

Benchmarking tailored formulations of multiphase flow in porous media

Álvaro Sáinz-García, E. Abarca, A. Nardi, F. Grandia

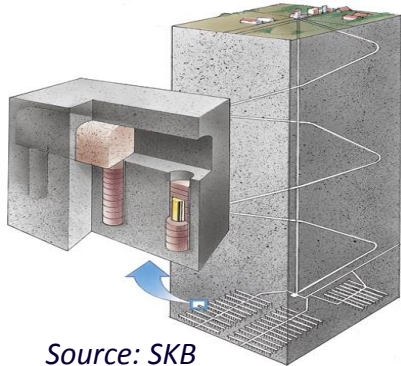
**COMSOL
CONFERENCE**
2015 GRENoble



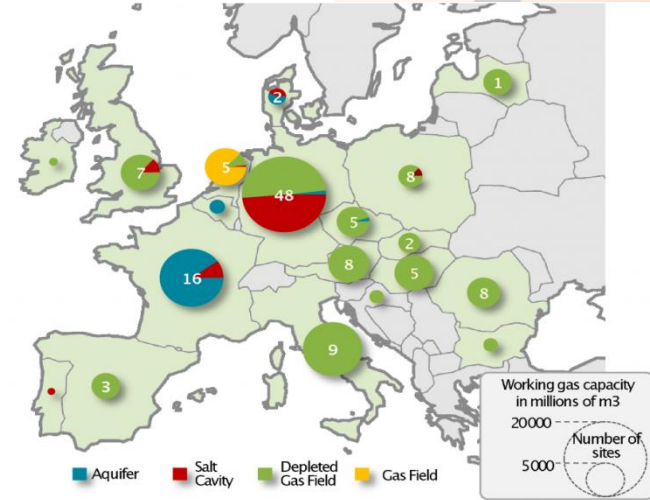
Multiphase flow situations

New uses of the underground

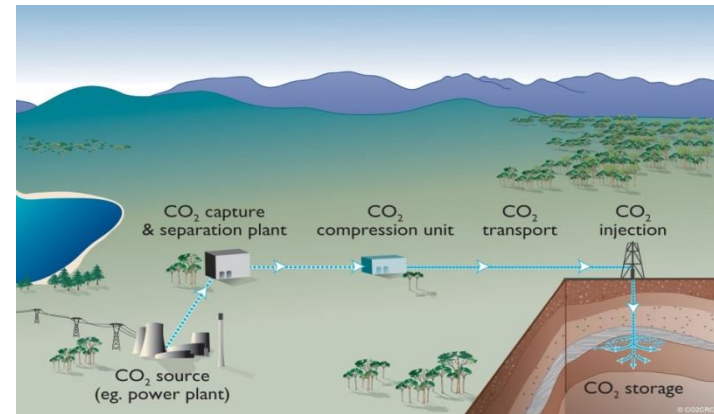
- Geological gas storage
 - Hydrogen storage
 - Carbon capture and storage
- Oil and gas industry
 - Enhance Oil Recovery
- Nuclear waste storage



Source: SKB



Source: GSE 2013



Source: CRC for GHG (CO2CRC)

Multiphase flow formulation

ADE system

Component mass conservation

$$\partial_t(\phi S_\alpha \rho_\alpha m_\alpha^k M^k) = -\nabla(\mathbf{q}_\alpha \rho_\alpha m_\alpha^k M^k - \phi S_\alpha \rho_\alpha \mathbf{D}_\alpha \nabla(m_\alpha^k M^k)) + Q_\alpha^k + T_\alpha^k$$

$$\mathbf{q}_\alpha = -\frac{k k_{r,\alpha}}{\mu_\alpha} (\nabla P_\alpha - \rho_\alpha \mathbf{g})$$

$\alpha =$ wetting & non-wetting phase

$k =$ components

Typically 4
equations

Constitutive equations

Total saturation

$$S_n + S_w = 1$$

Capillarity pressure

$$\begin{cases} P_{cap} = P_n - P_w \\ P_{cap} = P_{cap}(S_w) \end{cases}$$

Relative permeability $k_l^r, k_g^r = f(S_w)$

Gas volume

$$V_g = V_g(p_g, T, m_l^{CO_2}, m_s^{NaCl})$$

Liquid density

$$\rho_b = \rho_b(p_l, T, m_l^{CO_2}, m_s^{NaCl})$$

Viscosity

$$\mu_g = \mu_g(p_g, T, m_l^{CO_2}, m_s^{NaCl})$$

...

