
Increasing Heat Transfer in Microchannels with Surface Acoustic Waves*

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COMSOL
CONFERENCE
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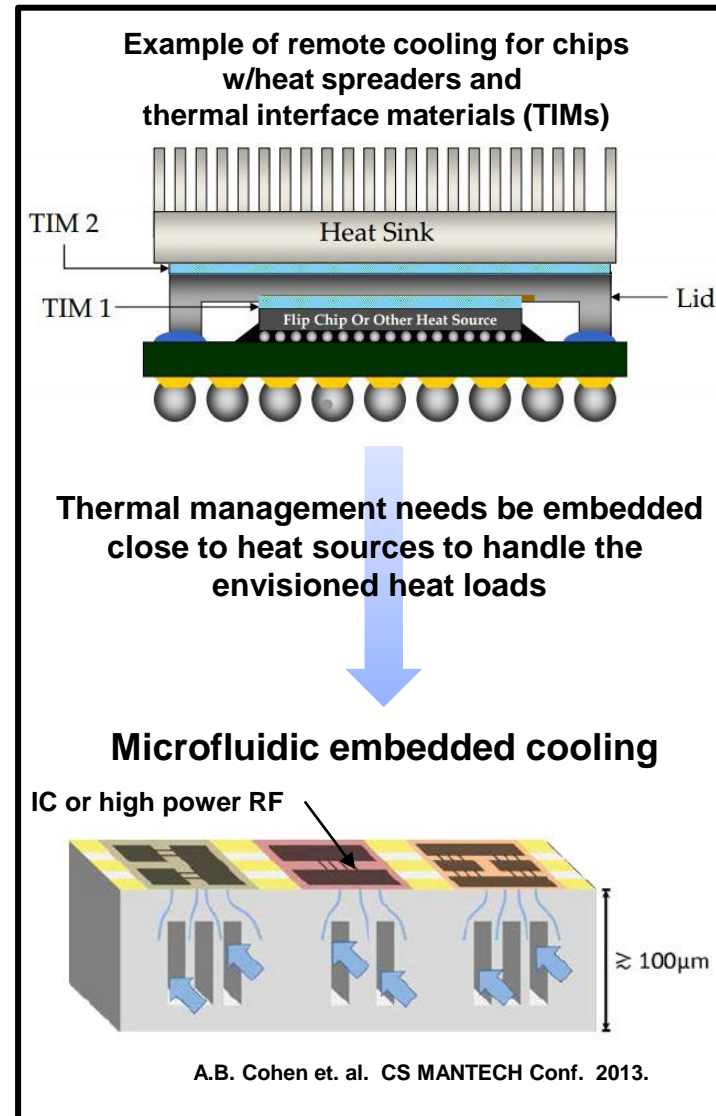
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Study Motivation

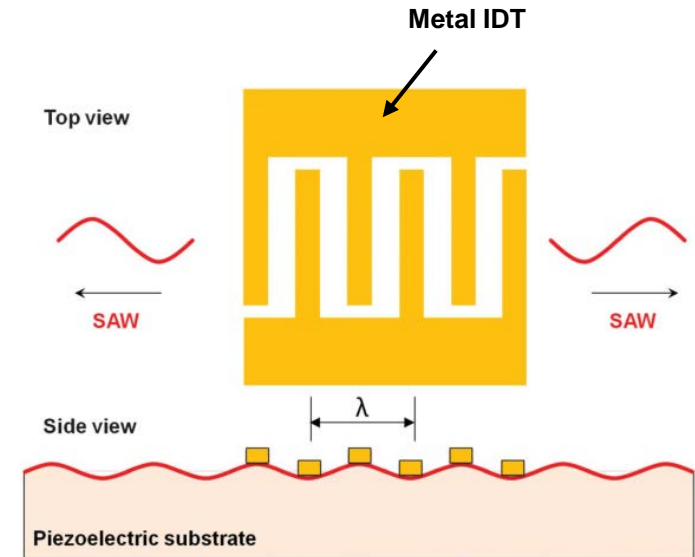
- As the trend in electronics continues towards higher integration density and higher power devices **current remote cooling technology will not be able to handle the predicted levels of heat removal**
 - Increased integration density of electronics, including 3D chip stack will push *localized heat fluxes* $> 1 \text{ kW/cm}^2$ and package-level *volumetric heat generation* $> 1 \text{ kW/cm}^3$
 - New paradigm for embedded cooling required
- Microfluidic cooling holds potential promise and a large amount of research and technology development has been going on (multiple DARPA programs since 2000)
 - Single-phase flow performance in microchannels limited to high flow rates due to laminar flow conditions and fixed thermal boundary layer
- In this numerical study surface acoustic waves (SAWs) are evaluated as a disruptive flow technology
 - SAWs coupled with single-phase microchannel flow
 - Goal is to drive circulating/chaotic flow to disrupt thermal boundary layers



