Turbulent Compressible Flow in a Slender Tube

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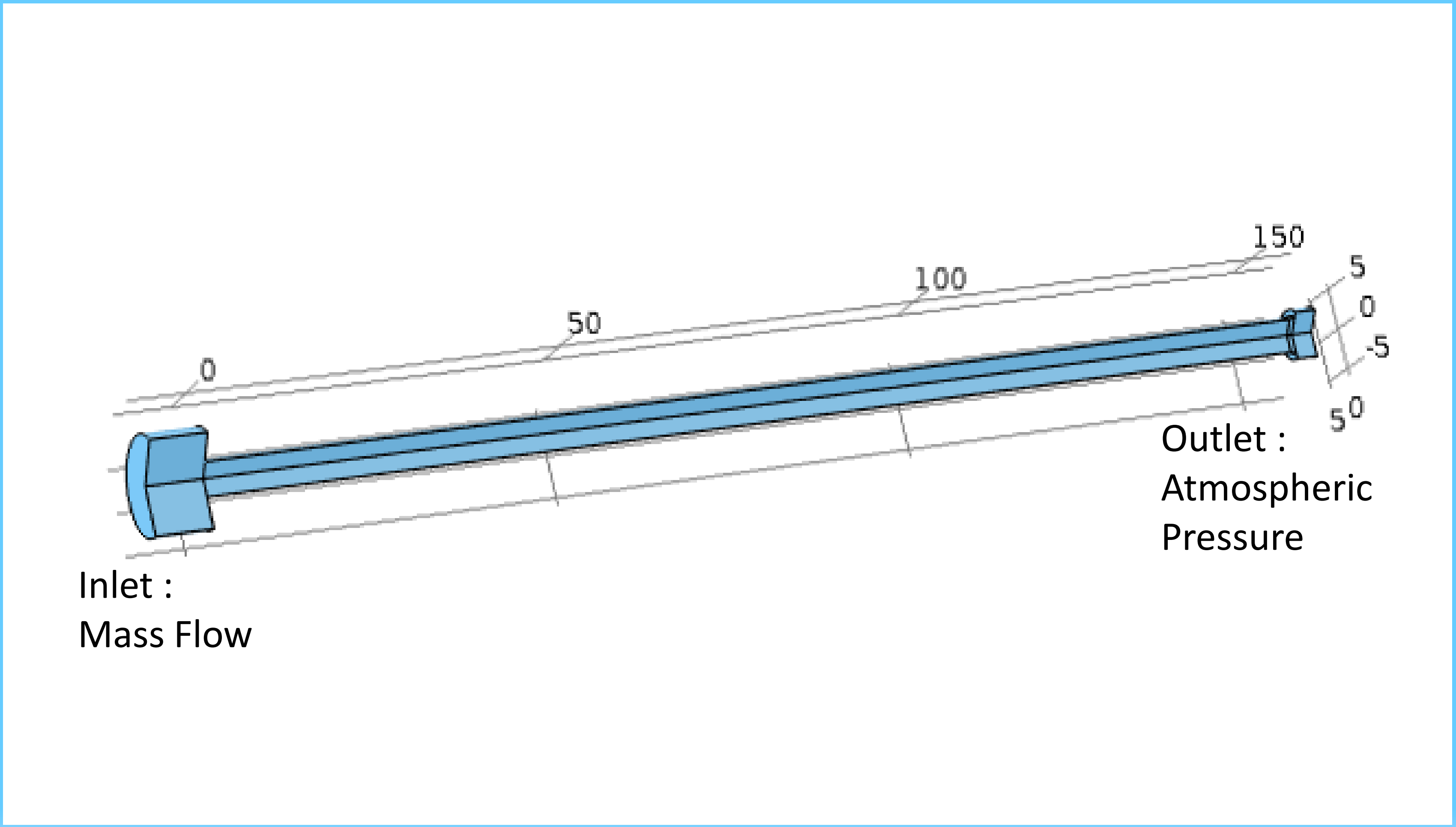
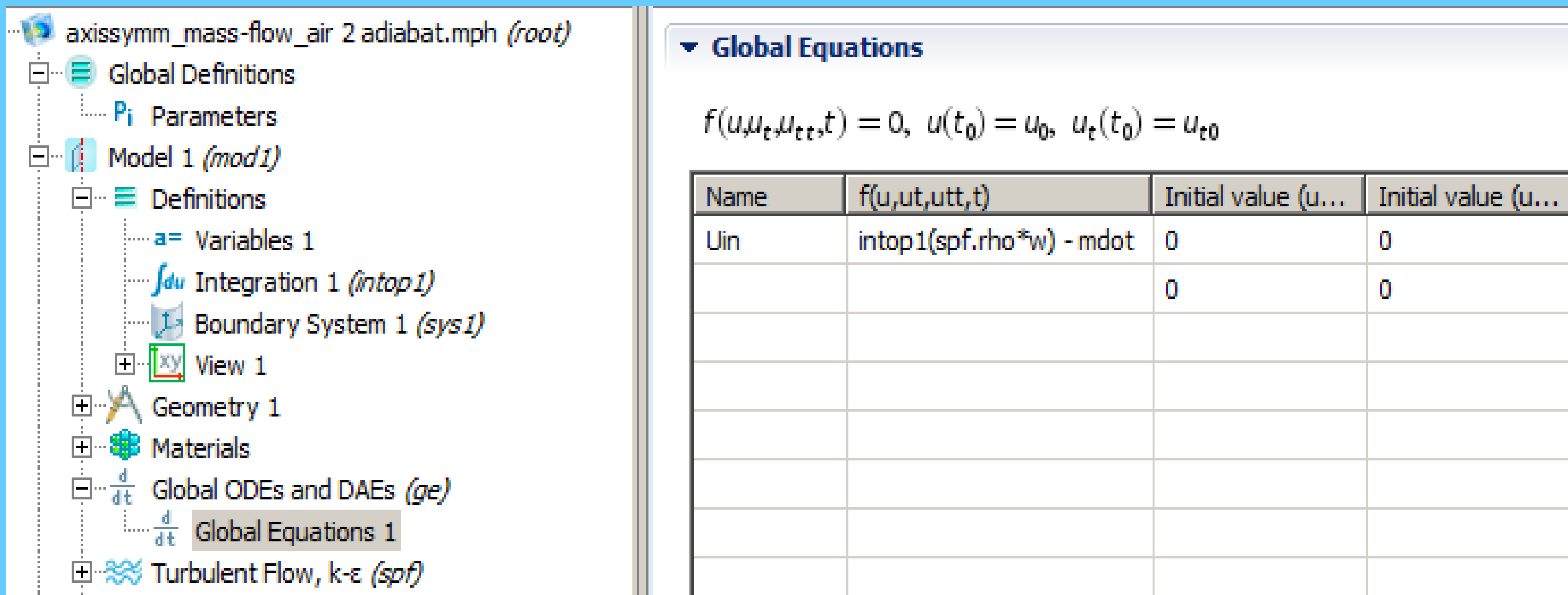
- compressible air-flow,
- COMSOL k-ε turbulence model,
- scalar integration variable,
- specified mass flow,
- experimental data,
- choked flow conditions,
- analytical approximation,