Multiphase flow formulation

Modeler alternatives

- How to represent the **capillary pressure**
 - Van Genuchten
 - Brooks & Corey
 - Specific interfacial area
- Which **unknowns** will be solved
 - Phase pressure & Phase saturation (S_α, P_α)
 - “Global” variables (total pressure or capillary pressure)
- Which **combination of governing equations** will be solved
 - Global equation, phase equation, linear combinations,...
- Which **state variables** will be used
 - Thermodynamic relations

Multiphase flow formulation

A²¹

COMSOL Implementation

Non-wetting + wetting eq.

$$0 = \nabla \left(k \lambda_n \left(\nabla P_w + \frac{dP_{cap}}{dS_n} \nabla S_n - \rho_n \mathbf{g} \nabla z \right) \right) + \nabla (k \lambda_w (\nabla P_w - \rho_n \mathbf{g} \nabla z)) + q_n + q_w$$

Non-wetting eq.

$$\phi \partial_t (S_n) = \nabla \left(k \lambda_n \left(\nabla P_w + \frac{dP_{cap}}{dS_n} \nabla S_n - \rho_n \mathbf{g} \nabla z \right) \right) + q_n$$

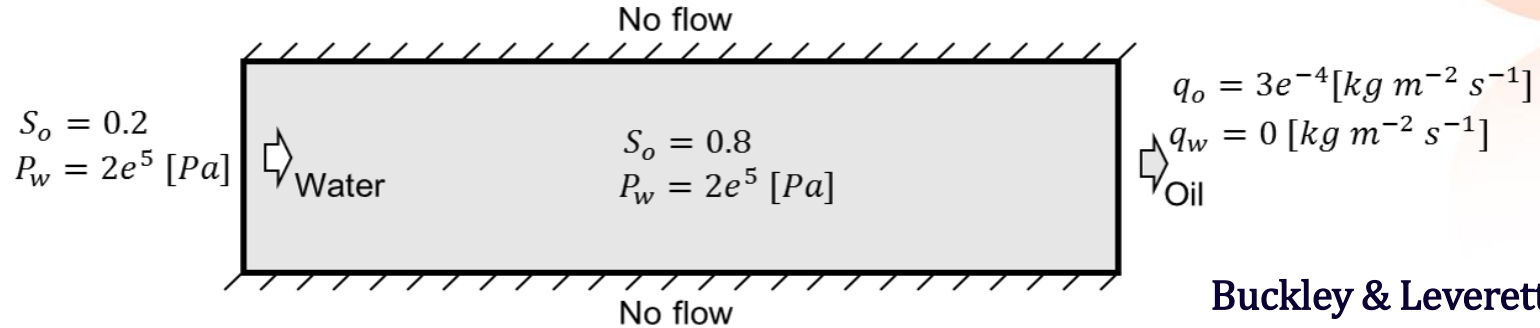
Matrix form:

$$\begin{pmatrix} 0 & 0 \\ 0 & \phi \end{pmatrix} \frac{\partial}{\partial t} \begin{pmatrix} P_w \\ S_n \end{pmatrix} + \nabla \cdot \left[-k \begin{pmatrix} \lambda & \lambda_n \frac{dP_{cap}}{dS_n} \\ \lambda_n & \lambda_n \frac{dP_{cap}}{dS_n} \end{pmatrix} \nabla \begin{pmatrix} P_w \\ S_n \end{pmatrix} - kg \begin{pmatrix} \lambda_w \rho_w + \lambda_n \rho_n \\ \lambda_n \rho_n \end{pmatrix} \right] = \begin{pmatrix} q_n + q_w \\ q_n \end{pmatrix}$$

