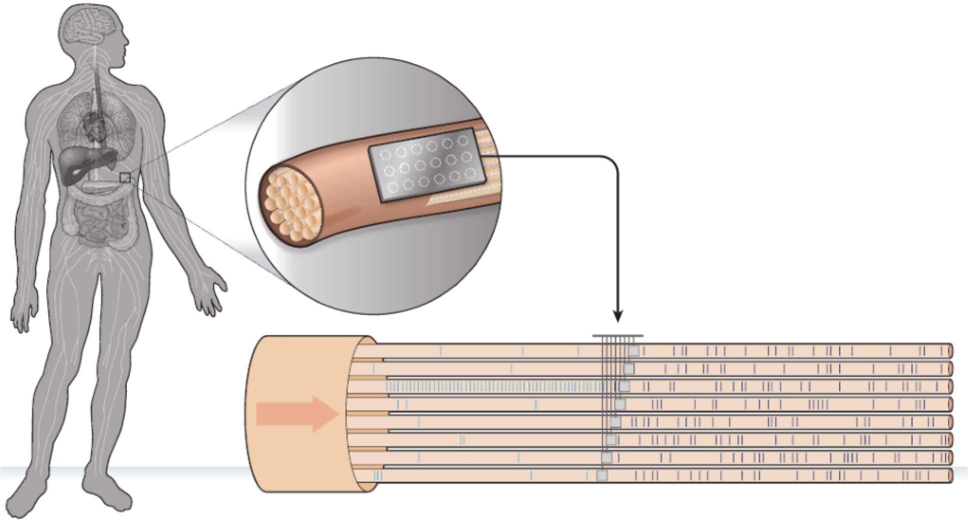
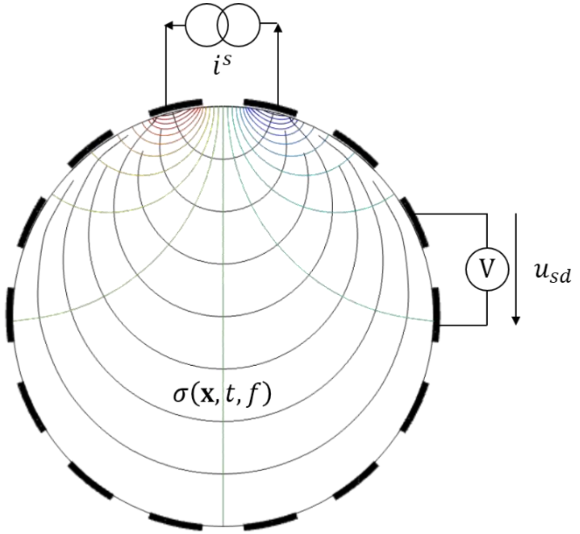
Flexible Numerical Platform For Electrical Impedance Tomography





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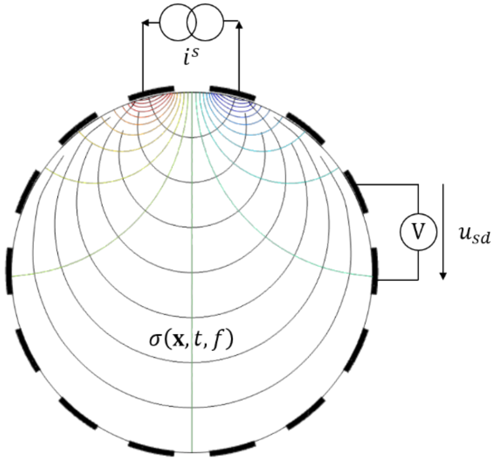
Context: Complete Electrode Model in EIT