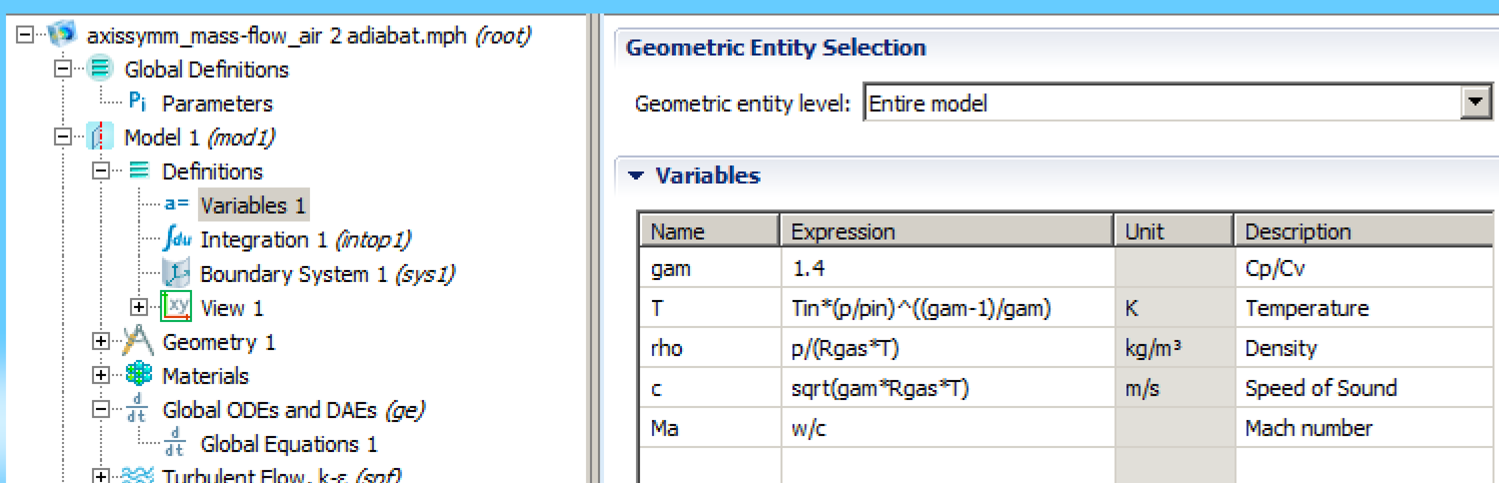
Test Section Geometry (4.2 x 150 mm Tube)