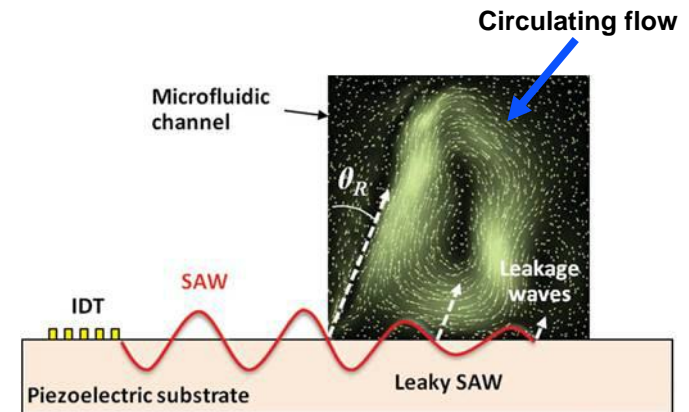


Surface Acoustic Waves (SAWs) and Acoustic Streaming

- **SAWs are generated by sinusoidal electrical potential applied to an interdigitated transducer (IDT) on piezoelectric substrate and propagate along the surface**
 - Used extensively in telecommunication
 - Signal processing and filtering
- **When a SAW traveling along the surface comes in contact with a fluid medium some of SAW's energy is refracted into the liquid**
 - **Acoustic streaming ("Leaky SAW")**
- **Non-linear acoustic interaction occurs within a thin viscous boundary layer ($<1\mu\text{m}$ for MHz frequencies)**
 - Bulk fluid motion arises from viscous interactions with the boundary layer
- **Frequency ranges: 1 to 100's MHz**
- **Substrate amplitude ranges: 0.1 nm to 10 nm**
- **Gaining attention in microfluidics**
 - Fluid mixing and particle and cell sorting



X. Ding, et. al. Lab Chip, 13, 3626-3649, 2013.



X. Ding, et. al. Lab Chip, 13, 3626-3649, 2013.



Numerical Modeling of Acoustic Streaming

- **Numerical simulation challenges:**
 - SAW operate in MHz range and behave with harmonic time dependences
 $\rightarrow e^{i\omega t}$
 - Viscous effects in a fluid occur on msec or larger time scales
 - This requires a time-averaged response of the acoustic oscillations
- **Perturbation expansion is employed on dependent variables u , p , and ρ to solve conservation of mass and momentum**
 - Acoustic velocity is of first-order (u_1)
 - Streaming velocity is assumed to be of second-order (u_2)
- **Computational methodology:**
 - Solve the first-order acoustic motion (u_1, p_1, ρ_1)
 - Use first-order solution as inputs for second order conservation equations
 - Time averaged equations need to be solved

Continuity:

$$\rho_o \nabla \cdot \langle \bar{u}_2 \rangle = -\nabla \cdot \langle \rho_1 \bar{u}_1 \rangle$$

Mass source term (kg/m²s)

Volume force Source terms (N/m³)

