

# Turbulent Compressible Flow in a Slender Tube

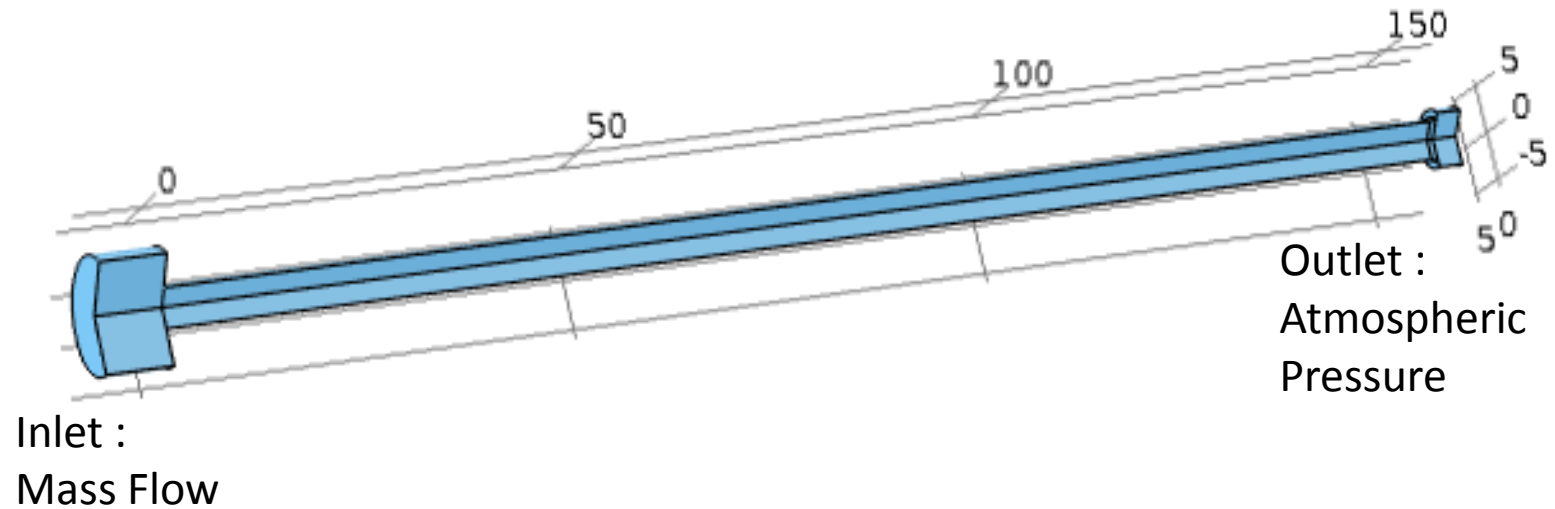
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Excerpt from the Proceedings of the 2012 COMSOL Conference in Boston

- compressible air-flow,
- COMSOL k- $\epsilon$  turbulence model,
- scalar integration variable,
- specified mass flow,
- experimental data,
- choked flow conditions,
- analytical approximation,

# Test Section Geometry (4.2 x 150 mm Tube)



## Scalar Integration Variable

**Global Equations**

$$f(u, u_t, u_{tt}, t) = 0, \quad u(t_0) = u_0, \quad u_t(t_0) = u_{t0}$$

Name	$f(u, u_t, u_{tt}, t)$	Initial value (u...)	Initial value (u...)
Uin	intop1(spf.rho*w) - mdot	0	0
		0	0

