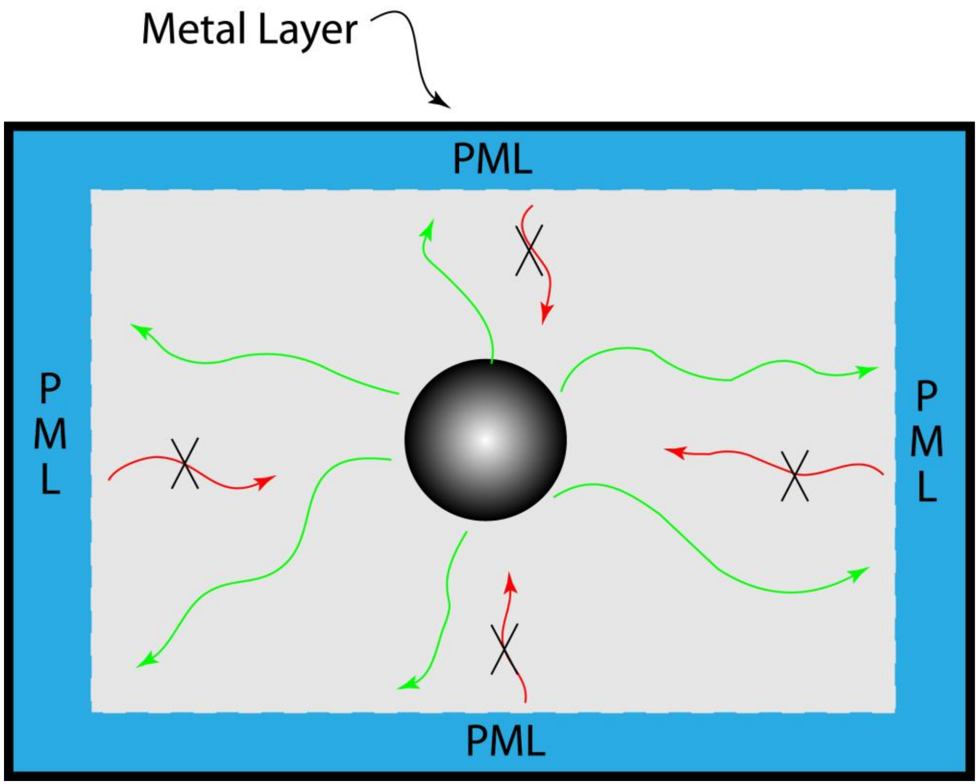
Perfectly Matched Layer (PML)

- Modes of an open optical micro-cavity radiate into surroundings
- PML acts as an artificial boundary to truncate the computation domain
- PML as an anisotropic absorber – modification of the diagonal permittivity and permeability tensors of the absorber



PML: Mathematical Details

Oxborrow's Master FEM Equation

$$\int_V \left((\vec{\nabla} \times \vec{\tilde{H}}^*)\epsilon^{-1}(\vec{\nabla} \times \vec{H}) - \alpha(\vec{\nabla} \cdot \vec{\tilde{H}}^*)(\vec{\nabla} \cdot \vec{H}) + c^{-2}\vec{\tilde{H}}^* \cdot \frac{\partial^2 \vec{H}}{\partial t^2} \right) dV$$

PML in cylindrical coordinates

$$\bar{\epsilon} = \epsilon\bar{\Lambda}, \bar{\mu} = \mu\bar{\Lambda},$$

$$\bar{\Lambda} = \left(\frac{\tilde{r}}{r}\right)\left(\frac{s_z}{s_r}\right)\hat{r} + \left(\frac{r}{\tilde{r}}\right)(s_z s_r)\hat{\phi} + \left(\frac{\tilde{r}}{r}\right)\left(\frac{s_r}{s_z}\right)\hat{z}$$

$$s_r = \begin{cases} 1 & 0 \le r \le r_m \\ 1 - jG\left(\frac{r - r_m}{t_p}\right)^2 & r > r_m \end{cases}$$

$$s_z = \begin{cases} 1 - jG\left(\frac{z_{ml} - z}{t_{zl}}\right)^2 & z < z_{ml} \\ 1 & z_{ml} \le z \le z_{mu} \\ 1 - jG\left(\frac{z - z_{mu}}{t_{zu}}\right)^2 & z > z_{mu} \end{cases}$$

$$\tilde{r} = \begin{cases} r & 0 < r < r_m \\ r - jG\left(\frac{(r - r_m)^3}{3t_r^2}\right) & r > r_m \end{cases}$$

AD Greenwood and JM Jin, IEEE Transactions on Antennas and Propagation, 47, 620-629 (1999).

