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Comparison of 2D Conduction Models for Vertical Ground Coupled Heat Exchangers

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Abstract: The effect of the infinite length approximation on evaluating the temperature T_b of the external surface of Borehole Heat Exchangers (BHEs) is determined by means of COMSOL Multiphysics (© COMSOL AB.). In detail, two 2D models of a BHE are compared: in the first model, the computational domain is represented by a cross-section of the geometry, where the BHE is replaced by a hole subjected to a constant heat flux. In the second model, the computational domain is represented by an axial-section of the geometry and, thus, the actual length of the BHE is considered. Also in this case, the internal structure of the BHE is not modeled and a constant heat flux on its surface is prescribed. The discrepancies in T_b due to these two models are investigated as a function of time and a dimensionless correlation is proposed to estimate the error made by neglecting the BHE length.

Keywords: Borehole Heat Exchanger (BHE), Finite element simulations, 2D conduction models.

1. Introduction

Heat pump systems coupled to ground heat exchangers (GCHPs) benefit from the fact that, despite the seasonal change of air temperature, the ground temperature at a depth of about 10 m remains almost constant throughout the year and close to the average annual temperature at the surface, with a further increase with depth due to the geothermal gradient (about 3°C in 100 m). This temperature stability ensures to GCHPs better performance, lower operating costs and reduced maintenance compared to conventional air-to-air heat pump systems and fossil-fuelled equipment, also helping in a reduction of CO_2 emissions.

A wide market penetration of GCHPs, and in particular of systems based on vertical ground heat exchangers (Borehole Heat Exchangers, BHEs), needs proper modeling and design criteria. In fact, since the BHE installation strongly affects the overall plant cost, the correct sizing of ground heat exchangers is a compulsory task in order to achieve the financial sustainability of the plant.

The main analytical models to determine the BHE thermal performance are based on the Infinite Line Source theory (ILS) [1] and on the Infinite Cylinder Source theory (ICS). Both models do not account for the real length of the BHE. The cylinder source method, introduced by Carslaw and Jaeger [2], developed by Deerman and Kavanaugh [3] and later refined by Kavanaugh and Rafferty [4], assumes the BHE as an infinite cylinder surrounded by a homogeneous medium with constant properties and considers heat transfer by pure conduction. It provides an analytical solution which depends both on time and distance from the cylindrical source. This model has been adopted as reference in the ASHRAE Handbook since 2003 [5] and, thus, it is commonly used.

On the other hand, the approximation made by neglecting the real length of the BHE in these models deserves attention. It is well known that ICS leads to relevant error when investigating long-term effects; indeed, the ASHRAE method introduces the correction factor called 'temperature penalty' [6, 7].

Another model, called Finite Line Source model (FLS), takes into account the finite length of the BHE and considers the ground surface as a boundary [8].

Philippe et al. [9] compare the results obtained using three analytical solutions (ILS, ICS and FLS) to study unsteady heat transfer around a single BHE and determine their best application ranges. The Authors suggest the use of ICS for time intervals up to 34 hours, as the thermal effects occurring at the extremities of the BHE are negligible in the first working hours.

In the present paper, two 2D numerical models to study the thermal performance of a BHE are compared, by means of the software package COMSOL Multiphysics (© COMSOL AB.). The first model considers a cross-section of the BHE and neglects axial heat conduction effects, as in the ICS analytical model; the second model represents the BHE by means of its axial-section. In both cases, the internal structure of the BHE is not modeled and a constant heat flux on its surface is prescribed. The discrepancies in the temperature T_b of the BHE external surface due to these two models are investigated as a function of time with reference to some values of the BHE length, of the ground thermal conductivity and diffusivity, and of the prescribed heat flux.

Finally, an overall dimensionless correlation is proposed to obtain an estimate of the approximation made by neglecting the BHE real length.

2. Numerical model

Let us consider a BHE, having an external radius $r_b = 0.05$ m, that is placed in a homogeneous soil with an undisturbed ground temperature T_g = 14°C.

When the long-term performance of BHEs is investigated, 3D models can become very demanding in terms of computational effort. Thus, in this case it is a common practice to consider 2D numerical models representing a section of the real geometry. Accordingly, in order to study the BHE long-term performance, let us assume only the ground as computational domain, *i.e.* let us replace the BHE with a hole subjected to a constant heat flux per unit length, q'.

