Ultrasensitive Mass Sensing Through Coupled Microelectromechanical Resonator Arrays

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Abstract

Microelectromechanical coupled resonating arrays (CRA) are being used for detecting analytes through mass-sensing. A CRA can be considered as a periodic ordered system. On mass or stiffness perturbation, system becomes disordered and system parameters viz., eigenvalues and eigenvectors undergo a change which is a measure of the perturbation. Eigenvalue sensitivity may be defined as the relative shift in eigenvalue per unit mass perturbation. If the CRA is an n-DOF system, any of the n resonators may be perturbed and any of the n eigenvalues may be measured. This gives rise to n x n possibilities, which are represented in an n x n eigenvalue sensitivity matrix. Using first order perturbation theory, a relation between eigenvalue sensitivity matrix and the eigenvector matrix has been derived in this article, stating that the sensitivity matrix can be approximated as square of the original eigenvector matrix. Since this is an approximation, there is bound to be an error in the measured mass w.r.t the actual mass, which can be modeled as measurement noise.

The highest value of sensitivity (theoretically equal to unity) is obtained in the absence of coupling, i.e. for a single DOF system. However, coupling offers other significant advantages. The next best value (equal to 0.67) is achieved for 3-DOF system, when the second resonator is perturbed and shift in the third eigenvalue is examined. For a fixed value of coupling coefficient (ratio of effective stiffness of single resonator to that of coupling element), sensitivity goes on decreasing with increasing number of resonators. Using COMSOL Multiphysics, this was verified by performing FEM analysis of overhang coupled cantilever array fabricated from Poly-Si. For the same inter-coupling distance, maximum sensitivity achieved with 3-DOF system was 0.67 fg/Hz and with 2-DOF system was 10 fg/Hz. As the coupling coefficient increases above 0.1, the sensitivity increases and error decreases. Using COMSOL Multiphysics, it was observed that sensitivity of disk resonators coupled at anti-nodal points is greater than that of an overhang coupled cantilever array. Error was found to decrease with a decrease in mass perturbation.

Sensitivities, orders of magnitude higher than their eigenvalue counterparts, were achieved by observing shifts in the eigenvectors of weakly coupled systems. This was a result of the vibration localization effect, observed in MEMS resonators. This was verified in COMSOL Multiphysics by performing the FEM analysis of arrays of electrically coupled clamped-clamped beams. Beam amplitudes were approximated as eigenvectors of the system. For a 3-DOF CRA, the maximum eigenvector sensitivity observed was 3.8395, an order of magnitude higher than its eigenvalue counterpart.

Reference

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[3] Bhaskar Choubey, et al. "Characterization of coupled micro/nanoresonators using inverse eigenvalue analysis." Applied Physics Letters 97.13 (2010): 133114.
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Figures used in the abstract

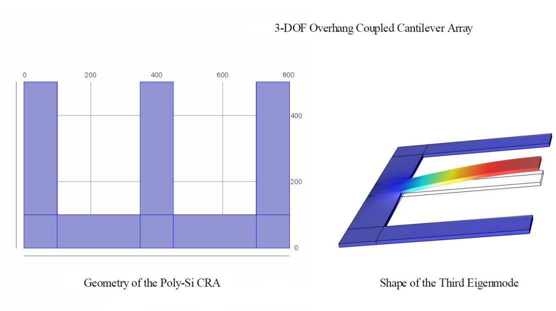

Figure 1: Geometry & Eigenmode: 3-DOF Cantilever Array.

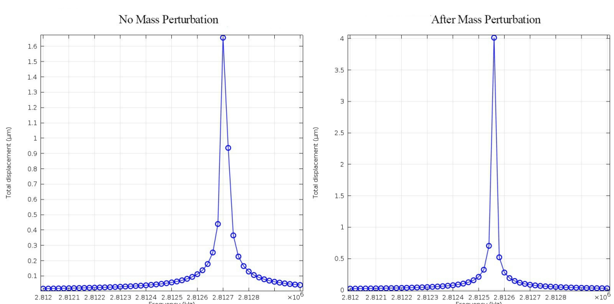

Figure 2: Frequency Response Function of a 2-DOF System: Before and After Perturbation.

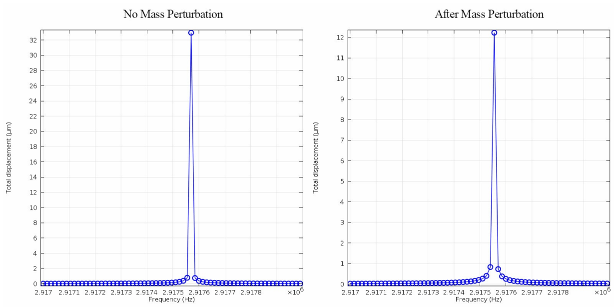

Figure 3: Frequency Response Function of a 3-DOF System: Before and After Perturbation.

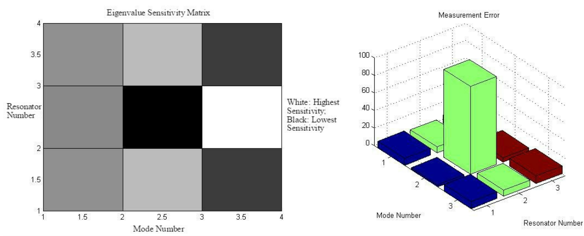

Figure 4: Eigenvalue Sensitivity & Error Matrices for a 3-DOF System.