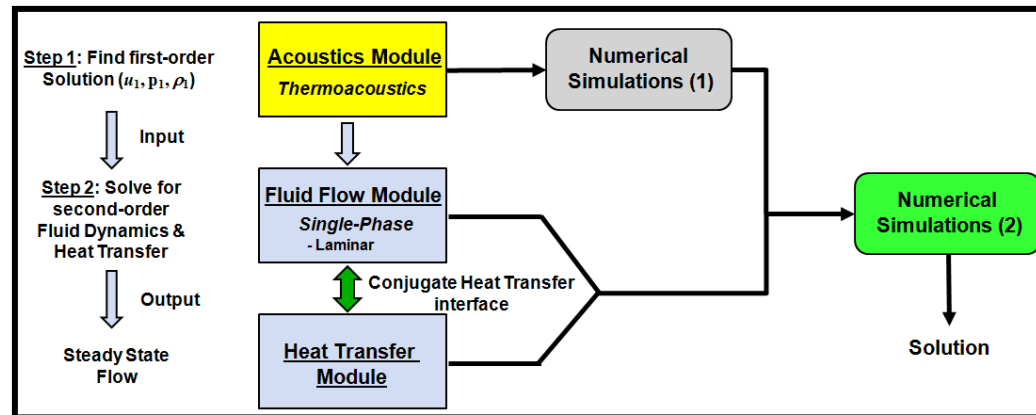
Momentum:

$$-\rho_o \left\langle \frac{\partial \bar{u}_2}{\partial t} \right\rangle + \mu \nabla^2 \langle \bar{u}_2 \rangle + \left(\mu_B + \frac{\mu}{3} \right) \nabla (\nabla \cdot \langle \bar{u}_2 \rangle) - \langle \nabla p_2 \rangle = \left\langle \rho_1 \frac{\partial \bar{u}_1}{\partial t} \right\rangle + \rho_o \langle (\bar{u}_1 \cdot \nabla) \bar{u}_1 \rangle$$

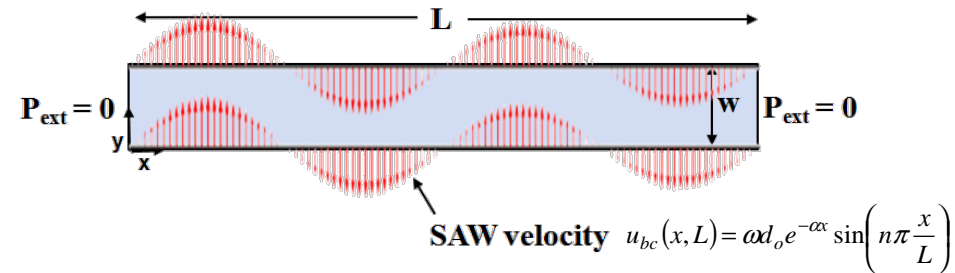


COMSOL Modeling

- COMSOL ver. 4.3b
- Step 1: solving first order equations using *Thermoacoustics interface*
 - Study 1: Frequency Domain
 - SAW introduced as velocity on channel walls
- Step 2: Solving second order equations using *Conjugate Heat transfer interface*
 - Study 2: Stationary (steady state)
 - Source terms (first-order results) added to mass and momentum equations
 - Momentum equation
 - Volume force, F directly added
 - Mass
 - Constitutive equation needs to be altered
 - Use “weak contribution”
 - No coupling of first-order terms to Energy equation
 - Time-averaged response of complex-values
 - Ex: Mass source term $= -\nabla \cdot \langle \rho_1 \bar{u}_1 \rangle \Rightarrow \langle \rho_1 u_{1,x} \rangle = \frac{1}{2} Re[conj(\rho_1) \cdot u_{1,x}]$



Step 1: First-Order boundary conditions



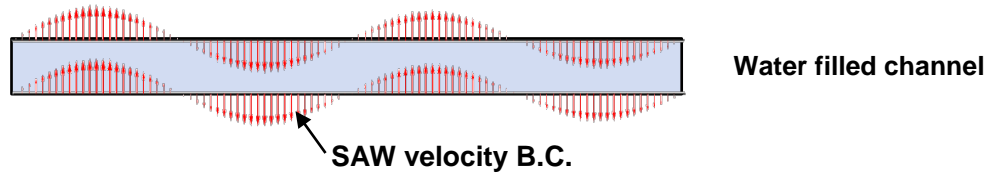
Step 2: Second-Order boundary conditions





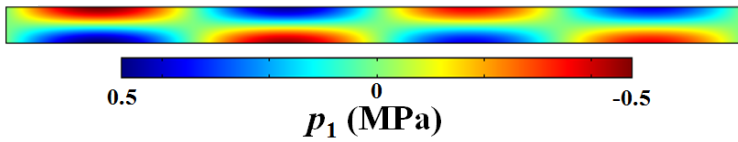
Model 1 Simulation Results: Laminar Flow Only