Signal



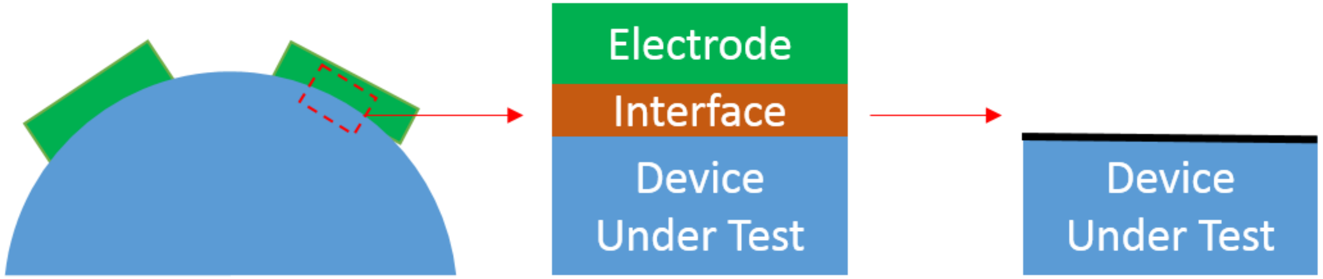
Source – detector transfer impedance

$$z_{sd} = \frac{u_{sd}}{i^s}$$

- Geometry (boundary shape & size)
- Electrode configuration
- Conductivity distribution $\sigma(\mathbf{x},t,f)$

Model: Electrode & Interface as a boundary condition

Current injection, potential measurements



$$\begin{cases} \nabla\cdot(-\sigma\nabla v) = 0 & \forall x \in \Omega \\ v + z_e\sigma\nabla v\cdot\mathbf{n} = v_e & \forall x \in \mathrm{E}_e \\ \displaystyle\int_{\mathrm{E}_e} \sigma\nabla v\cdot\mathbf{n}\,\mathrm{d}\Gamma = i_e & \forall e = 1 \dots E \\ \sigma\nabla v\cdot\mathbf{n} = 0 & \forall x \in \partial\Omega \backslash \displaystyle\bigcup_{e=1}^{E} \mathrm{E}_e \end{cases}$$

How to take into account contact impedances z_e?

$$z_e/S_e \gg z_{sd}$$

Context: Numerical platforms for EIT

Literature solutions

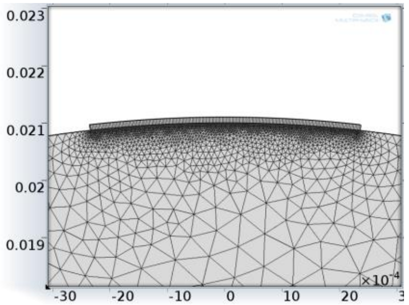
1. **Custom-built** FEM-based libraries, e.g. EIDORS

   Development & debugging costs, Simplistic computation assumptions, Augmented stiffness matrix

2. Model **electrode thickness** within generalist FEM packages, e.g. COMSOL

   High number of uninformative degrees of freedom

   How to recover sensitivity patterns?



How to perform efficiently EIT modeling using COMSOL ?

Forward problem

Data prediction z_{sd}

Solve a technical issue regarding electrodes-object interface

Sensitivity analysis

Parameter σ influence on z_{sd}

Adapt a mixed FEM-FVM framework

Inverse problem

Estimation of σ

Use COMSOL optimization toolbox

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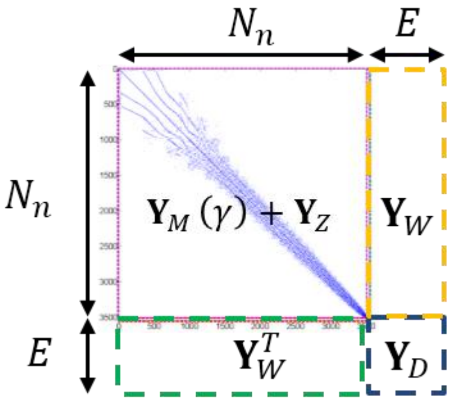
Methods: data prediction

Custom-built FEM libraries: specificity of the Complete Electrode Model

Test functions for electrode potentials along with electric potential discretization

$$-\int_{\Omega} \sigma \nabla v \cdot \nabla w \,\mathrm{d}\,\Omega + \sum_{e=1}^{E} z_e \int_{E_e} (v - v_e)(w - w_e)\,\mathrm{d}\,\Gamma = \sum_{e=1}^{E} i_e w_e \quad \forall (w, \mathbf{w}_e)$$