## Setting of Variables

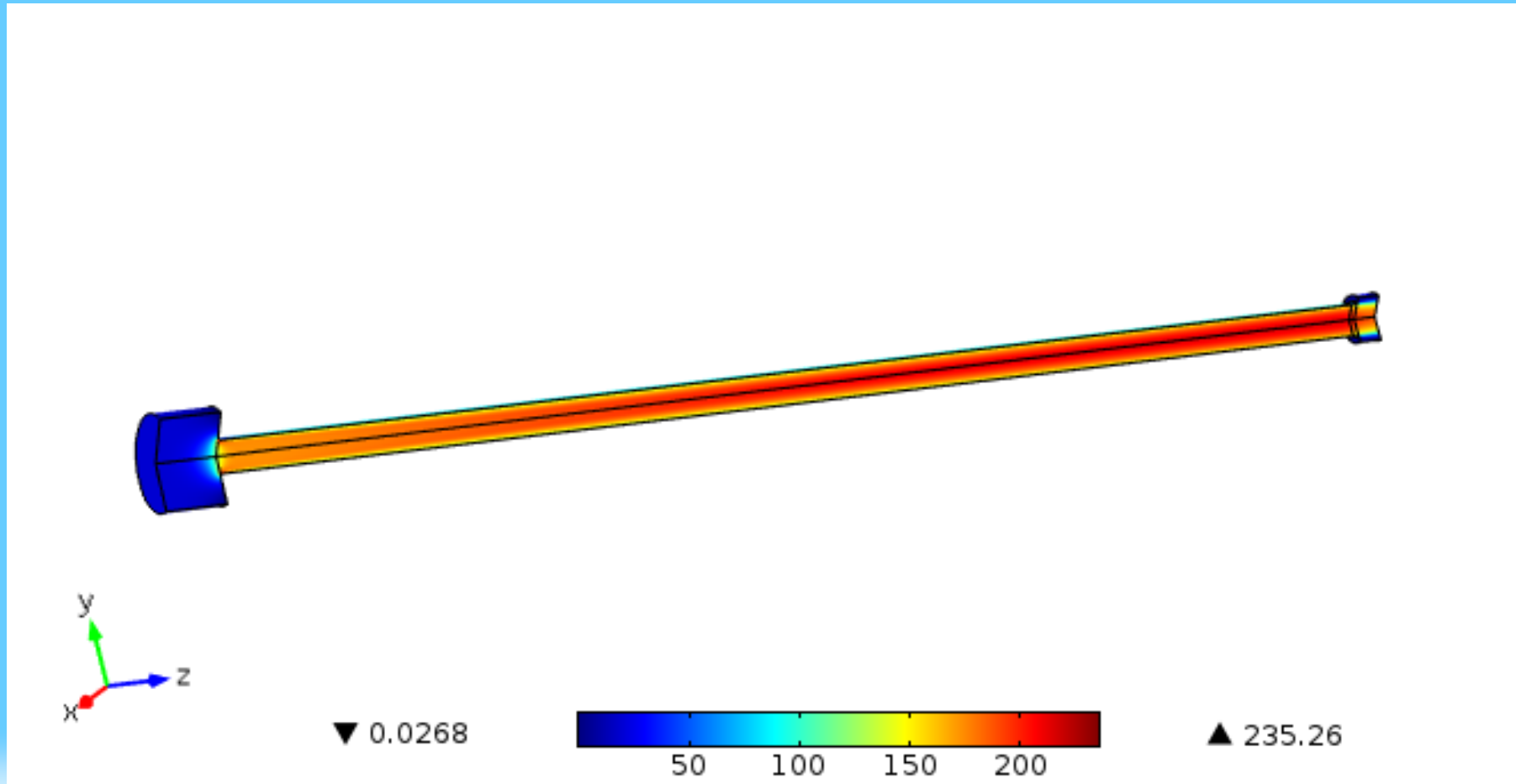
**Geometric Entity Selection**

Geometric entity level: Entire model

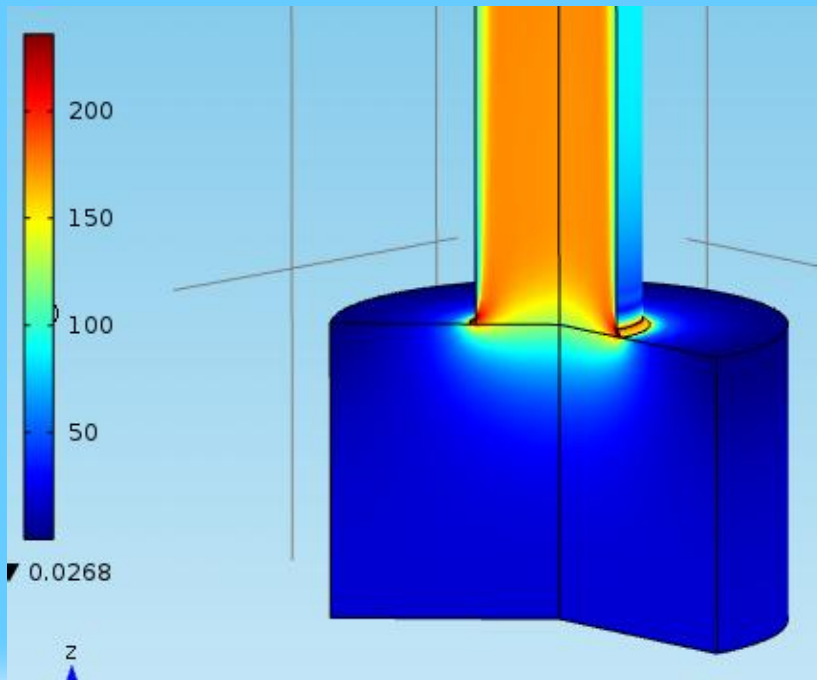
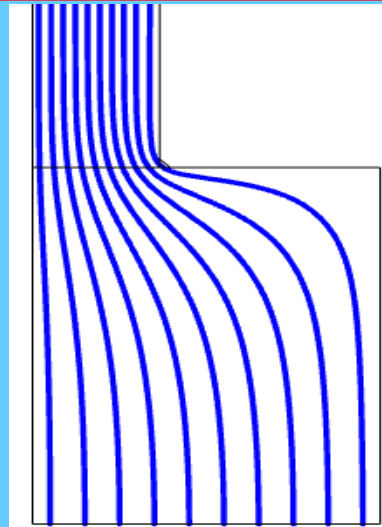
**Variables**

Name	Expression	Unit	Description
gam	1.4		Cp/Cv
T	$T_{in} * (p/p_{in})^{((gam-1)/gam)}$	K	Temperature
rho	$p / (R_{gas} * T)$	kg/m <sup>3</sup>	Density
c	$\sqrt{gam * R_{gas} * T}$	m/s	Speed of Sound
Ma	w/c		Mach number