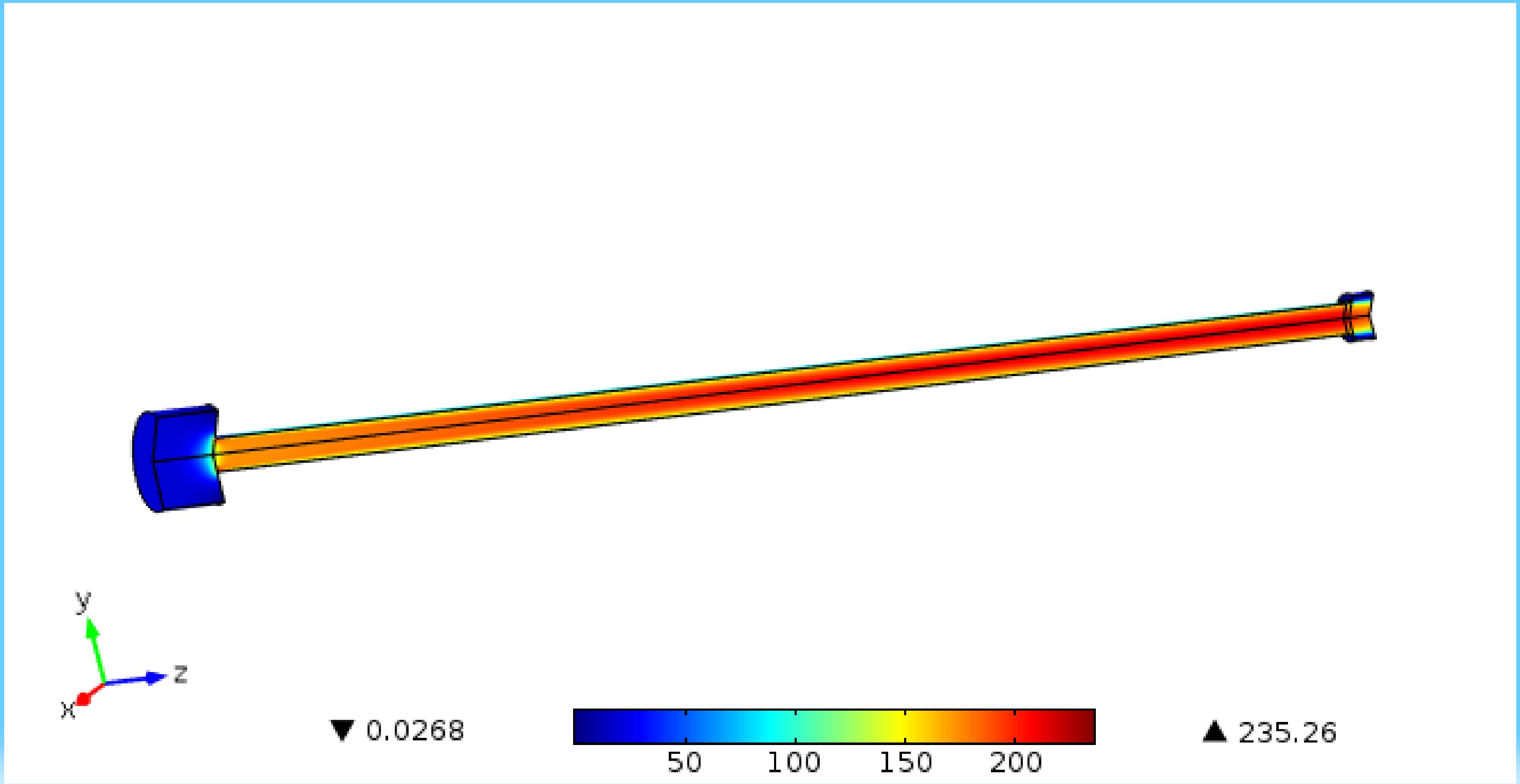
Scalar Integration Variable



Setting of Variables

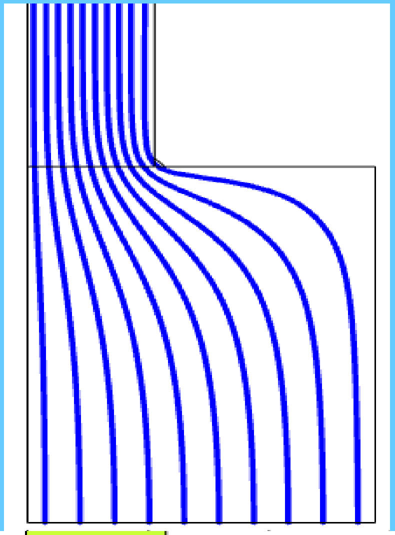


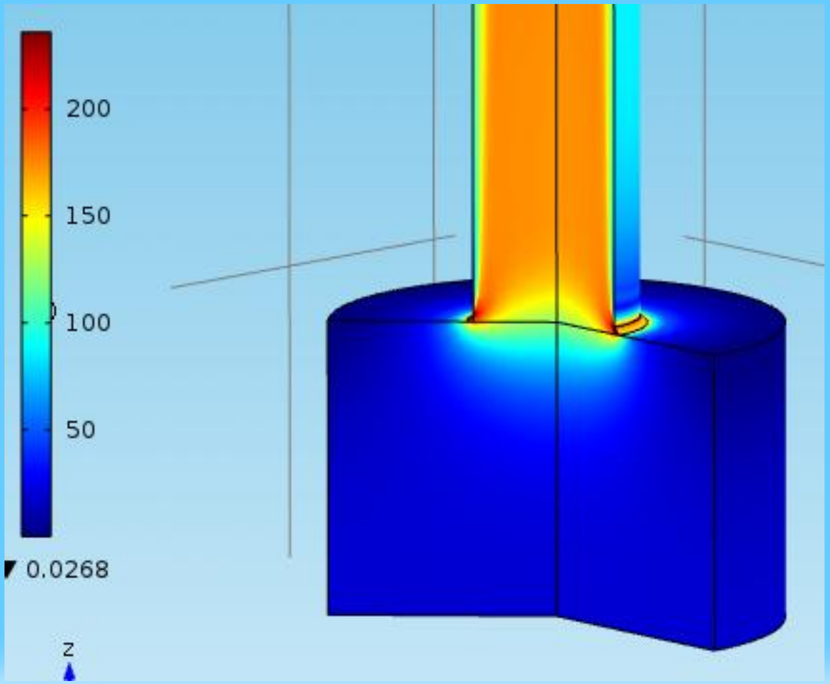
Axis-Symmetric COMSOL Model
(velocity [m/s] @ 10 kg/h mass flow)



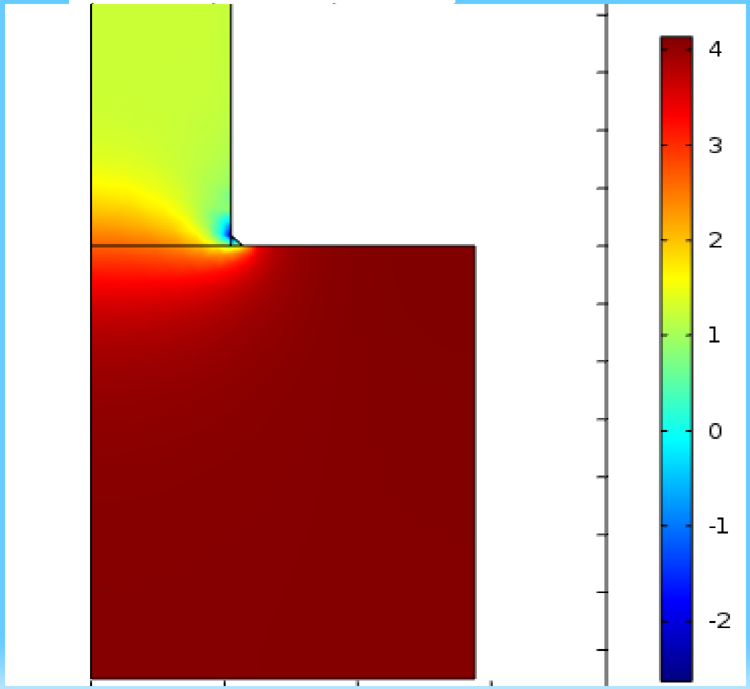
Inlet Conditions

(@ 10 kg/h mass flow)



