

# Extraction of Electrical Equivalent Circuit of One Port SAW Resonator using FEM based Simulation

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**Abstract:** The paper presents a method of extraction of electrical equivalent circuit of a one port surface acoustic wave (SAW) resonator from the results of simulation based on finite element method (FEM) using COMSOL Multiphysics. A one port SAW resonator consists of large number of periodic interdigital transducer (IDT) electrodes fabricated on the surface of a piezoelectric substrate. A section of aluminium IDT structure patterned on YZ lithium niobate piezoelectric substrate with periodic boundary condition is incorporated in the simulation. The equivalent circuit of a SAW resonator consists of motional resistance, capacitance and inductance connected in series, and static capacitance in parallel. The electrical resonance characteristics of SAW resonator obtained from frequency domain analysis provided by COMSOL Multiphysics are used to calculate the equivalent circuit parameters. The calculation of electrical equivalent circuit parameters is useful in designing matching circuits for SAW devices.

**Keywords:** Equivalent electrical circuit, FEM, IDT, MEMS, SAW device, SAW resonator.

## 1. Introduction

The surface acoustic wave (SAW) devices such as delay line, resonator, filter, oscillator, convolver, and correlator are used in communication and signal processing applications [1]–[4]. Other SAW devices such as sensor and actuator are used for industrial applications [5], [6]. The SAW technology plays important role in MEMS industries due to low cost, batch fabrication, light weight and inherent properties of its device substrate [2], [4]. The development of mobile phones stirred up more interest in the investigation of SAW devices. The SAW device consists of interdigital transducer (IDT) made of aluminum or gold material fabricated on the surface of the device substrate

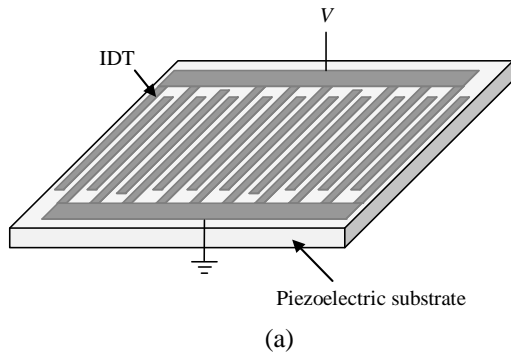
made of piezoelectric materials [3]. An IDT is a metallic comb-shaped periodic structure which converts electrical energy into acoustic energy and vice versa [3], [4]. The SAW devices are generally operated in two different ways: resonator and delay line [5]. A SAW delay line type device is a two port device, where two IDTs are fabricated at the two ends of the substrate separated by a few wavelengths. One IDT is called as a transmitter IDT and the IDT at the other end is called as a receiver IDT. The resonator devices are mainly of two types: one port resonator and two port resonator. In one port SAW resonator, a bidirectional IDT is fabricated with a set of reflectors on either side or a bidirectional IDT with large number of electrodes is fabricated without reflectors. In case of two port resonator, two sets of reflectors are fabricated on the outer sides of the two bidirectional IDTs. The function of IDT fabricated on the piezoelectric substrate in the SAW device is described by several methods of modeling such as discrete source or delta function method, impulse response method, piezoelectric permittivity method, matrix representation, coupling of mode (COM) method, numerical techniques using equations, and equivalent circuit method. The detailed description of the above methods is presented by many researchers [2], [3], [7]. The discrete source method or delta function method is one of the earliest and simplest methods and explains the shape of the IDT frequency response [3], [7]. It does not consider energy, capacitance, and electromechanical coupling coefficient of the material used in the device [3]. The impulse response method predicts the absolute amplitude, hence the power of SAW by considering the energy of the wave [3]. The piezoelectric permittivity method describes the electric potential associated with SAW in terms of actual charge density by considering the

surface piezoelectric permittivity and quasi-static approximation [2], [3], [8]. COM model is widely used in designing of SAW devices [2], [3], [9], [10]. This method is based on local analysis of way of two coupled waves evolve when propagating in both directions opposite to each other. The Mason equivalent circuit for surface wave transducers is well explained by smith *et al.* [11]. The equivalent electrical circuit at frequency near the resonance of one port SAW resonator can be presented by motional resistance, capacitance and inductance connected in series and static capacitance in parallel [7], [8]. It is called as Butterworth Van-Dyke model of piezoelectric resonator [8].

In this paper, we present a method of extraction of the electrical equivalent circuit from the finite element simulation of one port SAW resonator having large number of IDT electrodes fabricated on the surface of piezoelectric substrate. The electrical equivalent circuit parameters are calculated using electrical resonance characteristics obtained from the simulation of SAW resonator. The electrical equivalent circuit is useful in designing of matching circuits for SAW devices and also understanding about the electrical and resonance behavior of SAW devices in easy manner. The piezoelectric constitutive equations, electrical equivalent circuit parameters, simulation methodology, results and discussion, and conclusions are given in the following sections.