COMSOL Coefficient Form PDE

$$e_a \frac{\partial^2 \mathbf{u}}{\partial t^2} + d_a \frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot (-c \nabla \mathbf{u} - \alpha \mathbf{u} + \gamma) + \beta \cdot \nabla \mathbf{u} + a \mathbf{u} = \mathbf{f}$$

Benchmarking – Buckley-Leverett problem



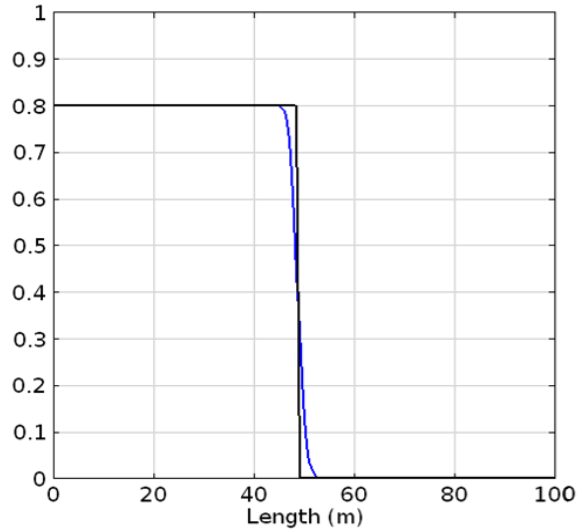
Buckley & Leverett, 1942

- 1D displacement of oil by water
- Immiscible fluids
- No capillary pressure
- Various fluid viscosities
- Gravity effects neglected
- Homogeneous porous media