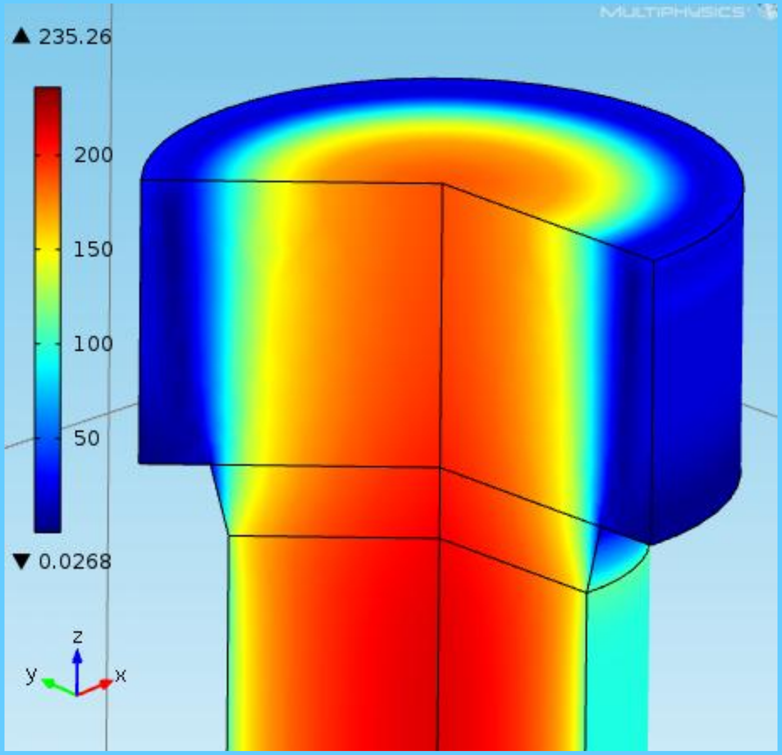


velocity [m/s]

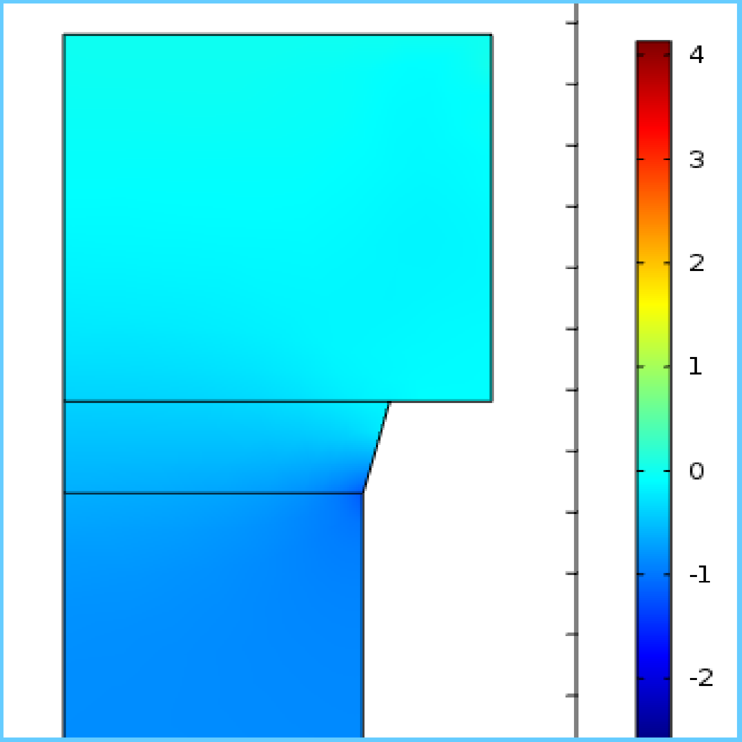


Pressure [psig]

Outlet Conditions
(@ 10 kg/h mass flow)

