Bridging the complexity gap in modern engineering education with COMSOL Multiphysics®

FH Bielefeld
University of Applied Sciences

COMSOL CONFERENCE
2014 CAMBRIDGE

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Introduction: Today multiphysics simulations play an important role in research and development in almost all disciplines. Therefore it is necessary to teach them at the universities to provide the industry with well-trained graduate students. Practically this means, that students should learn the handling of commercial multiphysics tools in order to set up reliable models. However, it is more important to train them how to interpret the numerical results comprehensively, i.e. qualifying them to draw the right conclusions for model optimizations and to estimate the influence of the used approximations on the obtained results.

Teaching Model: Heat Transfer in an Iron Rod

In our lectures about the finite element method we use a simple but rich enough example problem, which can be solved analytically. By comparing numerical and analytical results step by step even complex methodical and physical concepts are much easier to comprehend by the students. We use a one-dimensional heat transfer problem with an internal heat source (e.g. from Joule heating), a Neumann boundary condition (heat flux) and a Dirichlet boundary condition (fixed temperature).

$$q_0 = -k \left.\frac{dT}{dx}\right|_{x=0} \qquad T(x = L) = T_0$$

Corresponding differential equation:

$$-\frac{d}{dx}\left(k(x)\frac{d}{dx}T(x)\right) = Q$$

Analytical result:

$$T(x) = \frac{QL^2}{2k}\left(1 - \frac{x^2}{L^2}\right) + \frac{q_0 L}{k}\left(1 - \frac{x}{L}\right) + T_0$$

This way, the meaning and influence of different boundary conditions on the analytical as well as the numerical result can be studied in great detail.

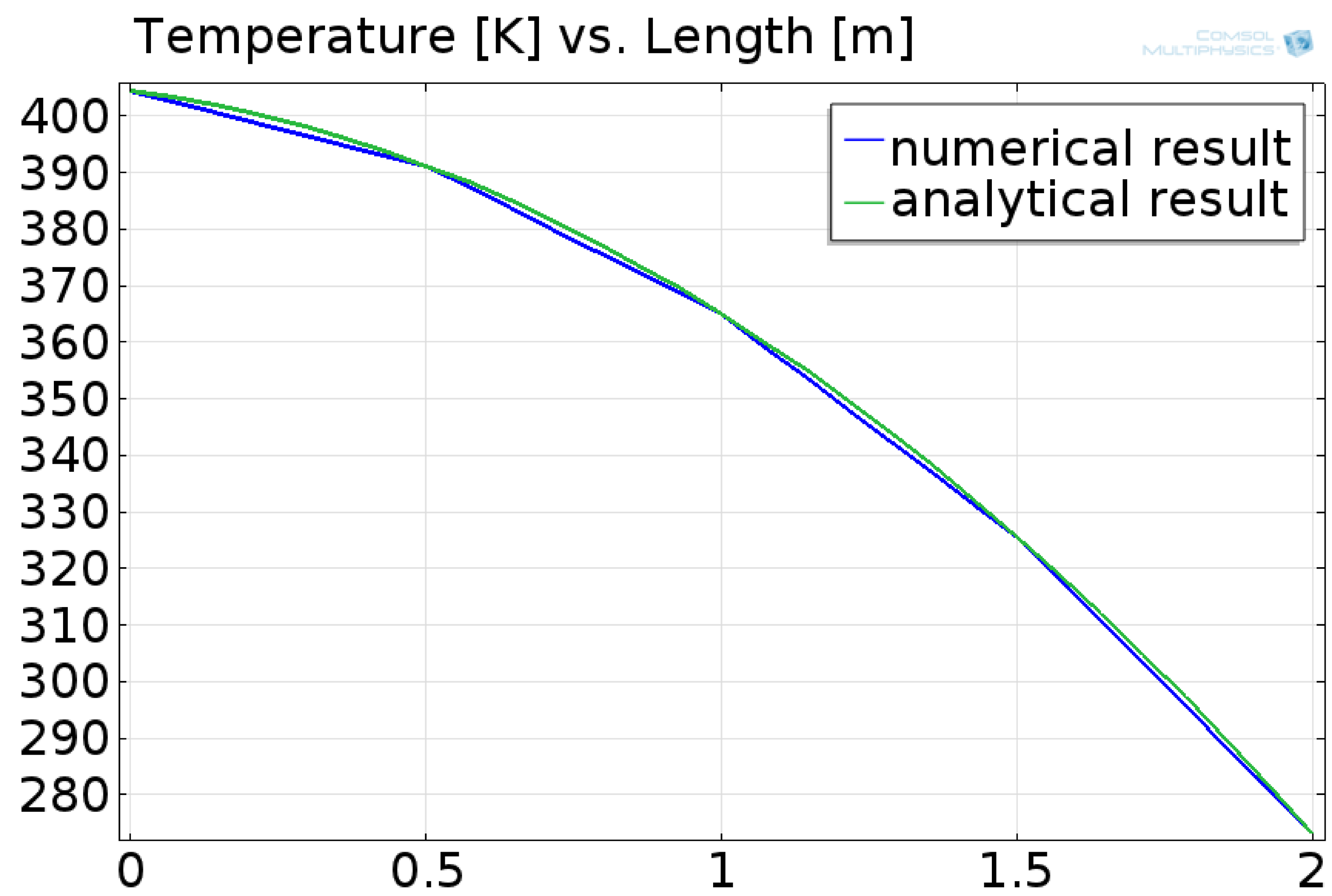

Figure 1. Comparison between the numerical (4 elements and linear shape functions) and the analytical results for $k = 76{,}2\,\frac{\mathrm{W}}{\mathrm{m \cdot K}}$, $L = 2\,\mathrm{m}$, $Q = 4000\,\frac{\mathrm{W}}{\mathrm{m}^3}$, $q_0 = 1000\,\frac{\mathrm{W}}{\mathrm{m}^2}$, and $T_0 = 273{,}15\,\mathrm{K}$.

In addition, the influence of the input parameters is discussed so that the students get a solid understanding of the underlying physical processes. Model extensions as well as model improvements by using a finer discretization and higher order shape functions can be demonstrated in the GUI of COMSOL Multiphysics® in a very comfortable way.

In this special case the analytical result can be exactly reproduced by the numerical calculation with only one element and quadratic shape functions.

$$\begin{pmatrix} 152{,}4 & -152{,}4 & 0 & 0 & 0 \\ -152{,}4 & 304{,}8 & -152{,}4 & 0 & 0 \\ 0 & -152{,}4 & 304{,}8 & -152{,}4 & 0 \\ 0 & 0 & -152{,}4 & 304{,}8 & -152{,}4 \\ 0 & 0 & 0 & -152{,}4 & 152{,}4 \end{pmatrix} \qquad \begin{pmatrix} 177{,}8 & -203{,}2 & 25{,}4 & 0 & 0 \\ -203{,}2 & 406{,}4 & -203{,}2 & 0 & 0 \\ 25{,}4 & -203{,}2 & 355{,}6 & -203{,}2 & 25{,}4 \\ 0 & 0 & -203{,}2 & 406{,}4 & -203{,}2 \\ 0 & 0 & 25{,}4 & -203{,}2 & 177{,}8 \end{pmatrix}$$

Figure 2. Stiffness matrix for (a) 4 elements and linear shape functions, (b) 2 elements and quadratic shape functions

Conclusions: On the basis of this simple one-dimensional model the generalization to two or three dimensions is straight forward and easy comprehensible so that the students finally are enabled to set up rather complicated multiphysics models during the accompanying practical lab course.

Excerpt from the Proceedings of the 2014 COMSOL Conference in Cambridge