Model parameters: $L = 1000 \mu\text{m}$
 $w = 50 \mu\text{m}$
 $f = 15 \text{ MHz}$
 $d_o = 0.1 \text{ nm}$
 $U_{in} = \text{no inlet flow}$

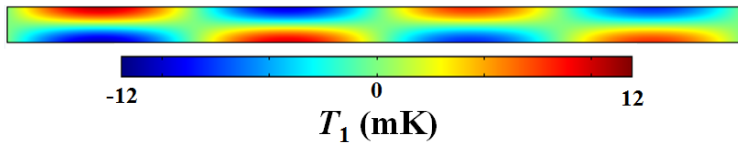


Contour Plots of First-Order Fields

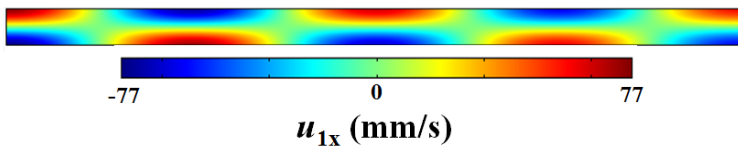
Pressure



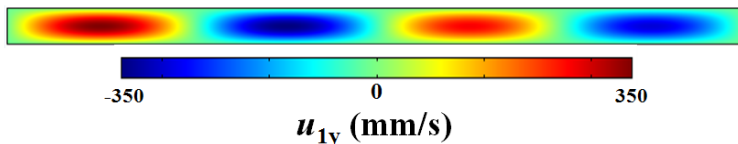
Temperature



Horizontal velocity

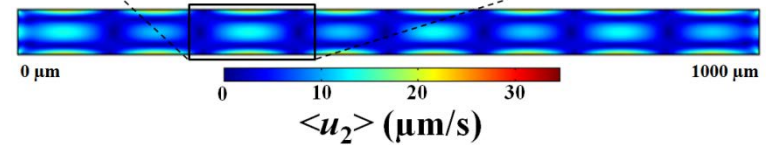
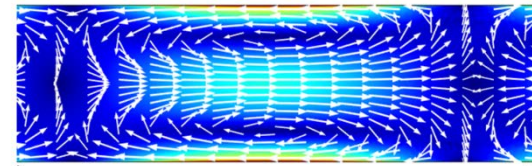