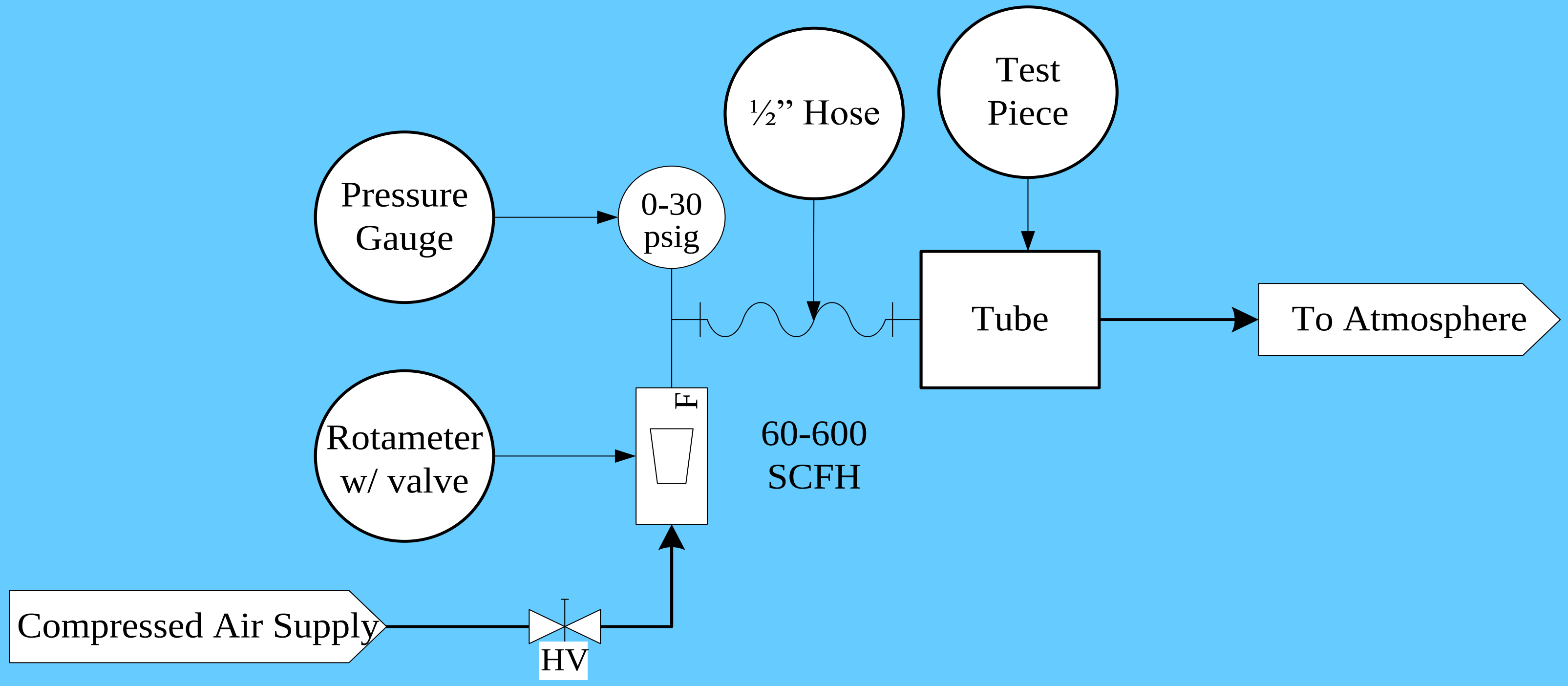


Flow Separation

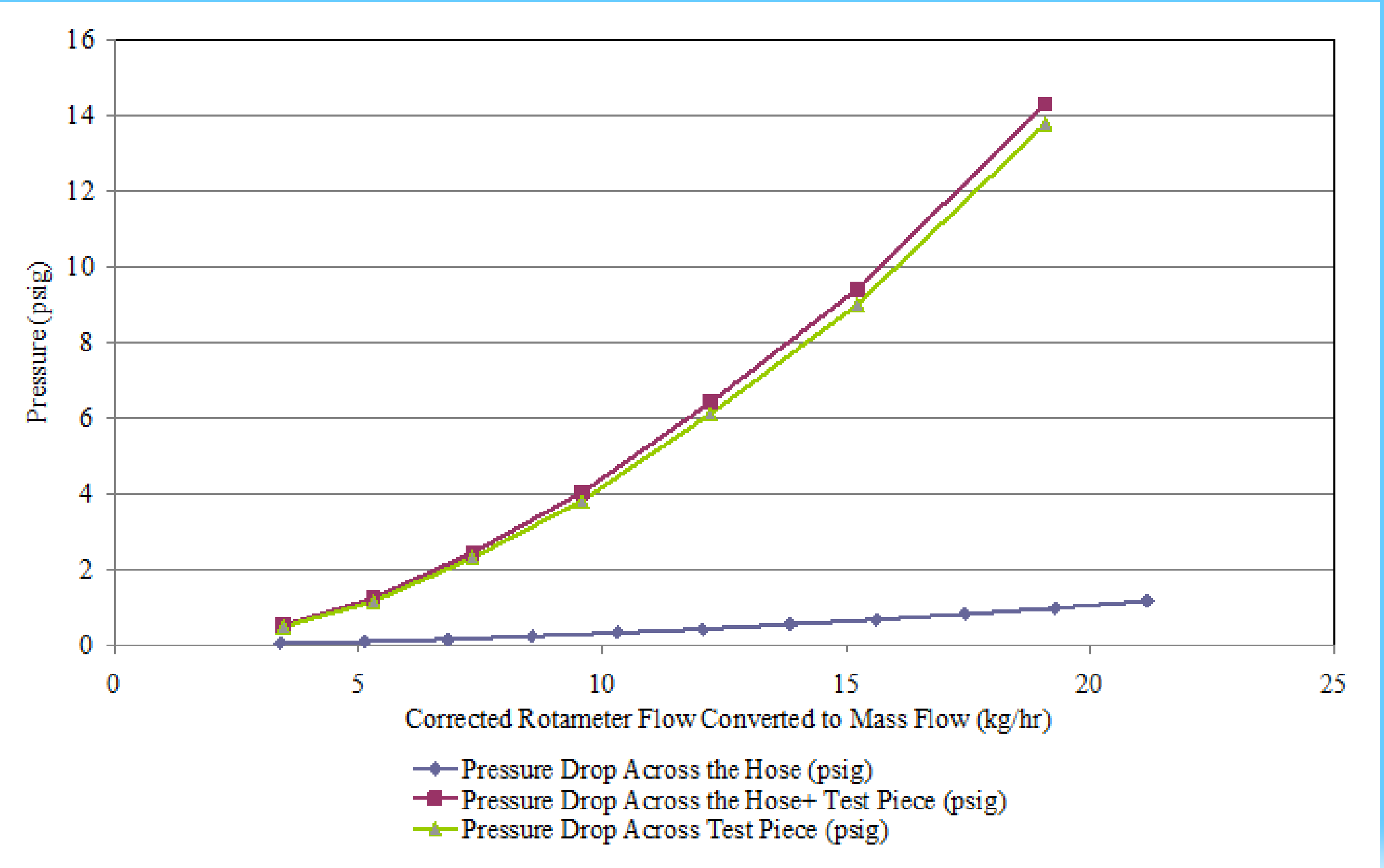


Pressure Recovery

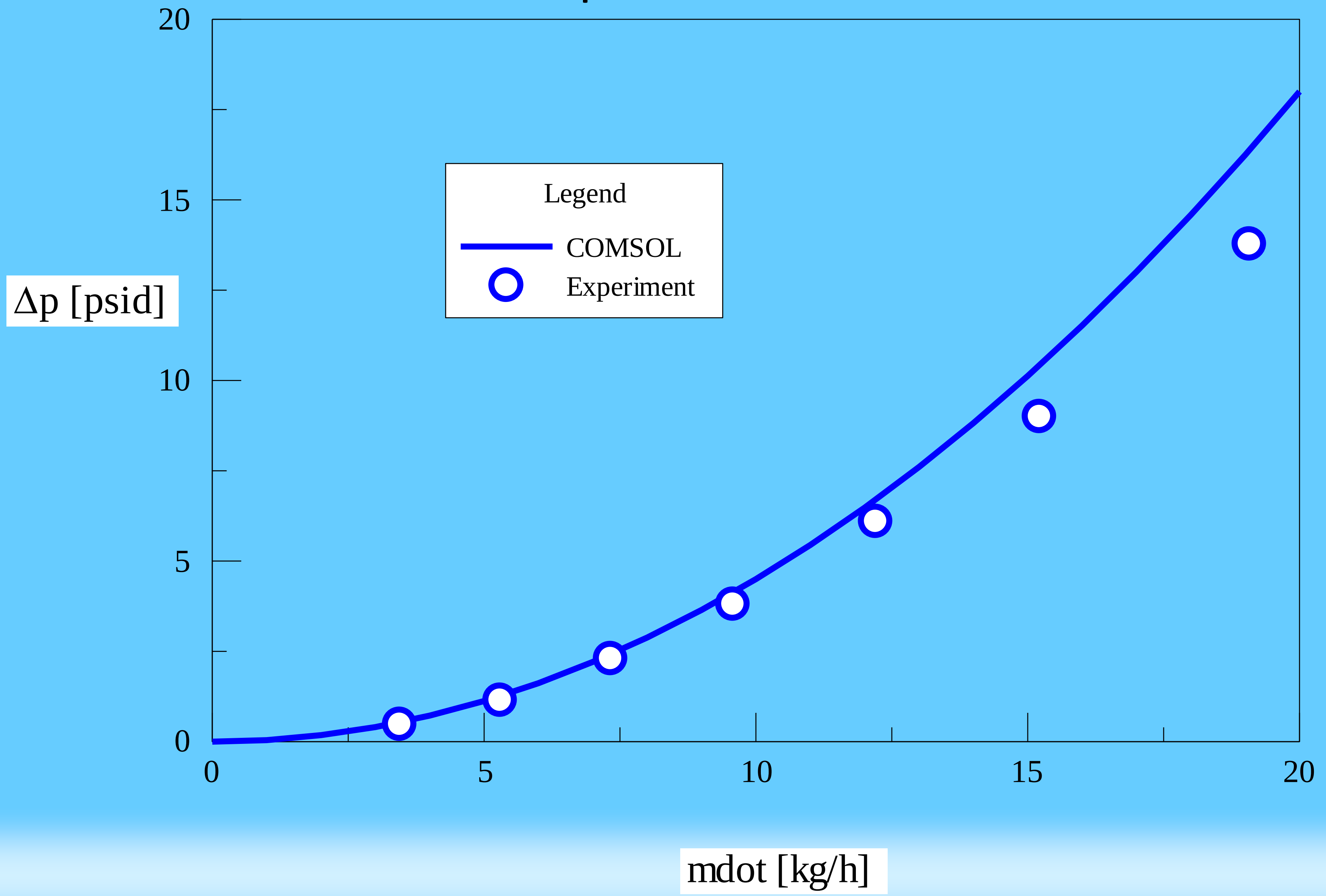
Experimental Setup