Let us first consider a 2D cross-section of the geometry, where the BHE is centered in a square portion of ground with a side of 800 m (see Fig. 1a). Then, let us consider a 2D axial-section of the geometry, where the BHE has a length H and the ground is a cylinder, coaxial with the BHE, with a radius of 400 m and a length of 400 m (see Fig. 1b). In this case, three different lengths H of the BHE are considered (50, 100 and 150 m).

Let us assume that groundwater movement has negligible effects, so that the problem under exam reduces to pure conduction heat transfer. Thus, the equation to be solved in the ground is the unsteady Fourier equation without heat generation

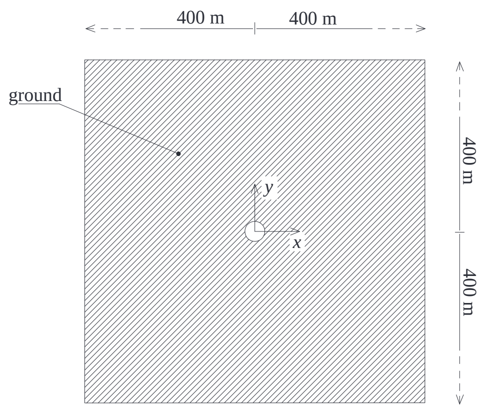
$$\rho_g \, c_g \frac{\partial T}{\partial \tau} = k_g \, \nabla^2 T \qquad (1)$$

where ρ_g, c_g and k_g are the density, the specific heat capacity and the thermal conductivity of the ground.

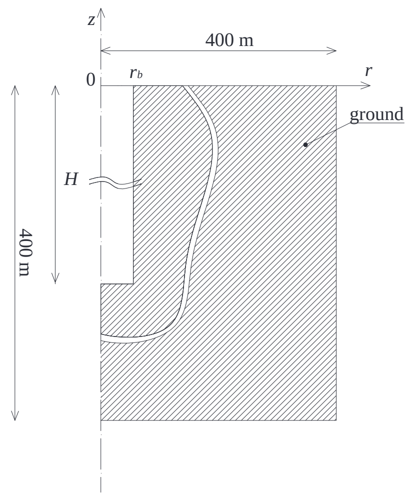
Adiabatic conditions at the ground external boundaries are imposed, while the boundary of the BHE is subjected to constant heat flux q', *i.e.*,

$$\vec{n} \cdot \left(k_g \nabla T \right) = \frac{q'}{2\pi r_b} \quad , \qquad (2)$$

where $\vec{n}$ is the outward normal unit vector.



(a)



(b)

Figure 1. Computational domains for 2D models of the BHE: (a) cross-section; (b) axial-section.

Table 1. Considered values of the main parameters.

Heat load q' [W/m]	Ground thermal conductivity k_g [W/m K]	Ground thermal diffusivity α_g [m^2/s]	BHE length H [m]
20; 40; 60	1.5; 2; 3	(0.5; 1; 1.5)·10^{-6}	50; 100; 150

Let us assume that the physical properties of the ground are temperature independent and that the heat load q' is a positive quantity if heat is supplied by the BHE and collected by the ground. Three values of the thermal conductivity k_g and thermal diffusivity $\alpha_g = k_g / (\rho_g\, c_g)$ of the ground are considered; moreover, three different values of q' are prescribed. Table 1 sums all the investigated cases; the total number of simulations performed is 108.

The mean temperature T_b of the boundary surface of the hole that replaces the BHE is evaluated as

$$T_b(\tau) = \frac{1}{2\pi r_b}\int_s T dl \quad , \tag{3}$$

for the BHE cross-section model and as

$$T_b(\tau) = \frac{1}{H}\int_s T dl \quad , \tag{4}$$

for the BHE axial-section model. In Eqs.(3) and (4), s is the proper BHE boundary surface.

A period of 100 years of operation is considered and non-uniform time steps are adopted in computations: 60 s for $0 < \tau < 3600\,\text{s}$, 3600s for $3600\,\text{s} < \tau < 86400\,\text{s}$, 86400s for $86400\,\text{s} < \tau < 2592000\,\text{s}$, 2592000s for $2592000\,\text{s} < \tau < 31536000\,\text{s}$, 31536000s for $31536000\,\text{s} < \tau < 3153600000\,\text{s}$.