Modified FEM Master Equation

$$\int_V \left((\vec{\nabla} \times \vec{\tilde{H}}^*)\bar{\epsilon}^{-1}(\vec{\nabla} \times \vec{H}) - \alpha(\vec{\nabla} \cdot \vec{\tilde{H}}^*)(\vec{\nabla} \cdot \vec{H}) + c^{-2}\vec{\tilde{H}}^* \cdot \bar{\mu} \cdot \frac{\partial^2 \vec{H}}{\partial t^2} \right) dV$$

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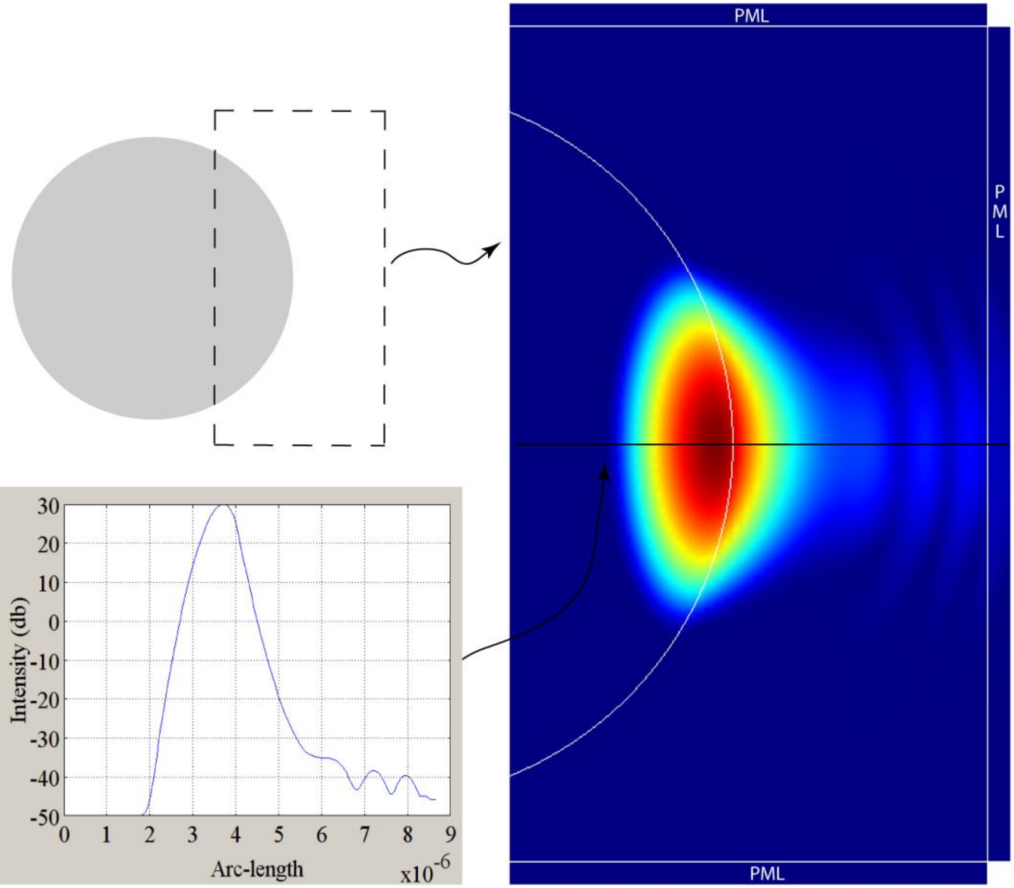
Our FEM model

- Full vectorial model – No transverse approximation
- PML along the computation box
- Important parameters can be extracted for various cavity geometries accurately:
  - Quality factors
  - Mode Volumes
  - Resonant Frequencies
  - Shapes of fundamental and higher order modes
- Quality factor of the disc resonator with the PML is

  1.60×10^7
  - No prior knowledge of the mode frequency is required
  - Quality factor for all modes simultaneously
  - One time execution of the model



Fundamental TE mode of a 12 microns silica micro-sphere in air (False Colors)