Experimental Data

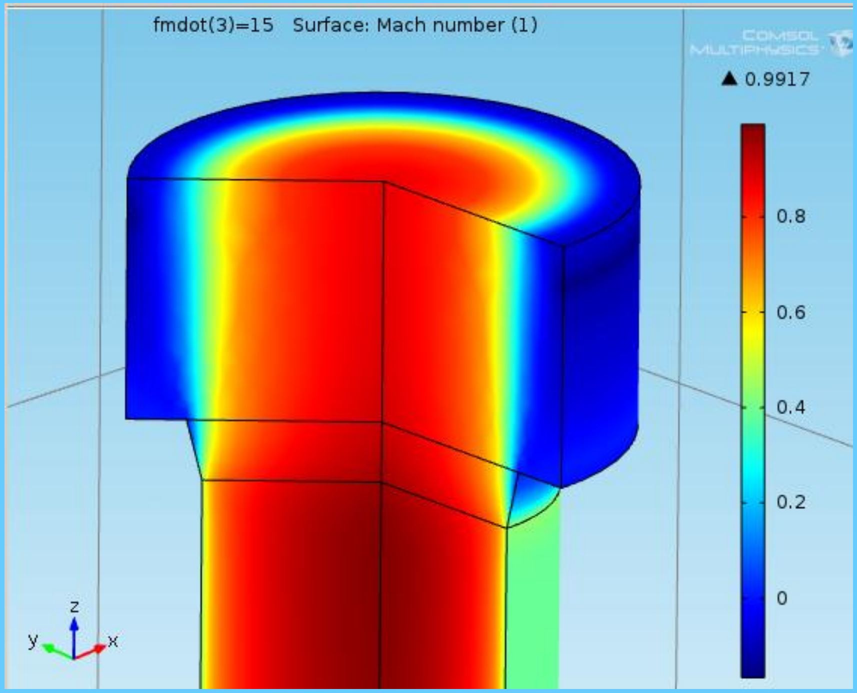


Comparison of Results

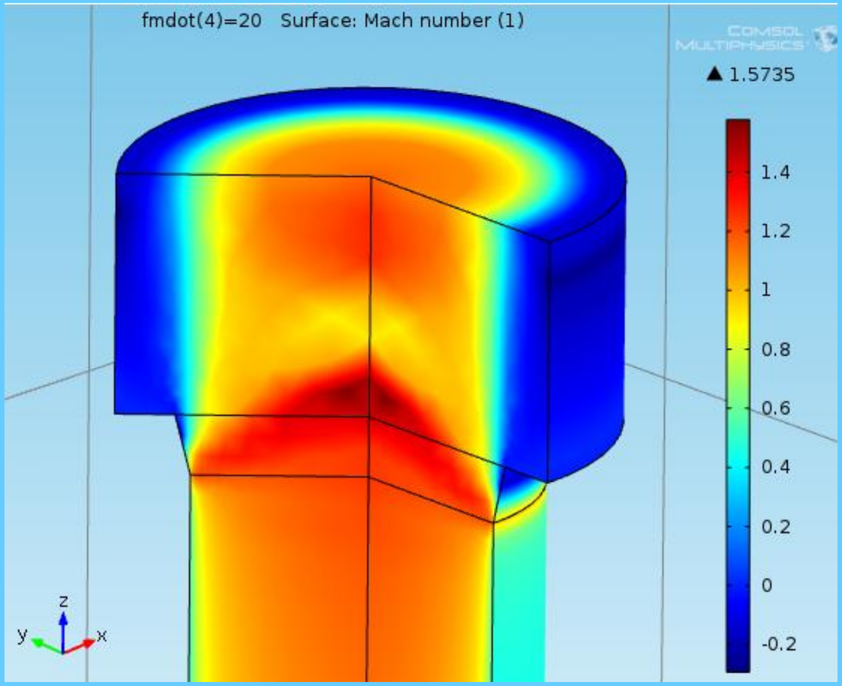


Outlet Mach Numbers

15 kg/h



20 kg/h



One-dimensional Theory.