Augmented stiffness matrix

$$\mathbf{Y} \in \mathbb{R}^{(N_n+E)\times(N_n+E)}$$

$$\mathbf{Y}\begin{bmatrix}\mathbf{v}_n\\ \mathbf{v}_e\end{bmatrix} = \begin{bmatrix}\mathbf{Y}_M(\sigma) + \mathbf{Y}_Z & \mathbf{Y}_W\\ \mathbf{Y}_W^T & \mathbf{Y}_D\end{bmatrix}\begin{bmatrix}\mathbf{v}_n\\ \mathbf{v}_e\end{bmatrix} = \begin{bmatrix}0_{N_n}\\ \mathbf{i}_e\end{bmatrix}$$

COMSOL implementation

$$-\mathbf{n}\cdot\mathbf{j} = \frac{v_e - v}{z_e}$$

$$v_e = \frac{1}{\|E_e\|}\int_{E_e} v\,\mathrm{d}\Gamma + z_e i_e$$

CEM as a single Neumann boundary condition

$$\boxed{-\mathbf{n}\cdot\mathbf{j} = \frac{1}{z_e\|E_e\|}\left(\int_{E_e} v\,\mathrm{d}\Gamma + z_e i_e - \|E_e\| v\right)}$$

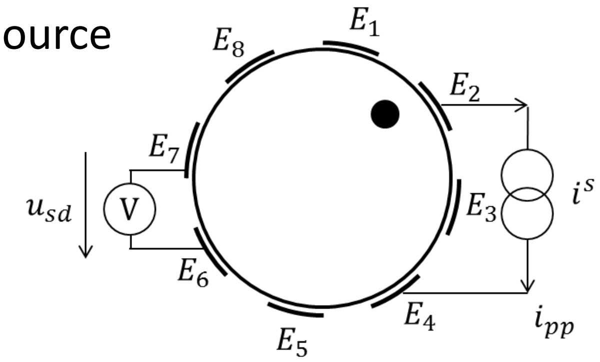
Equivalent formulation with a classic stiffness matrix

electrode potentials deduced during **post-processing**

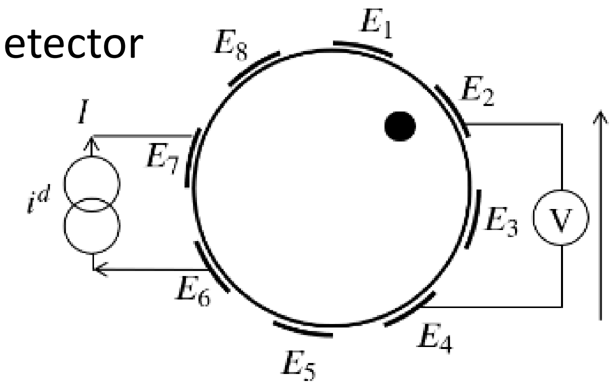
Methods: sensitivity analysis

Dual configurations

source



detector



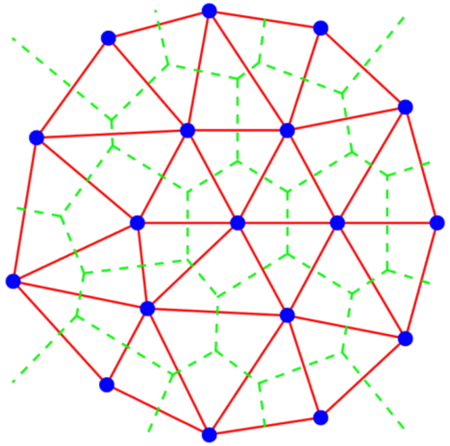
$$\frac{\partial u_{sd}}{\partial \sigma_c} = \frac{-1}{I} \int_{\Omega_c} \nabla v^s \cdot \nabla v^d \mathrm{d}\Omega$$

Voronoï cell-based control volumes

(see Fouchard et al. GRETSI 2015)

Nodal approximation of electric fields using COMSOL

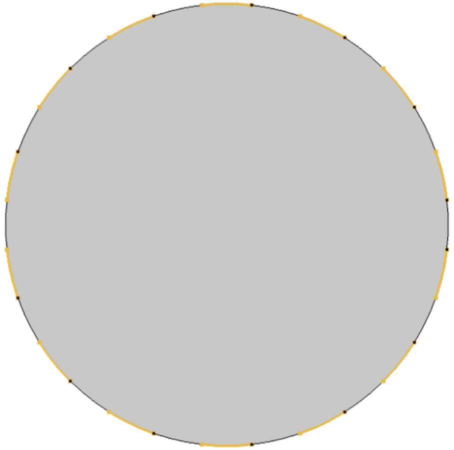
$$\boxed{[\mathbf{J}]^d_{sd,n} = \frac{\partial u_{sd}}{\partial \sigma_n} = \frac{-\|\Pi_n\|}{I} \nabla \mathbf{v}_n(i^s) \cdot \nabla \mathbf{v}_n\left(i^d\right)}$$