Vertical velocity

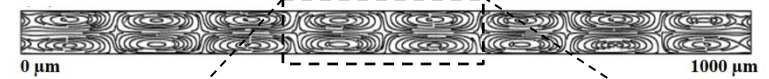


Time-averaged Second-order Results

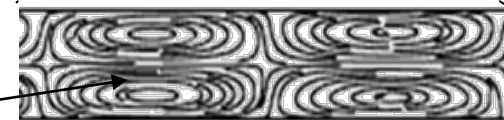
Velocity contour plot



Streamlines



Rayleigh streaming vortices

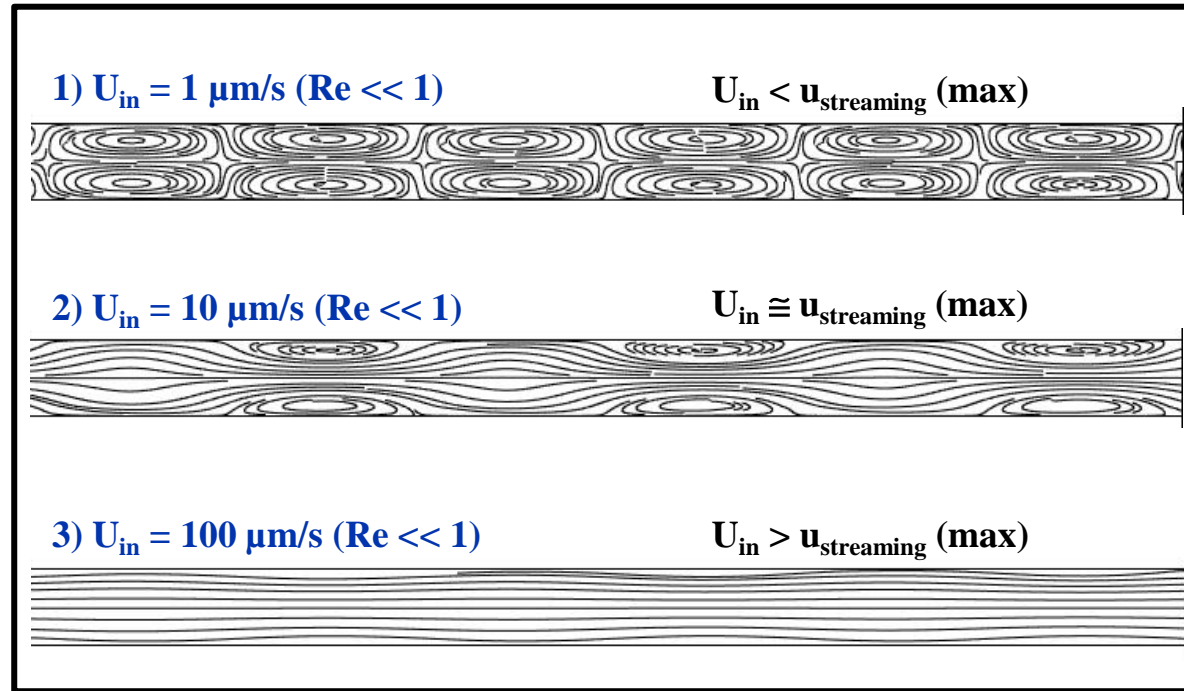




Model 1: Effect of Inlet Velocity

- Inlet flow included in 2nd order simulations
 - 1st order results the same for all cases
- $U_{in} \leq \text{Max acoustic streaming velocity}$
 - Circulating flow in the microchannel is created
- $U_{in} > \text{Max acoustic streaming velocity}$
 - Advection dominates flow and vortices are not generated
- Need to increase acoustic streaming velocity to have higher Reynolds number flows

Time-averaged 2nd Order Results: Streamlines for $\langle u_2 \rangle$

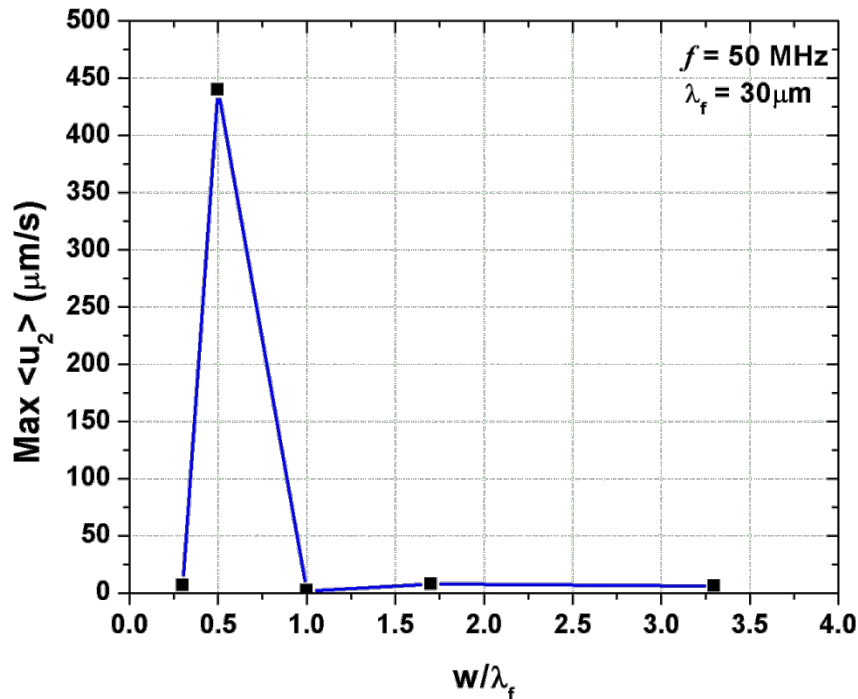


$L = 1000 \mu\text{m}$
 $w = 50 \mu\text{m}$
 $f = 15 \text{ MHz}$
 $d_o = 0.1 \text{ nm}$
 $U_{in} = \text{varied}$