In order to ensure the mesh independence of results, three unstructured meshes made of triangular elements are tested for each studied configuration. A comparison of the values of T_b obtained with different meshes at selected time instants is carried out. For the BHE cross-section model, the considered meshes have a number of elements in the range 28620-114480; for the BHE axial-section model, with H = 50 m, the considered meshes have a number of elements in the range 25909-103636; for the case H = 100 m, the range is 31630-126520; for the case H = 150 m, the range is 36902-147608.

Table 2 presents the mesh independence test for the BHE axial-section model, in the case H = 150 m, k_g = 2 W/m K, α_g = 1·10^{-6} m^2/s, q' = 40 W/m. In detail, the difference $T_b - T_g$ between the BHE boundary temperature and the undisturbed ground temperature is reported in Table 2, with reference to some selected time instants. The Table shows that the mesh made of 86948 elements gives results that are in good agreement with those obtained with the last finer mesh made of 147608 elements. Therefore, as a compromise between accuracy and computational effort, the mesh made of 86948 triangular elements is adopted for final simulations. On the other hand, for the case H = 50 m, the adopted mesh has 41860 triangular elements, while for the case H = 100 m, the adopted mesh has 64288 triangular elements. Finally, for the BHE cross-section model, the adopted mesh has 60410 triangular elements.

Table 2. Mesh independence, for the BHE axial-section model, in the case H = 150 m, k_g = 2 W/m K, α_g = 1·10^{-6} m^2/s, q' = 40 W/m.

mesh elements	**36902**	**86948**	**147608**
τ [s]	**$T_b - T_g$ [°C]**		
3600	2.7247	2.7687	2.7852
86400	6.6932	6.7726	6.7824
2592000	11.9223	11.9783	12.0804
31536000	15.8990	15.9976	16.0454
1.58E+08	18.4479	18.4772	18.5231
3.15E+08	19.4412	19.5296	19.5468
1.58E+09	21.8928	21.9553	21.9787
3.15E+09	22.8875	22.9501	22.9692

3. Results

In Ref. [10], the time evolution of the mean temperature T_b on the BHE surface, obtained by means of a numerical simulation of the BHE cross-section, is compared with that evaluated through the analytical solution of the ICS model. The results of the comparison, illustrated in Fig. 2, show an excellent agreement between the analytical solution and the numerical solution, with a mean standard deviation of 0.027°C. Thus, the adopted 2D cross-section model of the BHE is a very good approximation of the ICS analytical model.

Moreover, according to Ref. [11], a symmetric axial-section numerical model of the BHE agrees with the finite line source (FLS) model almost perfectly.

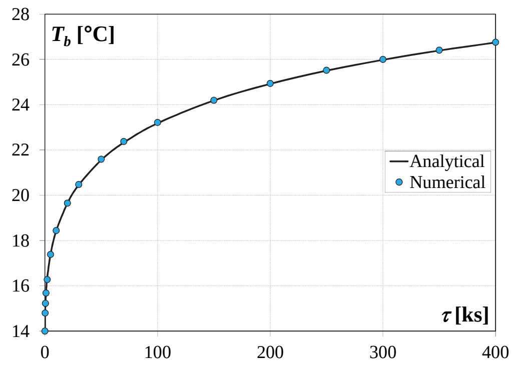


Figure 2. Comparison of the numerical BHE cross-section model with the ICS analytical solution [10].

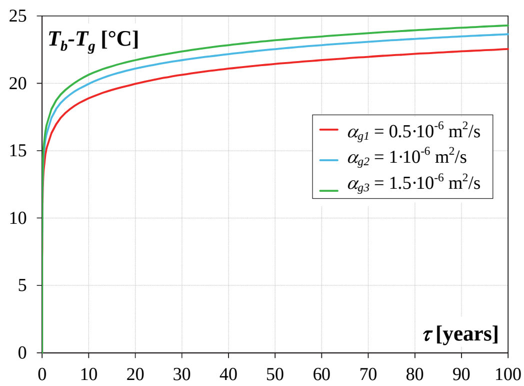


Figure 3. Plot of $T_b - T_g$ as a function of time for the BHE cross-section model, in the case $k_g = 2$ W/m K, $q' = 40$ W/m.

As described in the previous section, 108 simulations are performed. For each simulation, the difference between the temperature T_b of the BHE boundary and the undisturbed ground temperature T_g is determined as a function of time. An example of the plot of $T_b - T_g$ for the BHE cross-section model, in the case $k_g = 2$ W/m K, $q' = 40$ W/m, is represented in Fig.3. The Figure shows that $T_b - T_g$ is an increasing function of the ground thermal diffusivity α_g.