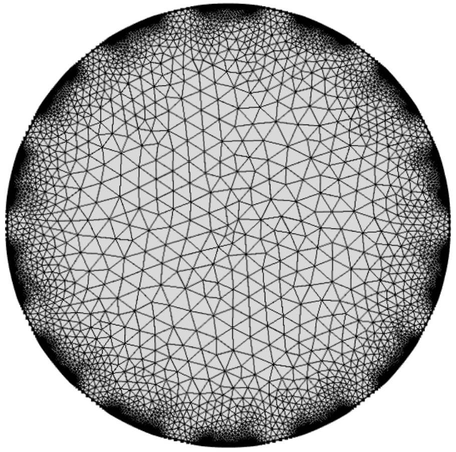


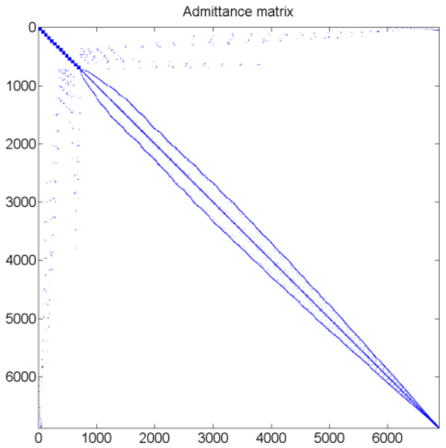
Forward solver for EIT*: solve identified literature issues towards standardization of numerical tools*