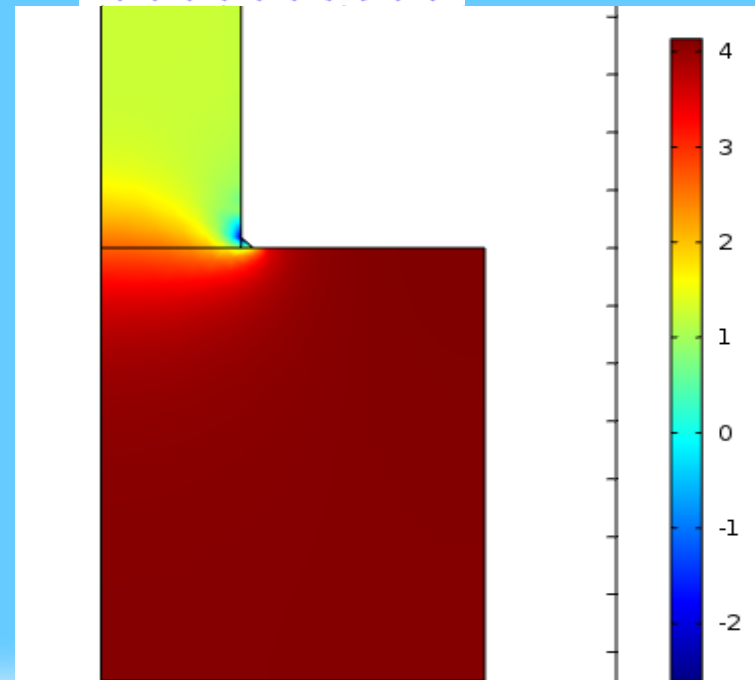
# Axis-Symmetric COMSOL Model (velocity [m/s] @ 10 kg/h mass flow)



# Inlet Conditions (@ 10 kg/h mass flow)

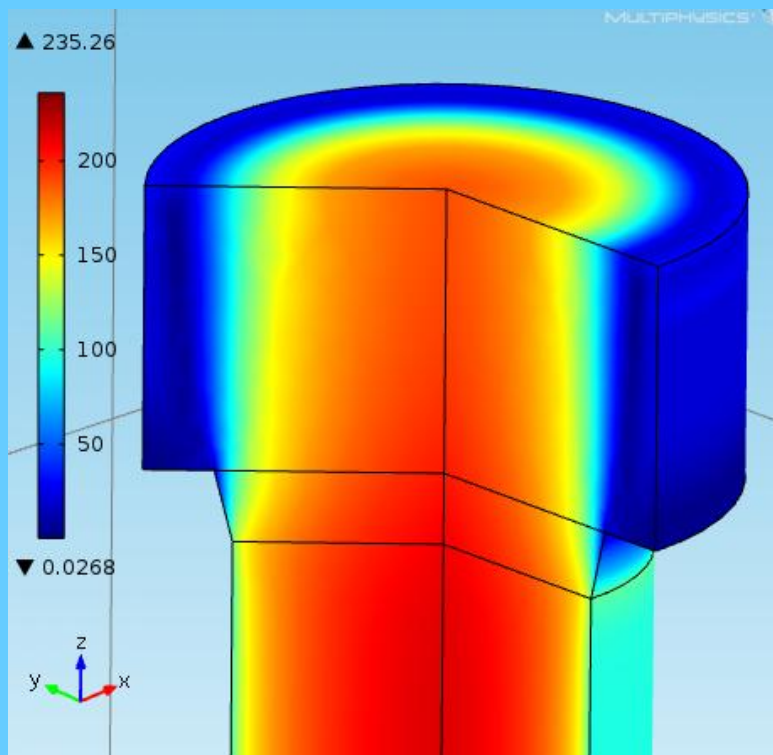


velocity [m/s]

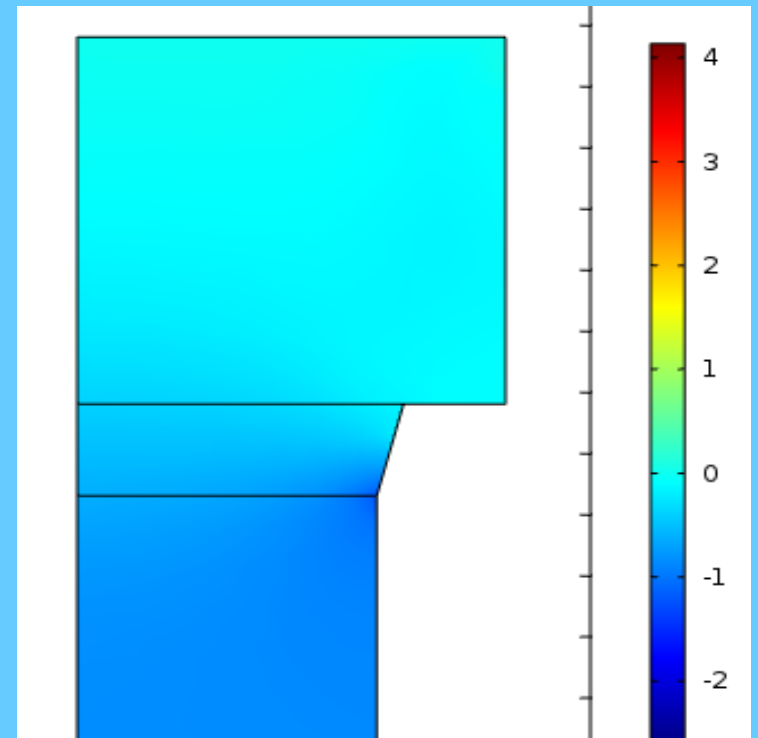


Pressure [psig]