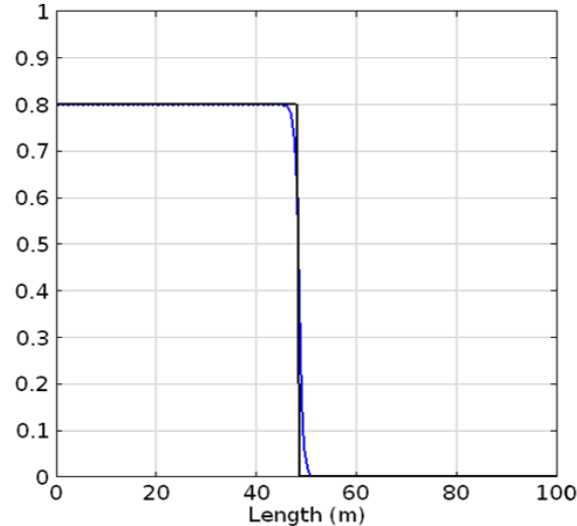
Benchmarking – Buckley-Leverett problem

Water Saturation after 300 days

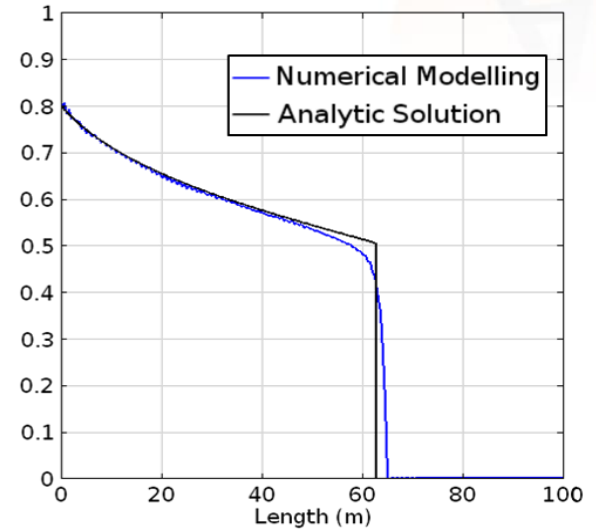
$$\mu_w / \mu_o = 1$$



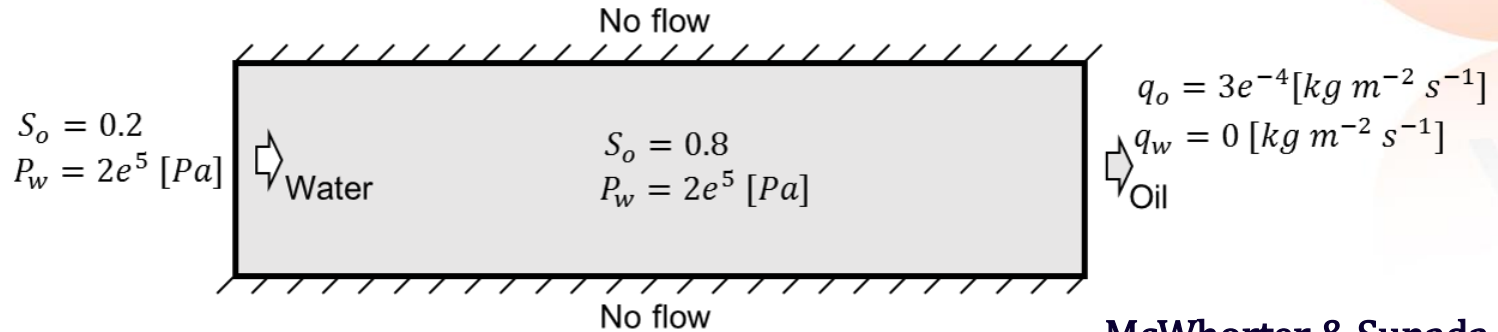
$$\mu_w / \mu_o = 2$$



$$\mu_w / \mu_o = 2/3$$