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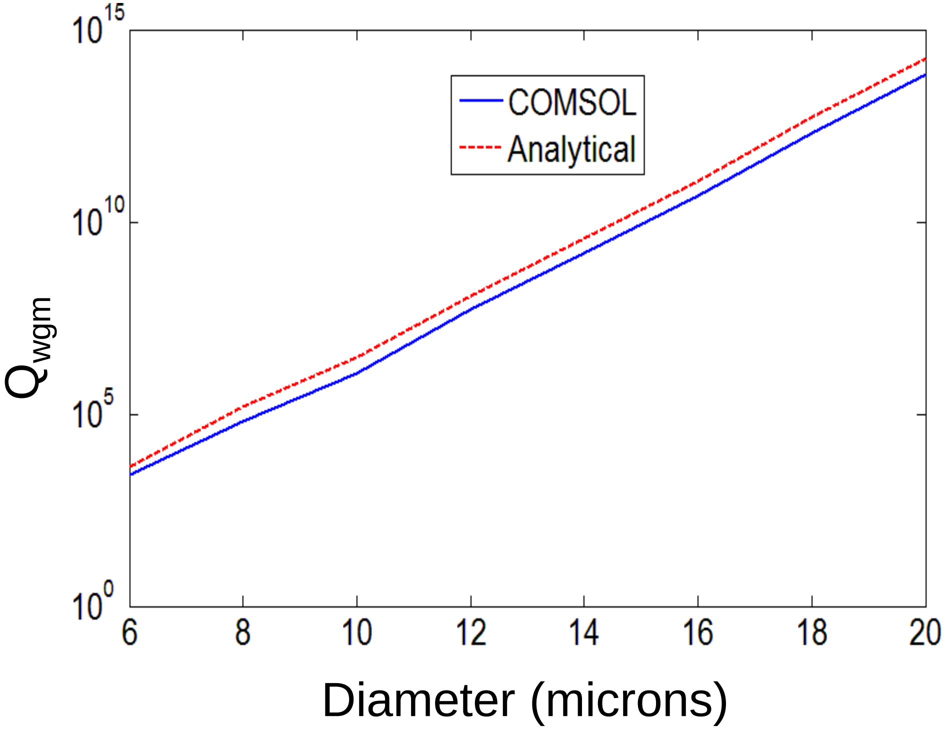
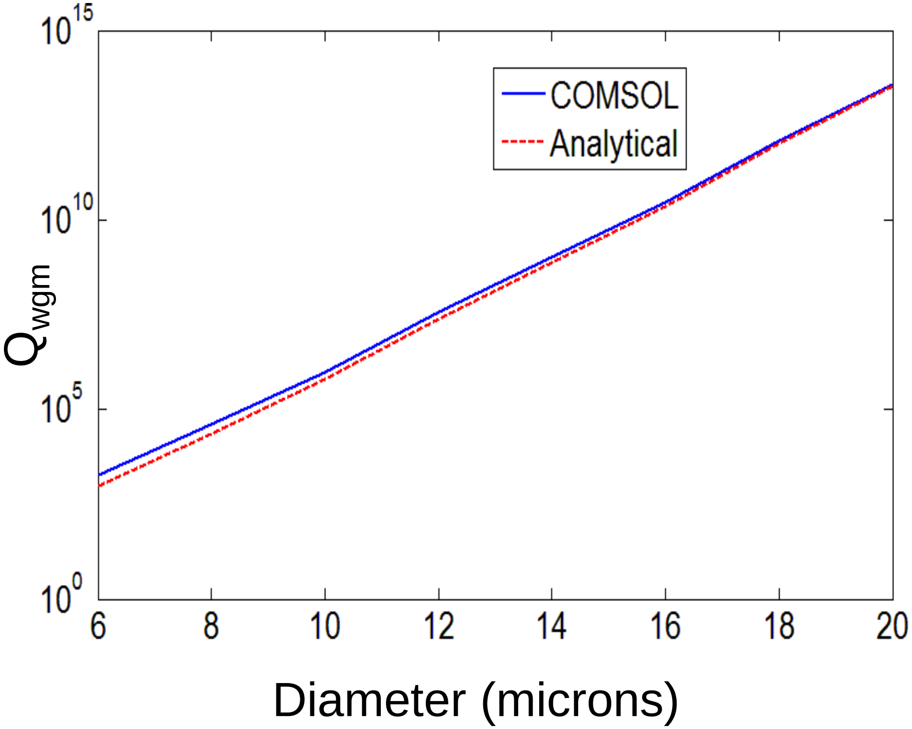
Results: silica microsphere



Quality factor of fundamental TE modes at 850nm



Quality factor of fundamental TM modes at 850nm

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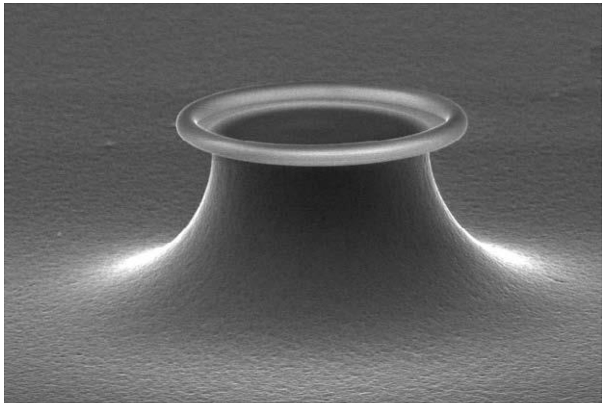
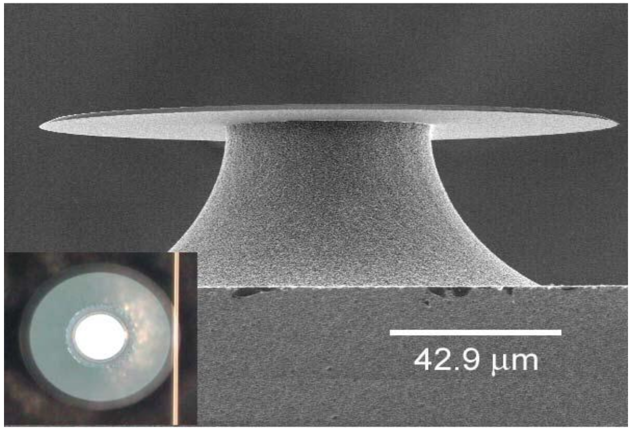
Conclusions

- Excellent agreement between the simulation and analytical results
- Third generation model - No transverse approximation and with PML
- Model is applicable to any axisymmetric micro-cavity geometries such as discs and toroids





T.J. Kippenberg, Ph.D. thesis, CALTECH (2004)