# Outlet Conditions (@ 10 kg/h mass flow)

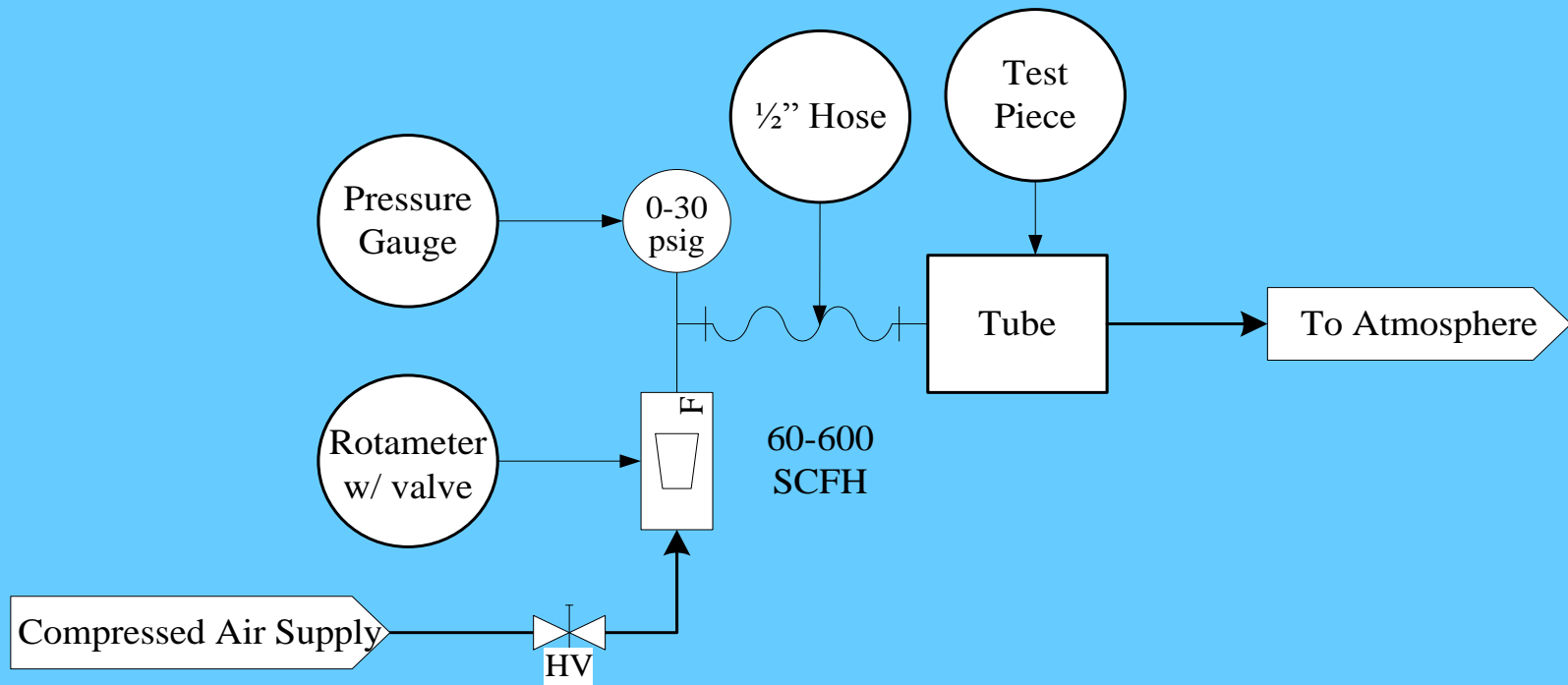


Flow Separation



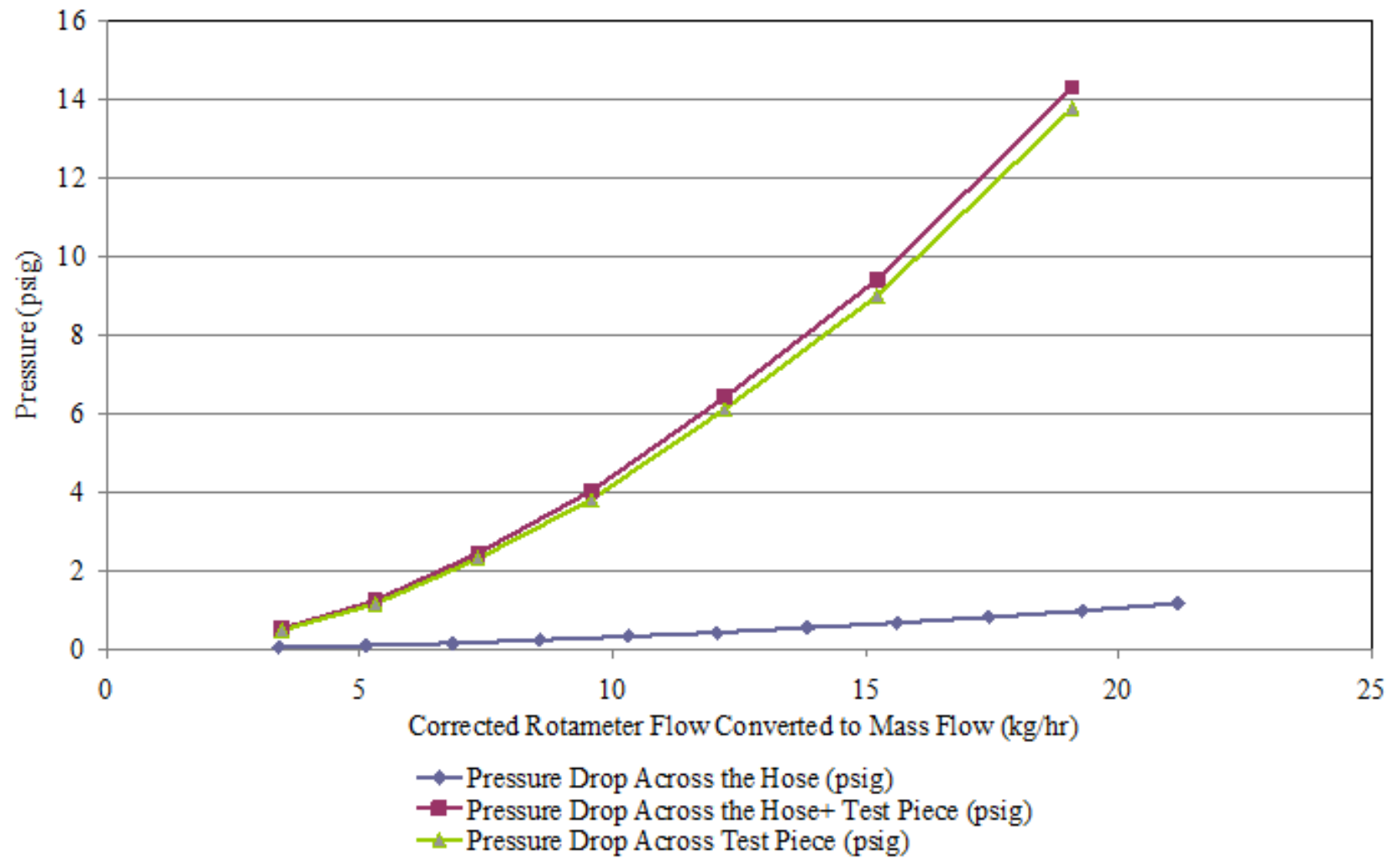
Pressure Recovery

# Experimental Setup

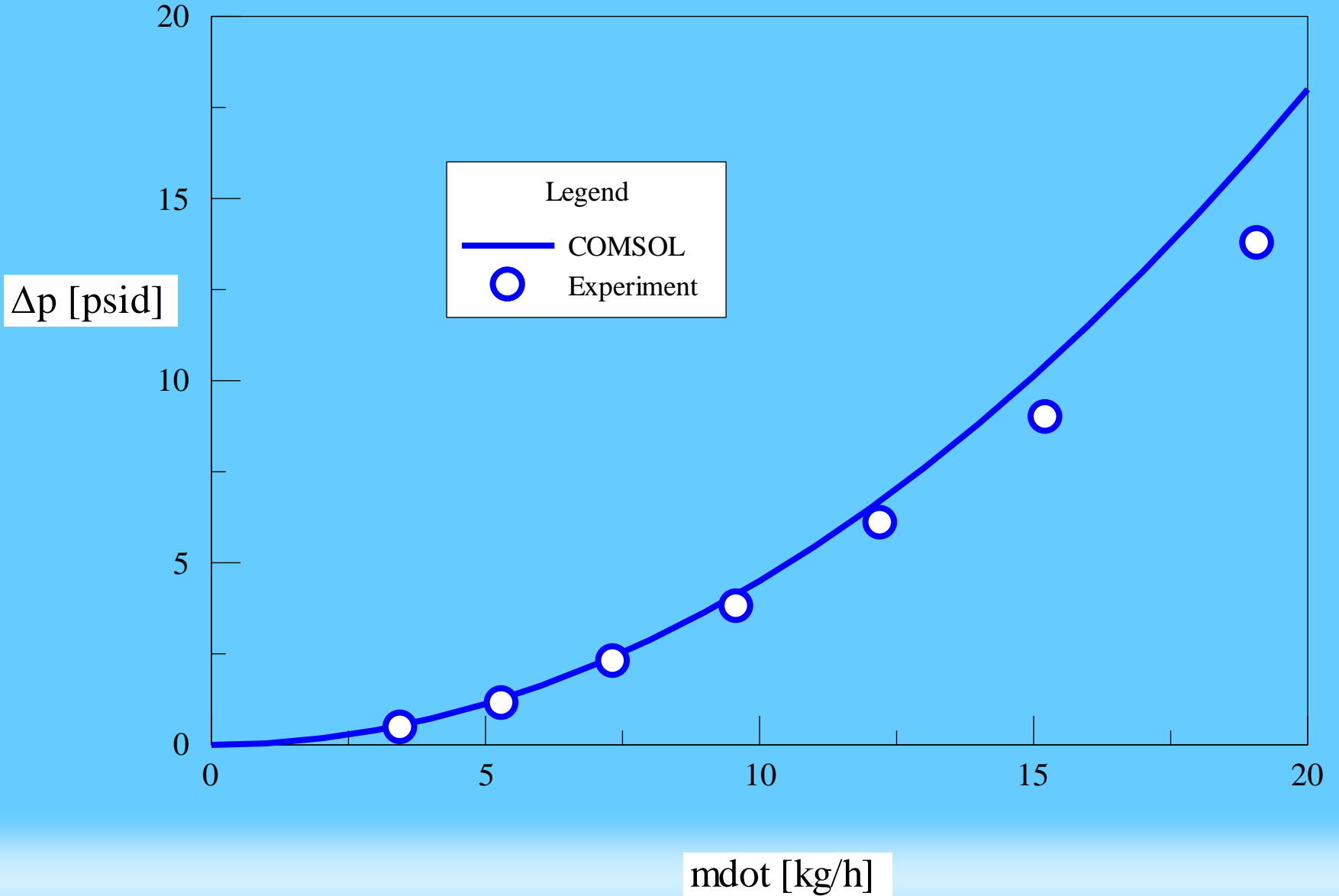




# Experimental Data

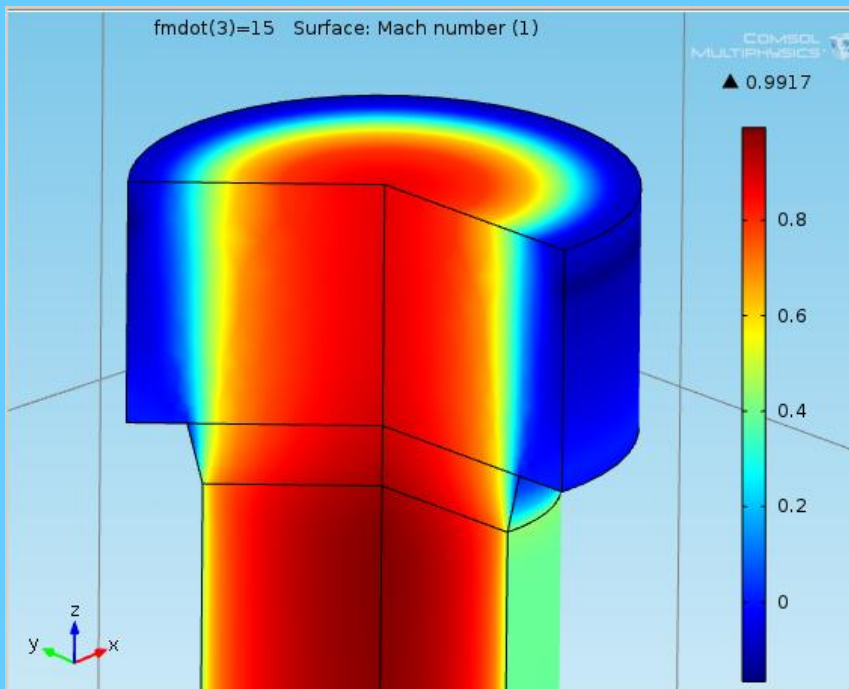