Benchmarking – McWorther problem

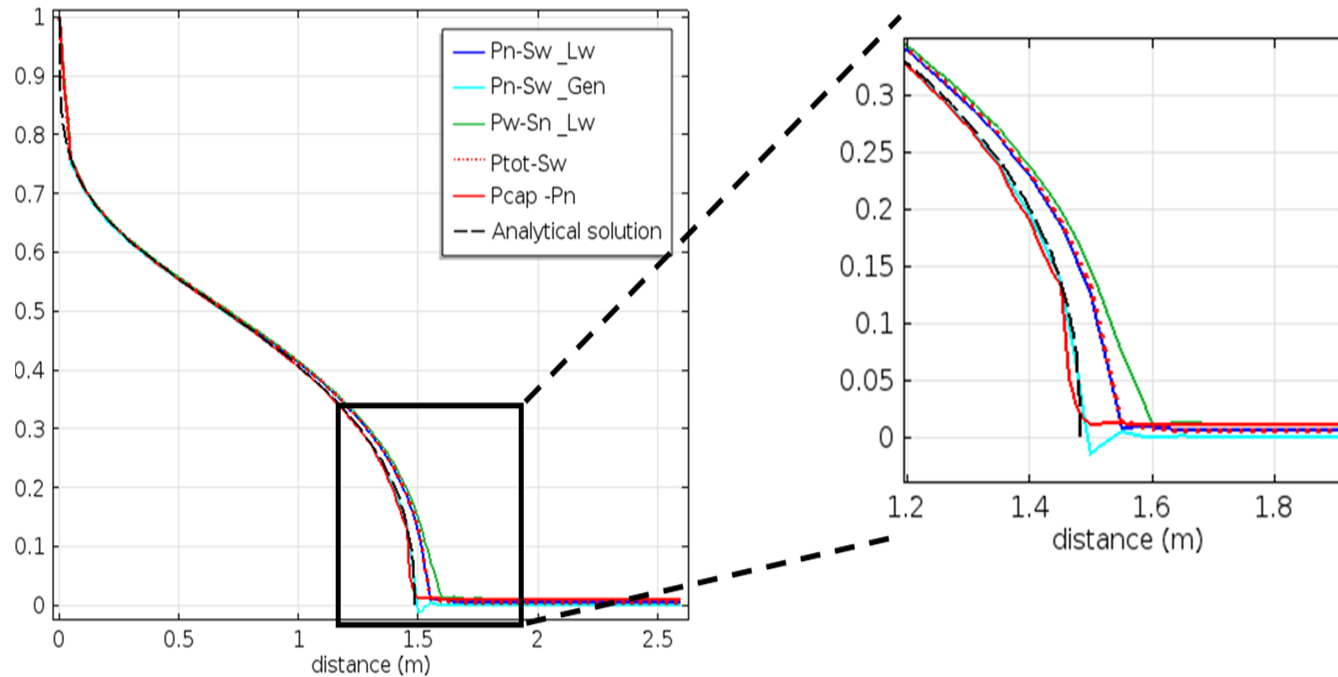


McWhorter & Sunada, 1990

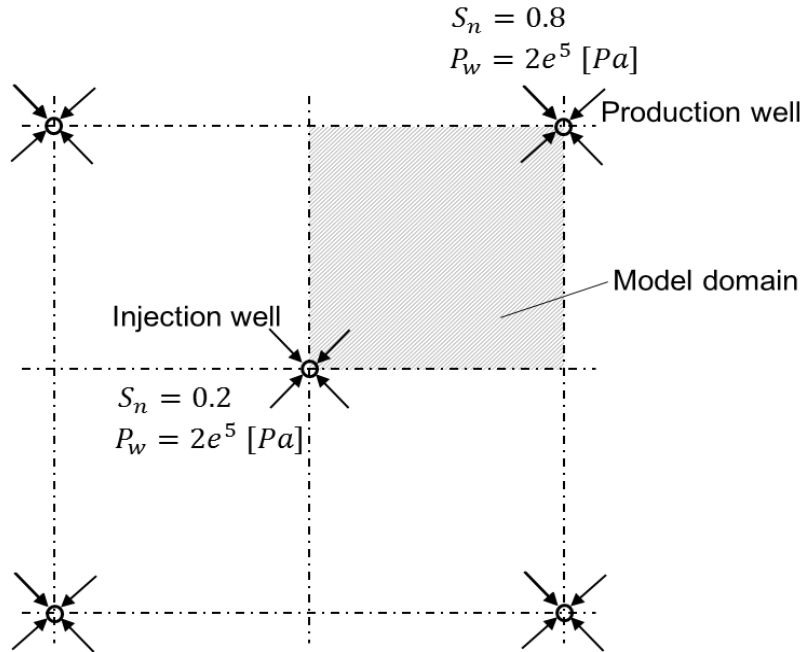
- 1D displacement of oil by water
- Immiscible fluids
- Capillary pressure
- Equal fluid viscosities
- Gravity effects neglected
- Homogeneous porous media