The aim of the present study is to estimate the error introduced by using a 2D cross-section numerical model of the BHE (approximation of the ICS model) instead of using a 2D axial-section model (approximation of the FLS model) which considers the real length of the BHE. Thus, let us define a parameter ΔT_b as

$$\Delta T_b = T_{b\ cs} - T_{b\ H} \quad , \tag{5}$$

where $T_{b\ cs}$ is the temperature of the BHE boundary evaluated with the cross-section model, whereas $T_{b\ H}$ is that evaluated with the axial-section model.

To generalize the results, one can introduce the non-dimensional parameter $\left(\Delta T_b \cdot k_g\right)/q'$ and the Fourier number Fo_H, defined as:

$$Fo_H = \frac{\alpha_g \cdot \tau}{H^2} \quad . \tag{6}$$

A plot of $\left(\Delta T_b \cdot k_g\right)/q'$ versus $\ln(9 \cdot Fo_H)$ for all the simulated cases, is represented in Fig. 4. The choice of the parameter on the *x*-axis originates from Eskilson's hybrid approach [12].

The data given by numerical simulations for all the considered values of the ground thermal conductivity and diffusivity, of the heat load per BHE unit length and of the length of the BHE, can be interpolated by the following polynomial function:

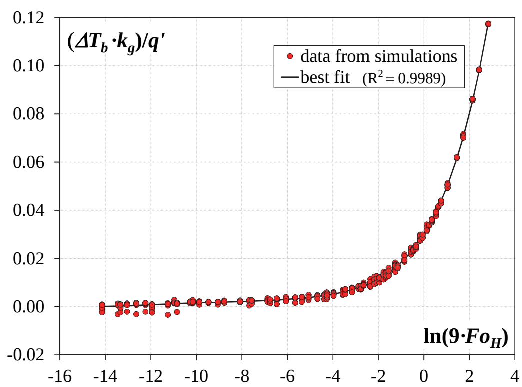


Figure 4. Collected data and best fit curve.

$$\frac{\Delta T_b \cdot k_g}{q'} = 3.01450 \cdot 10^{-2} + 1.49739 \cdot 10^{-2} \cdot x + \\ +3.71898 \cdot 10^{-3} \cdot x^2 + 5.33423 \cdot 10^{-4} \cdot x^3 + \\ +4.49233 \cdot 10^{-5} \cdot x^4 + 2.05832 \cdot 10^{-6} \cdot x^5 \\ +3.92593 \cdot 10^{-8} \cdot x^6 \quad , \tag{7}$$

where $x = \ln(9 \cdot Fo_H)$. The curve described by Eq.(7) is plotted as a solid line in Fig.4.

The non-dimensional parameter $\left(\Delta T_b \cdot k_g\right)/q'$ is an increasing function of time and ground diffusivity, while is a decreasing function of the BHE length. Similarly, Marcotte et al. [11] report that the importance of axial effects and the discrepancy between infinite and finite line source models increase with a Fourier number defined as $\alpha_g \cdot \tau / r_b^2$.

For given values of H, q', k_g and α_g, Eq. (7) allows one to determine, at different time instants, the error made by neglecting the real length of the BHE when using the cross-section numerical model (and, roughly, also the ICS model).

4. Conclusions

Two 2D numerical models to study the thermal performance of a BHE have been compared in order to estimate the error introduced by neglecting the real BHE length. According to the first model, that approximates the Infinite Cylindrical Source analytical model, the BHE is sketched by means of its circular cross-section, where the internal structure is neglected and the external boundary is subjected to a constant heat flux per unit length. On the other hand, the second model considers the axial-section of the BHE and, thus, accounts for the length of the BHE. This model, where the real structure of the BHE is again neglected by prescribing the heat flux directly on the external boundary, is a rough approximation of the Finite Line Source analytical model.

The software package COMSOL Multiphysics (© COMSOL AB.) has been used to perform several simulations with both models, by considering different values of the ground thermal conductivity and diffusivity, of the prescribed heat flux and of the BHE length. The results have been analyzed by introducing non-dimensional parameters and an overall interpolating function has been found. This interpolating function allows the estimate of the error introduced by neglecting the BHE length with the first model, as a function of time.

5. References

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