# Comparison of Results

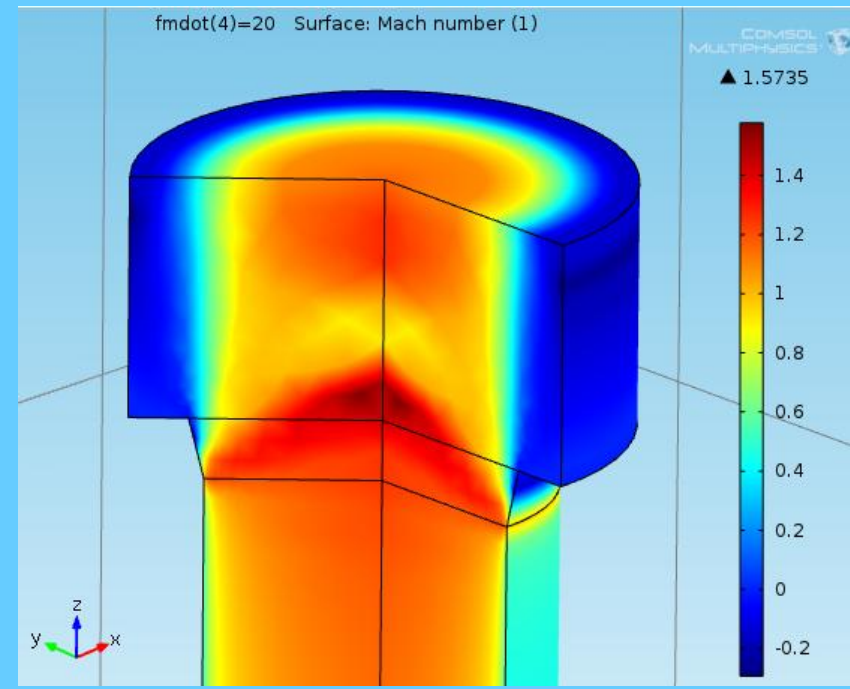


# Outlet Mach Numbers

15 kg/h



20 kg/h



# One-dimensional Theory.

$$\frac{\rho}{\rho_0} = \left(\frac{p}{p_0}\right)^{1/\gamma}$$

$$\frac{T}{T_0} = \left(\frac{p}{p_0}\right)^{(\gamma-1)/\gamma}$$

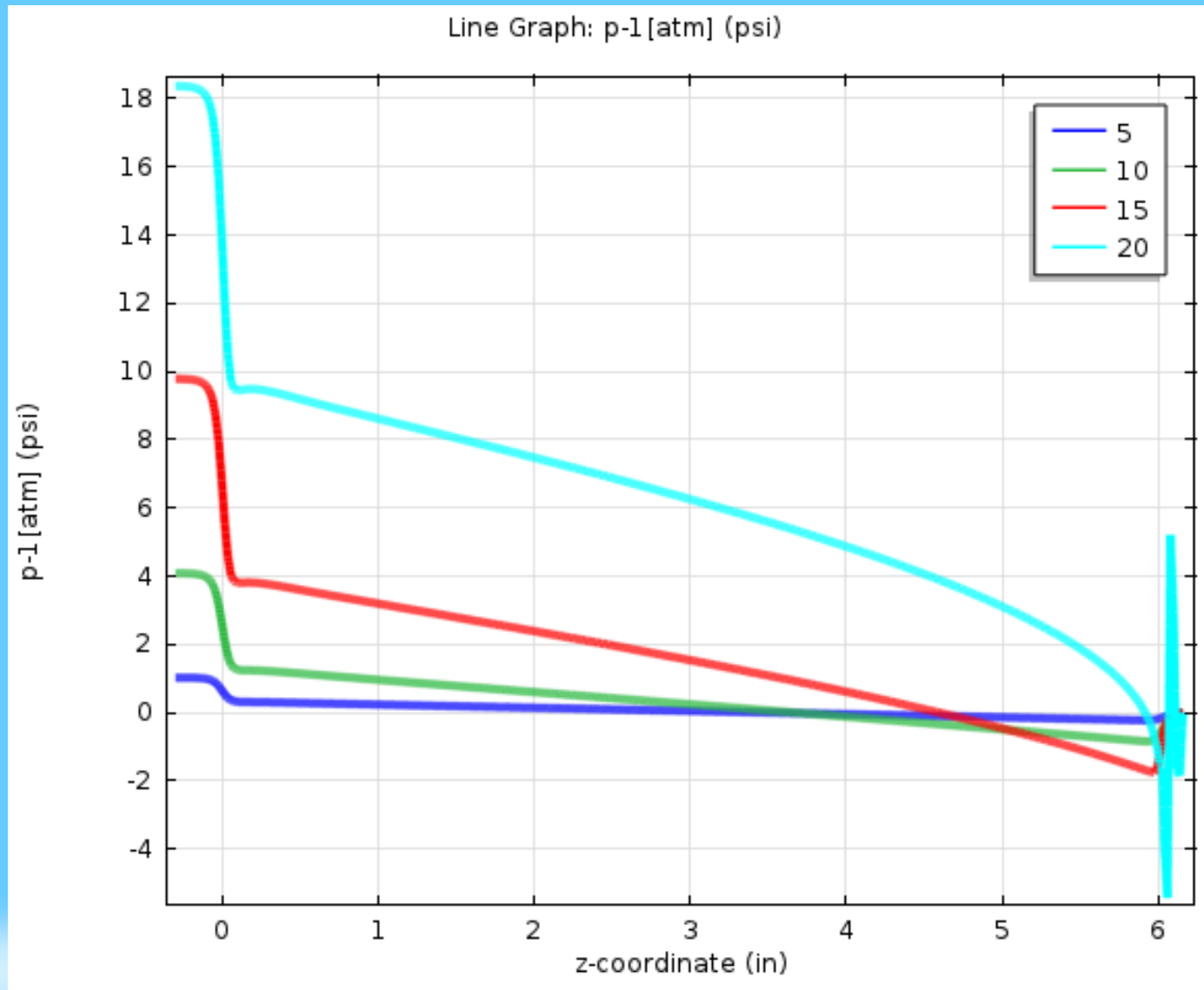
$$-\rho \frac{dp}{dx} = \frac{1}{2} \rho^2 V^2 f \frac{1}{D} = \frac{1}{2} G^2 f \frac{1}{D}$$

$$f = \frac{0.3164}{Re^{1/4}} = 0.3164 \left(\frac{\mu}{GD}\right)^{0.25}$$

$$-\frac{T_L}{T_0} + \frac{T_\delta}{T_0} \equiv -\left(\frac{p_L}{p_0}\right)^{\frac{\gamma-1}{\gamma}} + \left(\frac{p_\delta}{p_0}\right)^{\frac{\gamma-1}{\gamma}} = \frac{\gamma-1}{\gamma} \frac{G^2}{2\rho_0 p_0} f \frac{L-\delta}{D}$$

$$T_L = T_\delta - \frac{\gamma-1}{\gamma} \frac{V_0^2}{2R} f \frac{L-\delta}{D} = T_\delta - \frac{1}{2} \frac{\rho_0 V_0^2}{\rho_0 C_p} f \frac{L-\delta}{D}$$