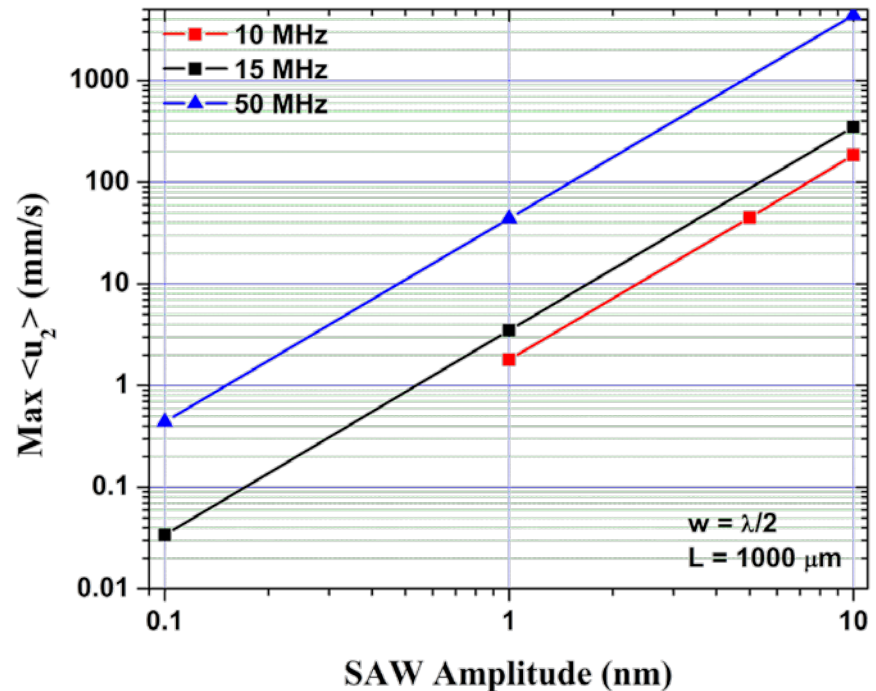


Effects on Acoustic Streaming Velocity

- Maximum acoustic streaming velocity occurs when:
 - Channel width $w = \lambda/2$
 - Wave length determined from fluid properties