Benchmarking – McWorther problem

Water Saturation after 2.78 hours



Benchmarking – Five spot problem

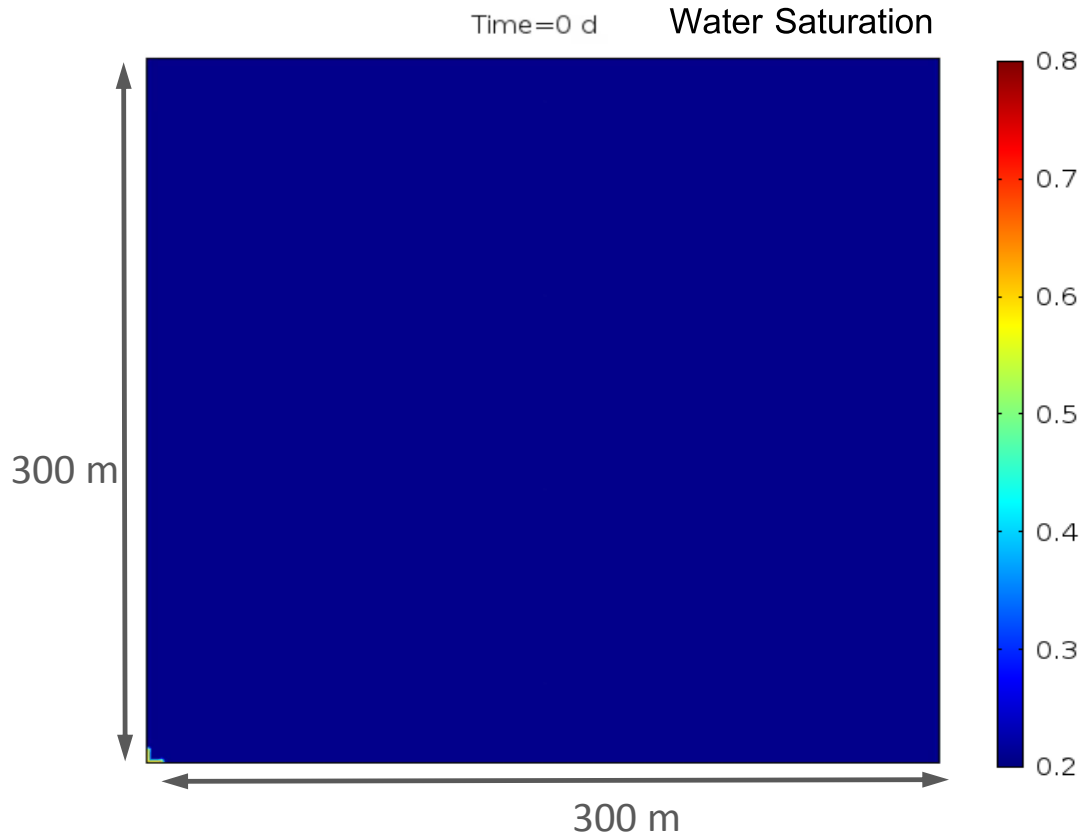


Chen, et al., 2006

- 2D displacement of oil by water
- Immiscible fluids
- No capillary pressure
- Equal fluid viscosities
- Gravity effects neglected
- Homogeneous porous media