# COMSOL Centerline Axial Pressure Profiles



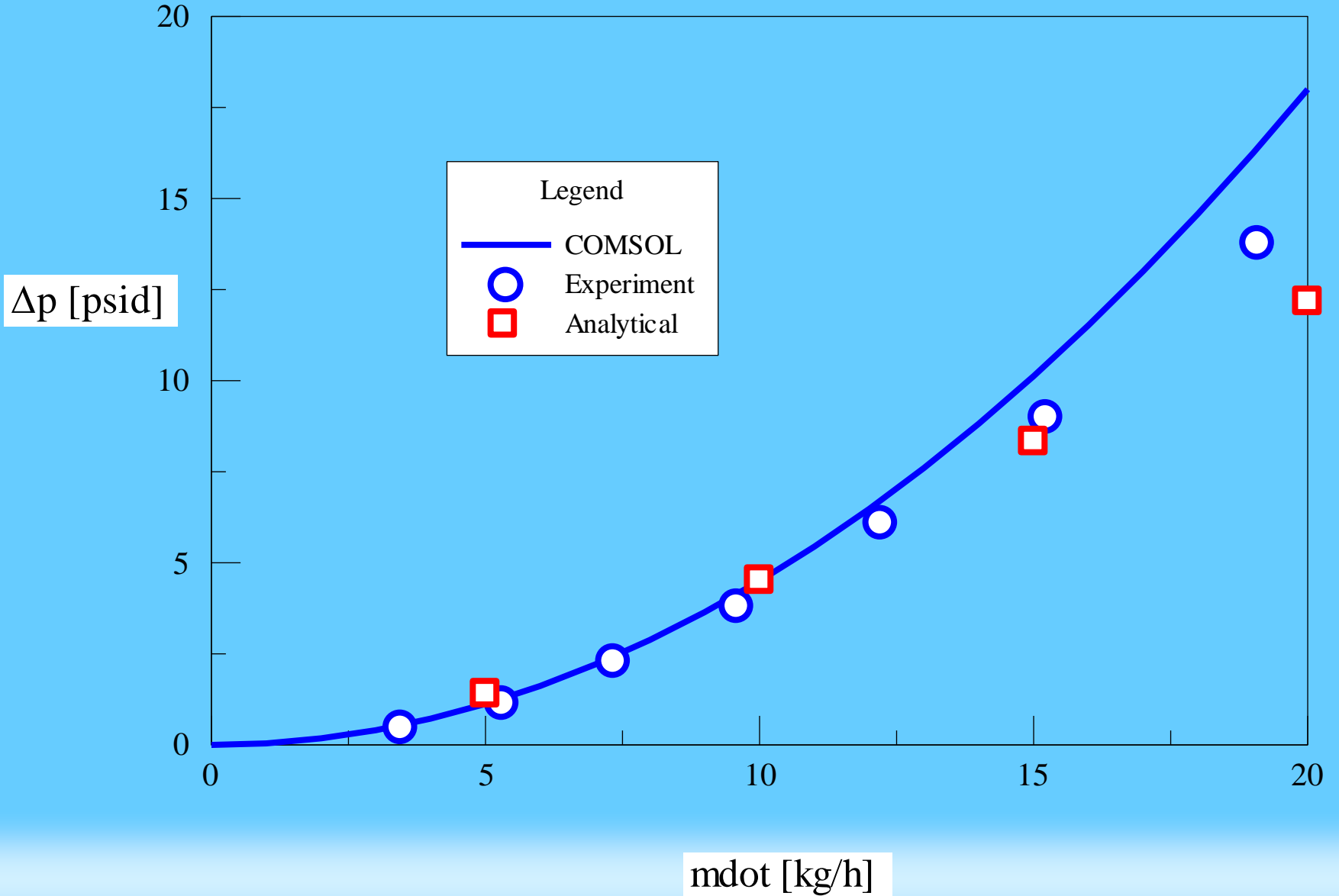
$$\Delta p_{ex} = p_L - p_{out} = \frac{1}{2} \rho_L V_L^2 (K_{ex} - 1) = \frac{G^2}{2\rho_L} (K_{ex} - 1)$$

$$\Delta p_{in} = p_{in} - p_\delta = \frac{1}{2} \rho_\delta V_\delta^2 (1 + K_{in}) \approx \frac{G^2}{2\rho_{in}} (1 + K_{in})$$

By comparison with the COMSOL data for 15 kg/h

$$K_{in} = 0.70 \text{ and } K_{out} = 0.65.$$

# Comparison of Results





Thank you