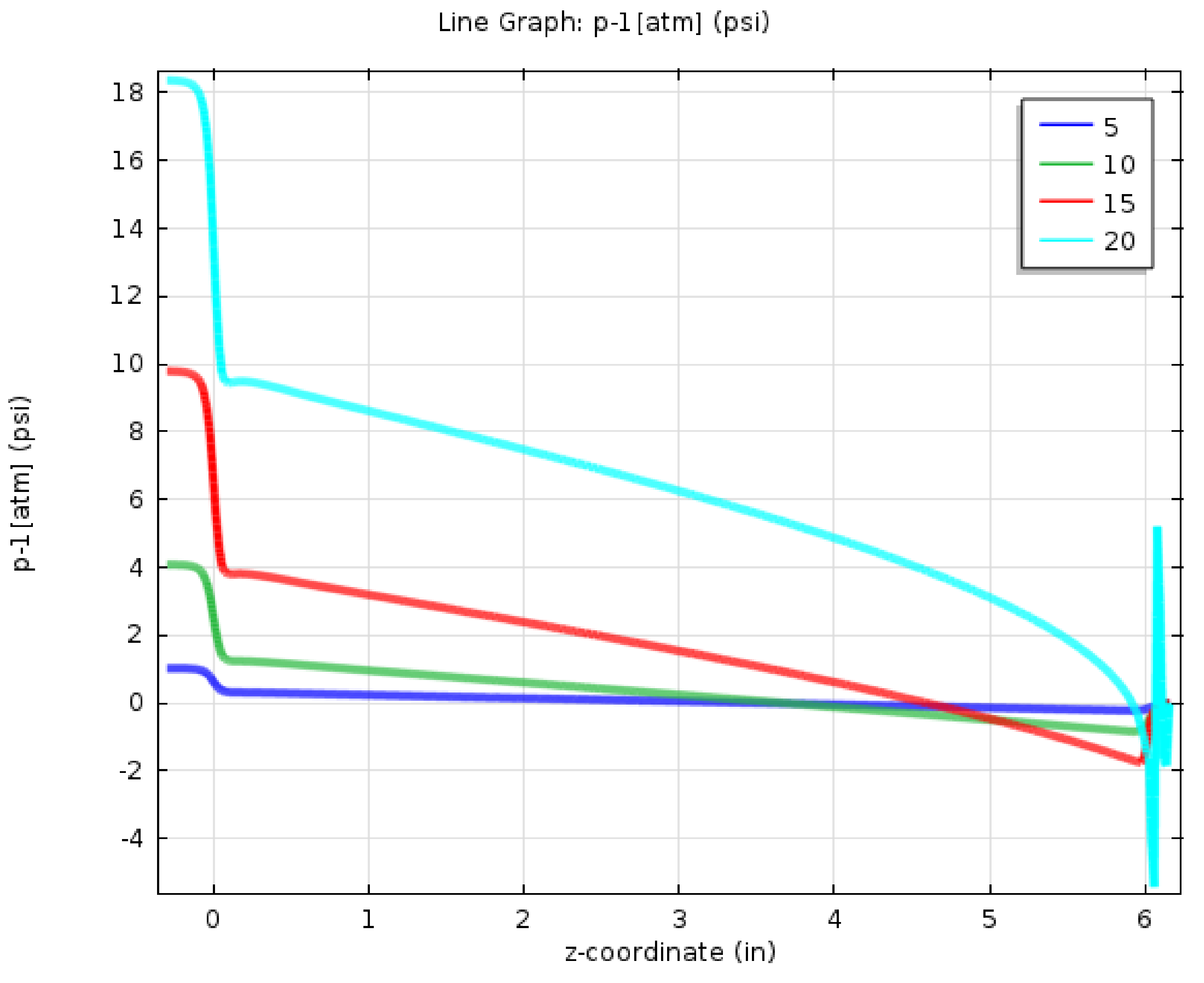
$$\frac{\rho}{\rho_0} = \left(\frac{p}{p_0}\right)^{1/\gamma} \qquad \frac{T}{T_0} = \left(\frac{p}{p_0}\right)^{(\gamma-1)/\gamma}$$