## 2. Piezoelectric Constitutive Equations

The electromechanical constitutive equations for piezoelectric substrate used in the simulation



are governed by the continuum equation of motion, Maxwell's equations under the quasi-static assumptions, the strain-mechanical displacement relations and appropriate boundary conditions [2], [8]. In a homogeneous piezoelectric substrate, the stress component  $T_{ij}$  at each point of substrate depends on the applied electric field  $\mathbf{E}$  or electric displacement  $\mathbf{D}$ . The stress tensor component  $T_{ij}$  can be expressed as [2]

$$T_{ij} = \sum_k \sum_l c_{ijkl}^E S_{kl} - \sum_k e_{kij} E_k$$

where,  $c_{ijkl}^E$  is the stiffness tensor for constant electric field,  $S_{kl}$  is the strain tensor,  $e_{kij}$  is the piezoelectric tensor, and  $E_k$  is the electric field.

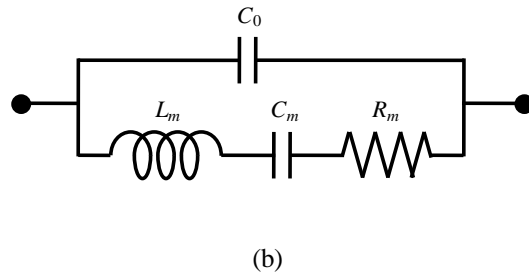
The electric displacement  $\mathbf{D}$  is determined by electric field  $\mathbf{E}$  and permittivity of the material  $\epsilon_{ij}$ . The electric displacement  $\mathbf{D}$  in piezoelectric substrate is also related to strain. It can be expressed as [2]

$$D_i = \sum_j \epsilon_{ij}^S E_j + \sum_j \sum_k e_{ijk} S_{jk}$$

where,  $D_i$  represents the electric displacement and  $\epsilon_{ij}^S$  represents the permittivity tensor for constant stress.

A quasi-static approximation is used to express the electric field since the elastic vibration travels much slower than the electromagnetic waves [2], [3], [8].

## 3. Electrical Equivalent Circuit Parameters



**Figure 1.** (a) Schematic of a one port SAW resonator with large number of IDT electrodes without reflectors and (b) electrical equivalent circuit of a one port SAW resonator.

The basic resonator modeling requires theory of acoustic wave propagation, material properties, geometric dimensions and electric field approximations [2], [8]. Another successful way, there is an electrical equivalent circuit model used to characterize the resonance behavior of the SAW resonators. The conceptual picture of a one port SAW resonator with large number of IDT electrodes without reflectors is shown in figure 1 (a) [12]. The corresponding electrical equivalent circuit at near resonance frequency is shown in figure 1 (b), where  $C_0$  is the static capacitance which represents the fixed dielectric capacitance of the resonator,  $L_m$  and  $C_m$  are the motional inductance and capacitance, which correspond to the contribution of inertia and elasticity respectively, and  $R_m$  is the motional resistance which corresponds to the contribution of the damping [7]. The motional resistance carries the information about acoustic energy dissipation in the SAW devices [8]. These parameters are calculated from resonance frequency  $f_r$ , anti-resonance frequency  $f_{ar}$ , conductance  $G$  and quality factor  $Q_r$  at resonance frequency of the SAW resonator [7]. The equations for the calculation of these parameters are given as

$$R_m = G_r = G|_{f=f_r}$$

$$C_m = \frac{1}{\omega_r Q_r G_r}$$

$$C_0 = \frac{C_m}{(\omega_{ar}/\omega_r)^2 - 1}$$

$$L_m = \frac{G_r Q_r}{\omega_r}$$

where  $G_r$  is the peak value of the conductance, and  $\omega = 2\pi f$  is the angular frequency of the SAW resonator.

The quality factor  $Q_r$  at resonance of SAW resonator is determined as [13]