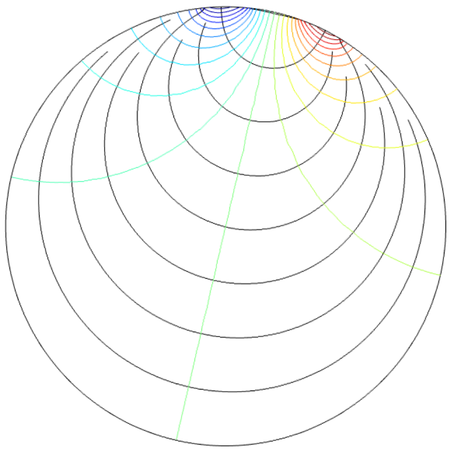
Results: forward solver

Overall behavior





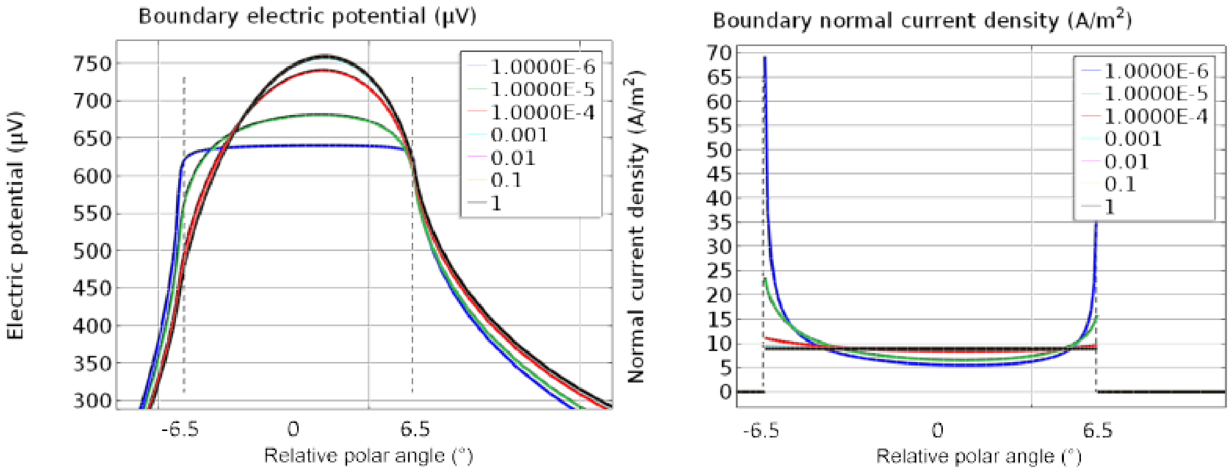




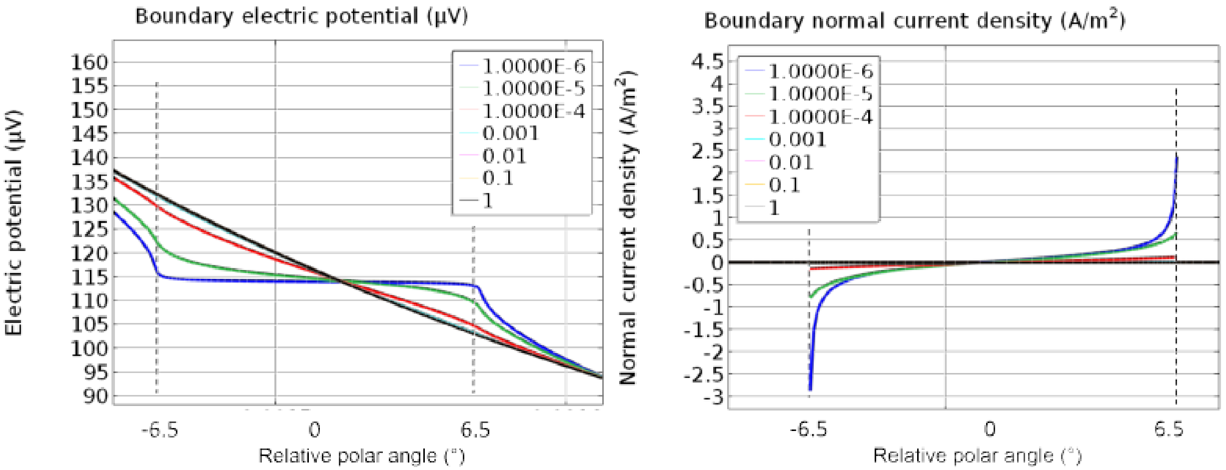
Influence of electrodes

edge effects with varying contact impedances z_e

strong field modifications in the boundary vicinity



Source electrode



Detector electrode

Results: solver, benchmarking EIDORS vs COMSOL

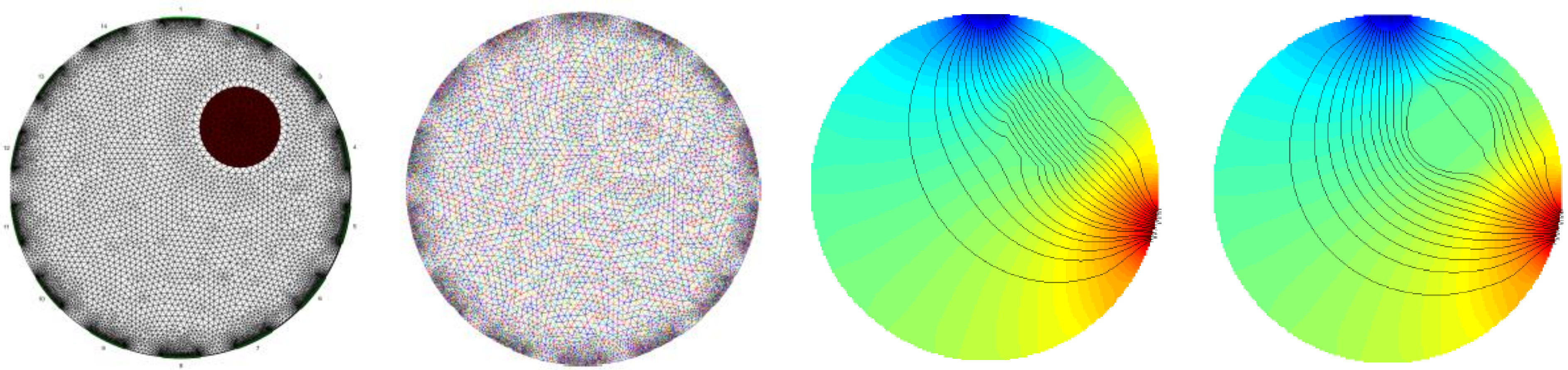
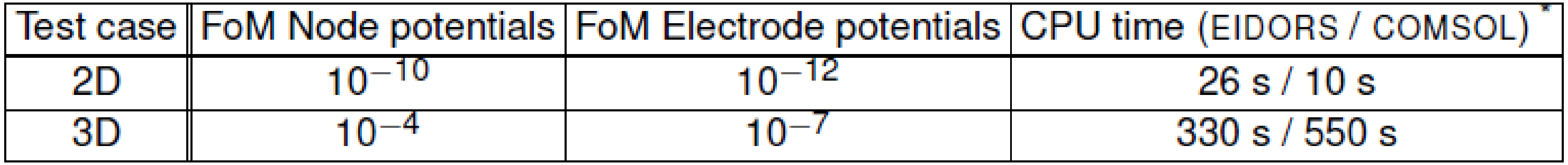
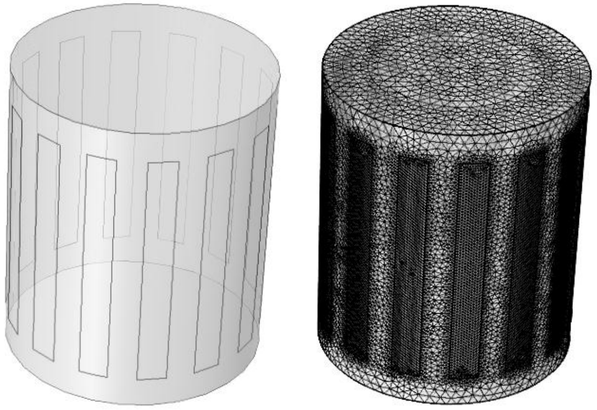


Figure of Merit: maximum of relative error

Results for varying z_e from 10^{-6} to $1\ \Omega.\mathrm{m}^2$



Test case	FoM Node potentials	FoM Electrode potentials	CPU time (EIDORS / COMSOL) *