$$-\rho\frac{dp}{dx} = \frac{1}{2}\rho^2 V^2 f\frac{1}{D} = \frac{1}{2}G^2 f\frac{1}{D} \qquad f = \frac{0.3164}{Re^{1/4}} = 0.3164\left(\frac{\mu}{GD}\right)^{0.25}$$

$$-\frac{T_L}{T_0} + \frac{T_\delta}{T_0} \equiv -\left(\frac{p_L}{p_0}\right)^{\frac{\gamma-1}{\gamma}} + \left(\frac{p_\delta}{p_0}\right)^{\frac{\gamma-1}{\gamma}} = \frac{\gamma-1}{\gamma}\frac{G^2}{2\rho_0 p_0}f\frac{L-\delta}{D}$$

$$T_L = T_\delta - \frac{\gamma-1}{\gamma}\frac{V_0^2}{2R}f\frac{L-\delta}{D} = T_\delta - \frac{1}{2}\frac{\rho_0 V_0^2}{\rho_0 C_p}f\frac{L-\delta}{D}$$

COMSOL Centerline Axial Pressure Profiles



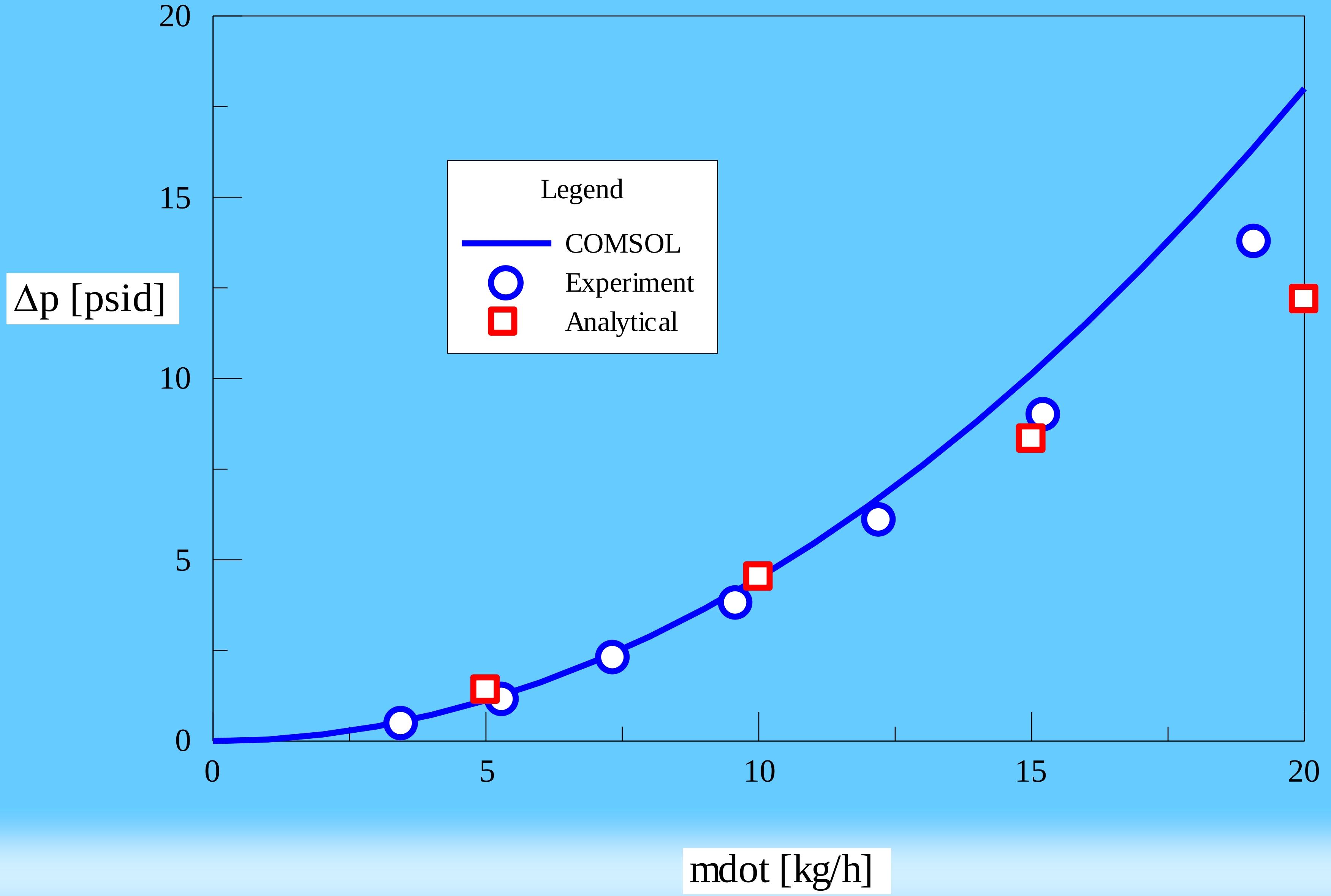
$$\Delta p_{ex} = p_L - p_{out} = \frac{1}{2}\rho_L V_L^2 (K_{ex} - 1) = \frac{G^2}{2\rho_L}(K_{ex} - 1)$$

$$\Delta p_{in} = p_{in} - p_\delta = \frac{1}{2}\rho_\delta V_\delta^2 (1 + K_{in}) \approx \frac{G^2}{2\rho_{in}}(1 + K_{in})$$

By comparison with the COMSOL data for 15 kg/h

K_{in} = 0.70 and K_{out} = 0.65.

Comparison of Results



Thank you