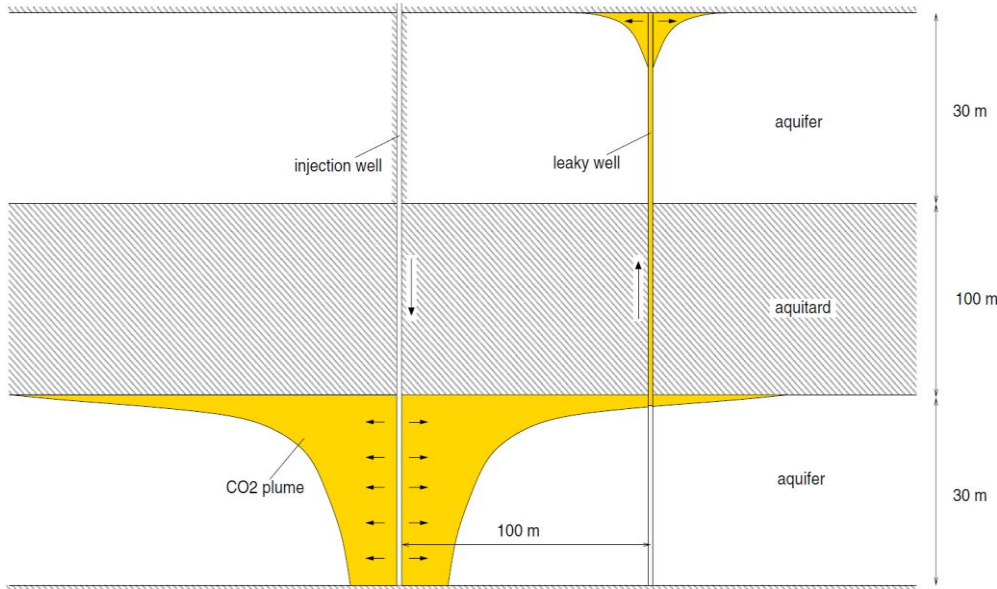
Multiphase flow

Benchmarking – Five spot problem



Multiphase flow

Benchmarking – CCS well leakage

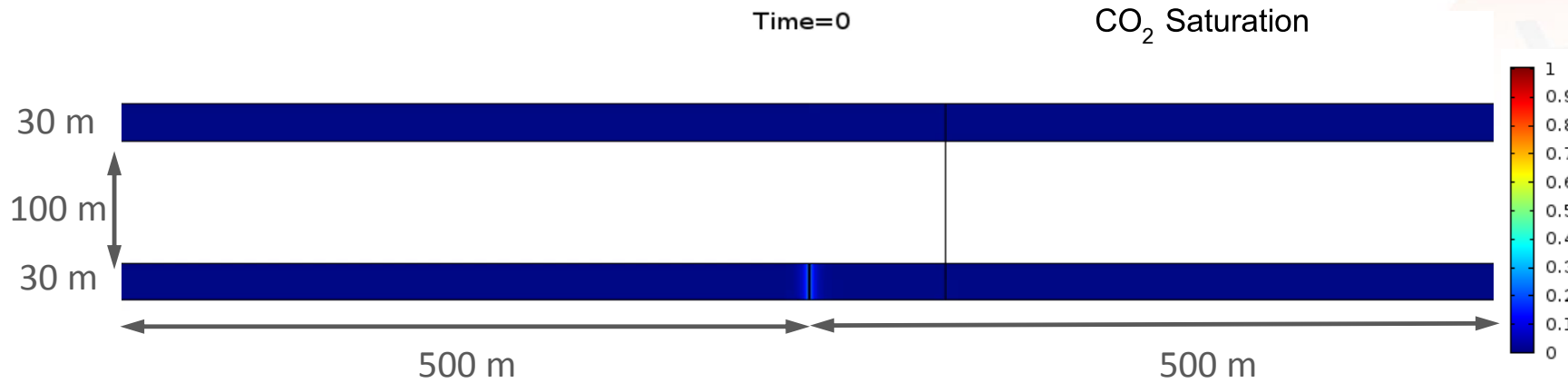


Ebigbo et al., 2007

- 2D model simplification of the 3D problem
- Immiscible fluids
- No capillary pressure
- ρ, μ are constant
- isothermal

Multiphase flow

Benchmarking – CCS well leakage



Summary

- Formulations reproduced multiphase physical processes
- equations of state of oil, water, brine and CO₂^{sup}
 - easily extended to any other fluid
- The manner the equations are combined matters
- **Each formulation has its own benefits and drawbacks**
- The preferred may vary depending on the physics and numerical methods

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SCIENTIFIC AND STRATEGIC ENVIRONMENTAL CONSULTING

ESPAÑA

AMPHOS 21 CONSULTING S.L.

Paseo de García Faria, 49-51

08019 BARCELONA

Tel.: +34 93 583 05 00

Fax : +34 93 307 59 28

CHILE

AMPHOS 21 CONSULTING CHILE Ltda.

Av. Nueva Tajamar 481 of. 1005 (Torre Sur)

Las Condes 7550099

SANTIAGO DE CHILE

Tel.: +562 27991630

PERÚ

AMPHOS 21 CONSULTING PERU S.A.C.

Av. del Parque Sur 661, San Borja

Lima 41

Tel.: +511 5921275

FRANCE

AMPHOS 21 CONSULTING FRANCE S.A.R.L.

14 Avenue de l'Opéra

75001 PARIS

Tel.: +33 1 69345030



www.amphos21.com

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