2D	10^{-10}	10^{-12}	26 s / 10 s
3D	10^{-4}	10^{-7}	330 s / 550 s

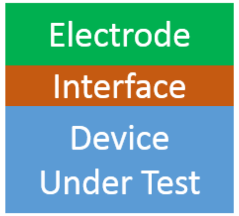


CPU time

EIDORS: assembly of stiffness matrix & solving only

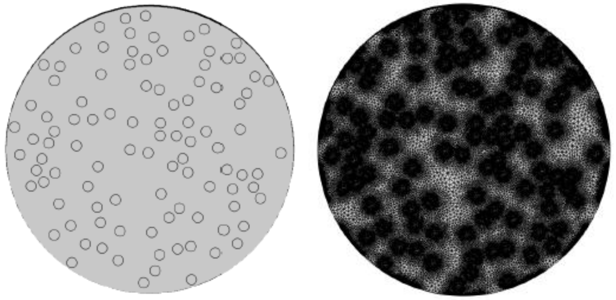
COMSOL: also includes meshing

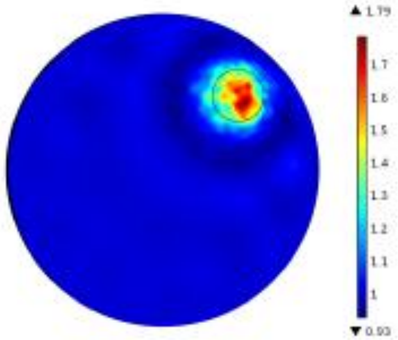
Discussion & Perspectives


Electrode
Interface
Device Under Test

Versatile forward solver

1. **Means of taking into account interface effects** (metallic electrodes)
2. **Consistent numerical approximation** with previous developments
3. **Complete framework for EIT**, with inverse problem capabilities (*poster*)
4. Extensions towards other electromagnetic situations
   e.g. tDCS, EEG, DBS (forward & inverse problems)