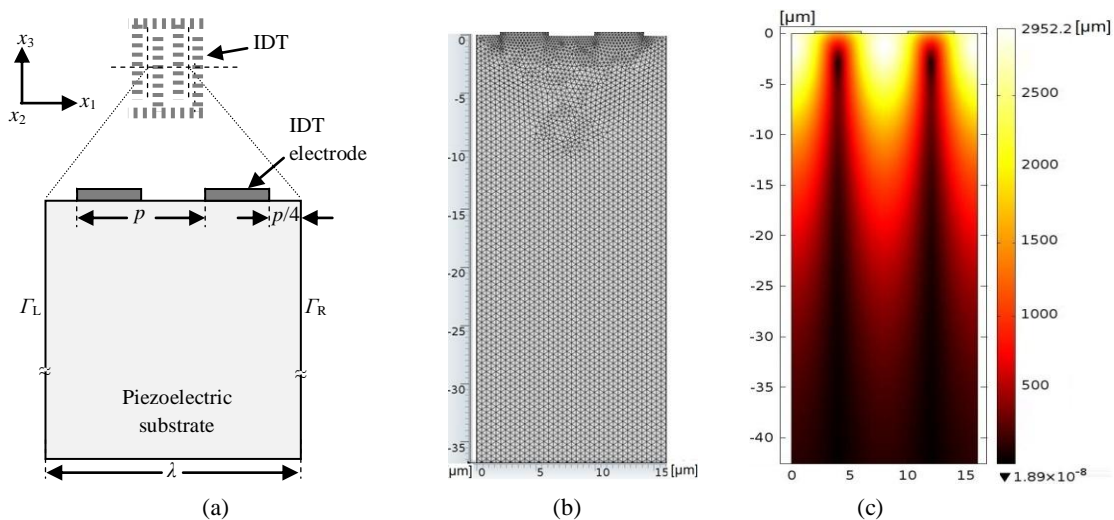
$$Q_r = \frac{f_r}{\Delta f}$$

where  $\Delta f$  is the bandwidth at half of the peak-conductance.

The resonance frequency  $f_r$ , anti-resonance frequency  $f_{ar}$ , conductance  $G$  and quality factor  $Q_r$  at resonance frequency are determined from the electrical resonance characteristics of SAW resonator obtained from the frequency domain analysis provided by COMSOL Multiphysics.

#### 4. Simulation Methodology

A one port SAW resonator with metallic IDT having large number of electrodes fabricated on piezoelectric substrate is considered for the simulation. Due to periodic nature of IDT structure, a section of IDT electrodes is incorporated and appropriate boundary



**Figure 2.** (a) 2D geometry of one port SAW resonator with one periodic section of IDT electrodes, (b) picture of mesh applied to one port SAW resonator and (c) displacement profile at resonance frequency of 212.88 MHz.

conditions are applied in the simulation [12], [14]. The 2D geometry of the SAW resonator with one periodic section of IDT electrodes used for simulation is shown in figure 2 (a). The dimensions are as follows: depth of the piezoelectric substrate  $160 \mu\text{m}$  ( $10 \lambda$ ) in  $-x_3$  direction, width of the piezoelectric substrate  $16 \mu\text{m}$  ( $1 \lambda$ ) in  $-x_1$  direction, period of IDT electrode ( $p$ )  $8 \mu\text{m}$ , width of IDT electrode ( $p/2$ )  $4 \mu\text{m}$ , electrode thickness  $0.2 \mu\text{m}$  in  $x_3$  direction. The YZ lithium niobate (YZ LiNbO<sub>3</sub>) is used for piezoelectric substrate and aluminium metal is used for IDT electrodes. The material properties such as values of elastic constants, permittivity constants, stress constants and density are used from Warner *et al.* [15].

The bottom boundary of the piezoelectric substrate is fixed and all other boundaries are traction free. The periodic boundary condition is applied to left ( $\Gamma_L$ ) and right ( $\Gamma_R$ ) sides of the device as shown in figure 2 (a). The applied periodic boundary condition is given as [12],

[14]