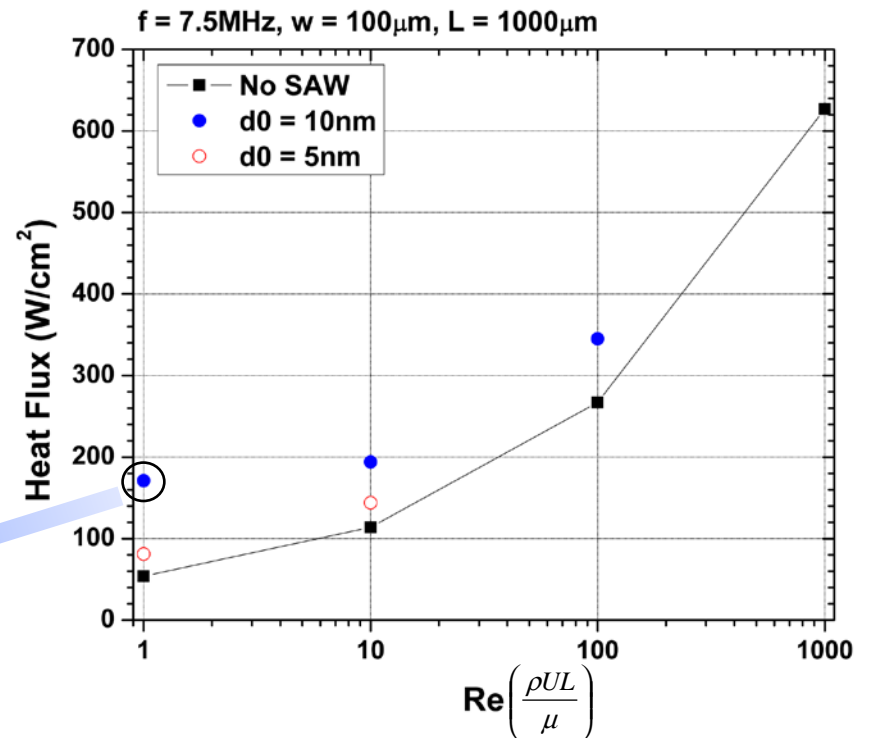
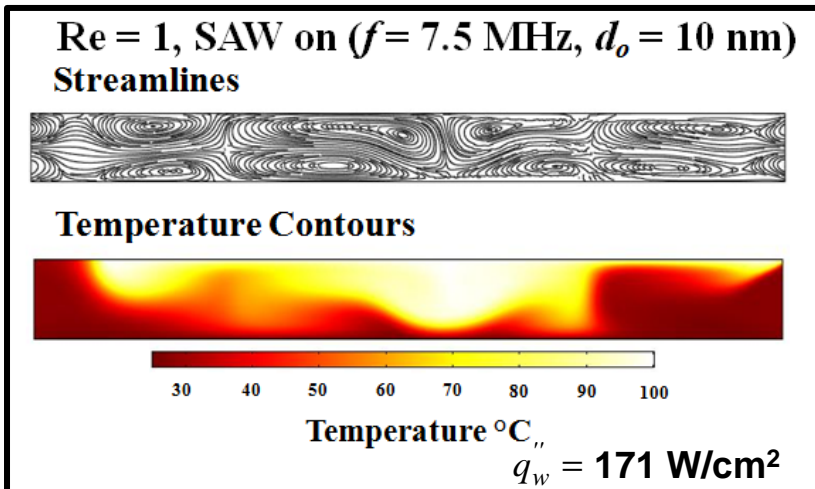
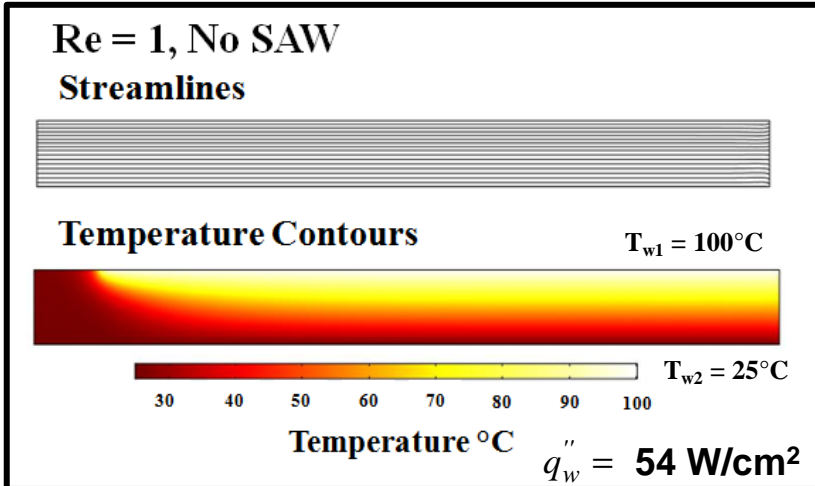
- Acoustic streaming velocity dependent on SAW amplitude d_0
- Streaming velocity is quadratic in SAW amplitude for all frequencies





Heat Transfer Results

Model parameters: $L = 1000 \mu\text{m}$
 $w = 100 \mu\text{m}$
 $f = 7.5 \text{ MHz}$
 $d_o = 10 \text{ nm}$
 $U_{in} = 0.01 \text{ m/s}$ ($Re = 1$)



- Results show under the right condition there is an increase in heat transfer due to SAWs
 - Stream velocity < Acoustic streaming velocity



Summary

- **Developed the framework to numerically simulate Acoustic Streaming in microchannels from surface acoustic waves**
- **Coupled three physics together**
 - **Acoustics, fluid flow and heat transfer**
- **Results indicate that SAW can enhance heat transfer**
 - **Bulk stream velocity < Acoustic streaming velocity**
- **Considerably more analysis work needs to be done**
 - **Need to increase vorticity strength in order for concept to be useful in microfluidic cooling**
 - **Identified potential ways to achieve this condition**
 - **Pulse the SAW and frequency or amplitude modulate**
 - **Pulse the inlet velocity**
 - **Explore different frequencies and geometries**
 - **Launch SAW from different locations in channel**
 - Perpendicular to the flow**



Thank you!