Future developments towards

1. Incorporation of **advanced regularization strategies**
2. **Adaptive** forward / inverse **meshing** schemes
3. Multispectral capabilities
4. Reduce **CPU time** for inverse problems & **3D models**
5. **Multiphysics coupling** for multimodality imaging

Thank you for your attention

Acknowledge Eric Favre
& COMSOL support

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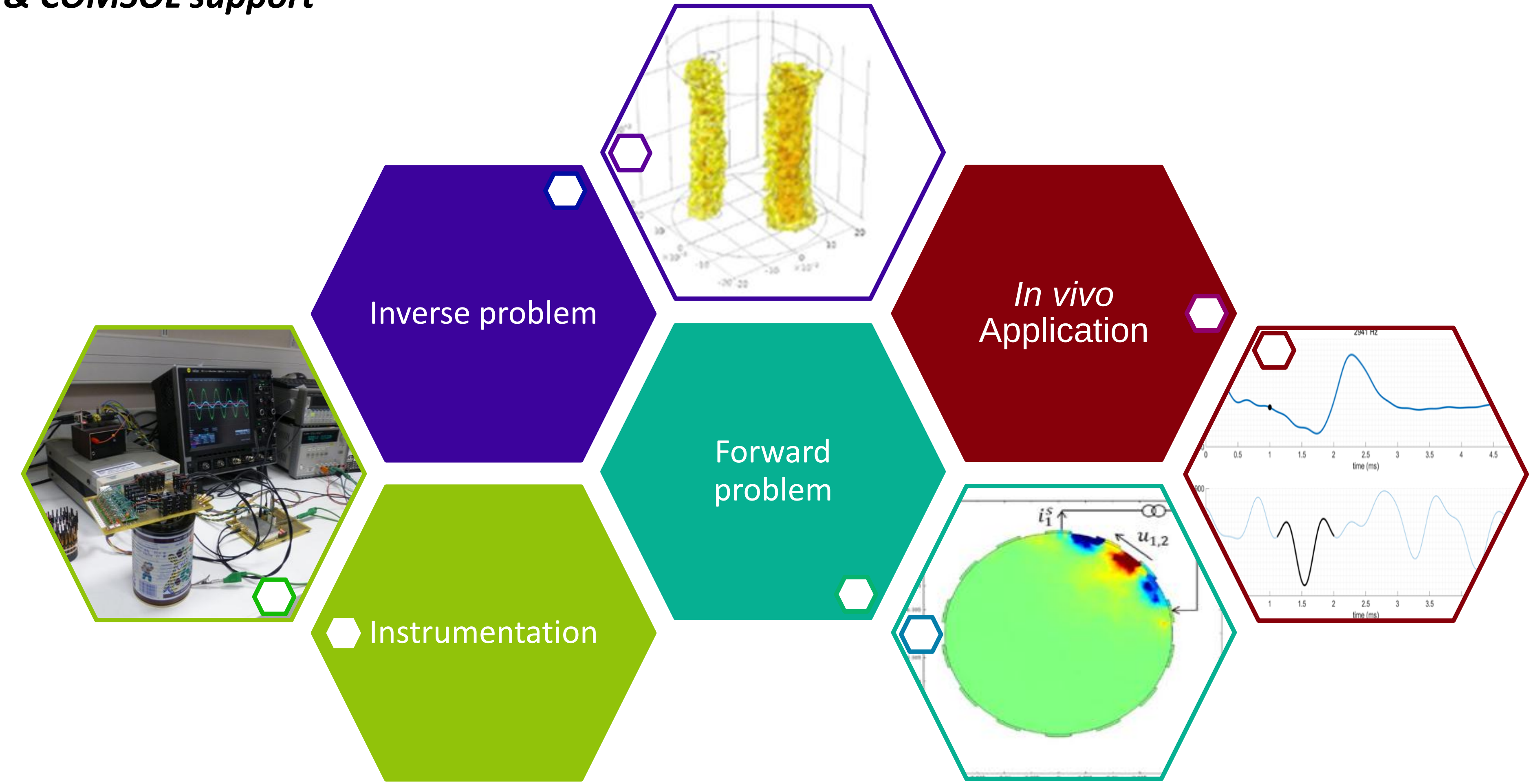

Looking for a job ☺, after my defense (Nov 6, 2015)