Questions?



Backup Material



Model Material Parameters

Water @25°C

Density	ρ_f	998	kg/m³
Speed of sound	c_f	1495	m/s
Dynamic viscosity	μ	8.90e-4	Ps s
Thermal diffusivity	D	1.43e-7	m²/s

Solid*

Density	ρ_s	4650	kg/m³
Speed of sound	c_s	3990	m/s

*Single crystal Lithium Niobate



Physics

First-order equations

a) Thermodynamic heat transfer equation for T_1

$$\frac{\partial T_1}{\partial t} = D \nabla^2 T_1 + \frac{\alpha T_0}{\rho_o C_p} \frac{\partial p_1}{\partial t}$$

b) Kinematic continuity equation in terms of p_1

$$\frac{\partial p_1}{\partial t} = \frac{1}{\gamma \kappa} \left[\alpha \frac{\partial T_1}{\partial t} - \nabla \cdot u_1 \right]$$

c) Momentum equation for velocity field u_1

$$\rho_o \frac{\partial u_1}{\partial t} = -\nabla p_1 + \mu \nabla^2 u_1 + \left(\mu_B + \frac{\mu}{3} \right) \nabla (\nabla \cdot u_1)$$

Equations solved with *Thermoacustics Physics* interface in COMSOL

Second-order equations

a) Continuity equation

$$\rho_o \nabla \langle u_2 \rangle = -\nabla \cdot \langle \rho_1 u_1 \rangle$$

b) Momentum equation

$$\begin{aligned} \mu \nabla^2 \langle u_2 \rangle + \left(\mu_B + \frac{\mu}{3} \right) \nabla \nabla \cdot \langle u_2 \rangle - \nabla \langle p_2 \rangle \\ = \langle \rho_1 \frac{\partial u_1}{\partial t} \rangle + \rho_o \langle (u_1 \cdot \nabla) u_1 \rangle \end{aligned}$$

c) Energy equation

$$\rho_o C_p \langle u_2 \rangle \nabla T_2 = \nabla \cdot (K \nabla T_2) + Q$$

$\langle x \rangle$ = time average quantity over full oscillation period

Equations solved with *Conjugate Heat Transfer* interface in COMSOL



Time Averaging of Complex Variables

- For complex-valued fields $A(t)$ and $B(t)$ with harmonic time dependence, the time average is the real part:

$$\langle A(t)B(t) \rangle = \frac{1}{2} \text{Re}[\text{conj}(A(0)) * B(0)]$$

Ex: Mass source term: $= -\nabla \cdot \langle \rho_1 \bar{u}_1 \rangle$

$$= -\left(\frac{\partial \langle \rho_1 u_{1x} \rangle}{\partial x} + \frac{\partial \langle \rho_1 u_{1y} \rangle}{\partial y} \right)$$

$$\langle \rho_1 u_{1x} \rangle = \frac{1}{2} \text{Re}[\text{conj}(\rho_1) \cdot u_{1x}]$$

$$\langle \rho_1 u_{1y} \rangle = \frac{1}{2} \text{Re}[\text{conj}(\rho_1) \cdot u_{1y}]$$



SAW Streaming Analysis

- **SAW can be generalized as periodic i.e. harmonic**
- **The fluid motion induced by harmonic forcing in general has two components**
 - **Harmonic component**
 - This is the motion of the fluid from the acoustic response
 - **Steady component**
 - This is the streaming response
- **The computational challenge is how to separate the two components**
- **To solve for the conservation of mass and momentum a perturbation expansion is done on dependent variables u, p, ρ**

$$\begin{aligned}u &= u_o + \varepsilon u_1 + \varepsilon^2 u_2 + O(\varepsilon^3) \\p &= p_o + \varepsilon p_1 + \varepsilon^2 p_2 + O(\varepsilon^3) \\ \rho &= \rho_o + \varepsilon \rho_1 + \varepsilon^2 \rho_2 + O(\varepsilon^3)\end{aligned}$$

$\varepsilon = \frac{U}{c_o}$ **Smallness factor (acoustic Mach #)**
 $\varepsilon \ll 1$