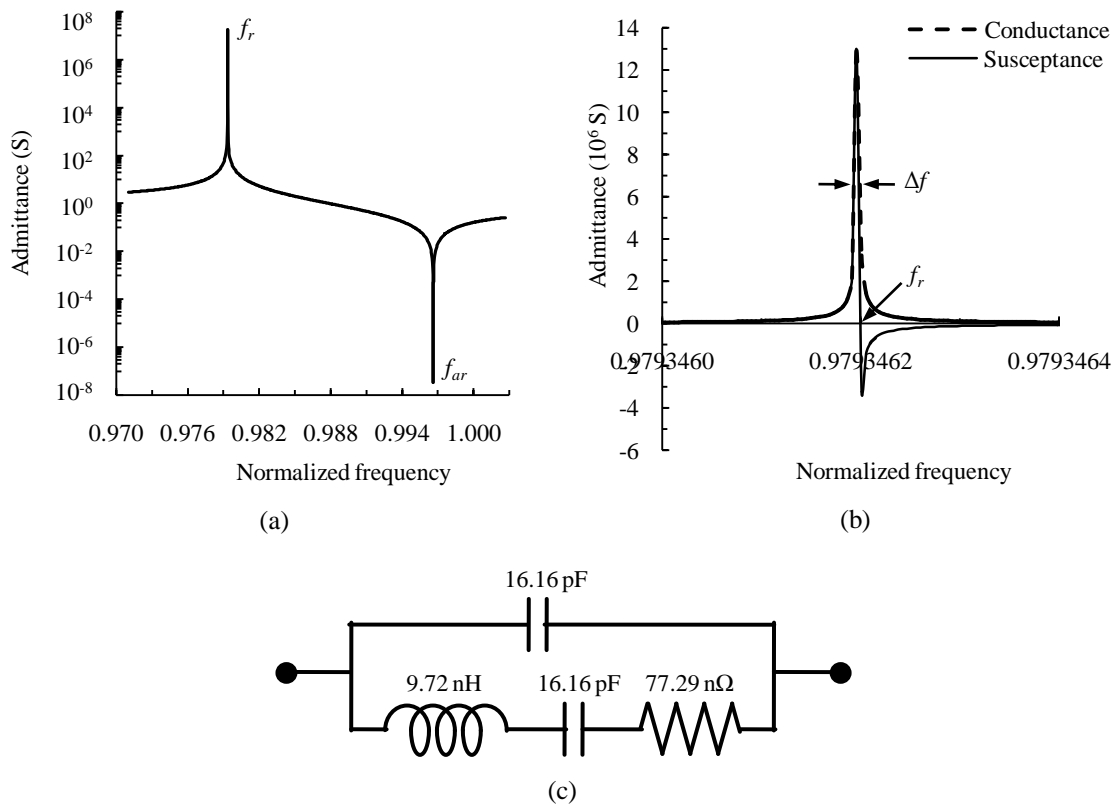
$$\Gamma_L(u, v, V) = \rho \Gamma_R(u, v, V)$$

$$\rho = (-1)^n, \quad n = 2a/\lambda$$

where  $u$ ,  $v$  and  $V$  are the  $x$ -displacement in  $x_1$  direction,  $y$ -displacement in  $x_3$  direction and applied electric potential, respectively, and  $a$  is the width of the piezoelectric substrate in multiples of  $\lambda$ .

The electric potential of 1 V is applied to the IDT electrode. Extremely fine mesh with minimum element quality of 0.5 is used. The number of degree of freedom solved is in the order of around  $2 \times 10^5$ . The picture of mesh applied to the SAW resonator in the simulation is shown in figure 2 (b).

The simulation is carried out using MEMS module provided by COMSOL Multiphysics [16]. The frequency domain analysis is used for calculation of electrical resonance characteristics of SAW resonator.



**Figure 3.** Results of simulation of one port SAW resonator: (a) plot of magnitude of harmonic admittance per period versus normalized frequency and (b) plot of harmonic admittance per period versus normalized frequency of one port SAW resonator, and (c) electrical equivalent circuit of a one port SAW resonator with parameters value.

## 4. Results and Discussion

The results of simulation of a one port SAW resonator are shown in figure 2 (c) and figure 3 (a) and (b). Figure 2 (c) shows the displacement profile at resonance frequency. Figure 3 (a) and (b) shows the electrical resonance characteristics of SAW resonator. The plot of harmonic admittance per period as a function of normalized frequency is obtained for an electric potential of 1 V applied to the IDT electrodes. The normalized frequency is expressed as  $\eta = 2pf/v_0$ , where  $v_0 = 3477.91$  m/s is the free surface velocity of the piezoelectric substrate calculated from eigenmode analysis. Figure 3 (a) shows the plot of magnitude of harmonic admittance as a function of normalized frequency. From figure 3 (a), the resonance and anti-resonance frequencies of the SAW resonator are observed at 212.88 MHz and 216.64 MHz, respectively. The value of admittance is high due to long aperture of IDT electrodes. Figure 3 (b) shows the plot of harmonic admittance as a function of normalized frequency. From figure 3 (b), the quality factor  $Q_r$  is calculated which is  $168.15 \times 10^6$ . The electrical equivalent circuit parameters calculated from the results of simulation of one port SAW resonator are given in Table 1.

Table 1 Electrical equivalent circuit parameters

Parameter(s)	Value
Motional resistance	77.29 nΩ
Motional capacitance	16.16 pF
Motional inductance	9.72 nH
Static capacitance	16.16 pF

The extracted electrical equivalent circuit of the simulated one port SAW resonator having large number of IDT electrodes is shown in figure 3 (c).

## 5. Conclusions

The paper has presented the extraction of electrical equivalent circuit of one port SAW resonator with large number of IDT electrodes. The admittance value of admittance is high due to long aperture and no damping in the simulation. As the damping is not applied in the simulation of the SAW resonator, the value of

motional resistance corresponding to damping is observed at 77.29 nΩ. The electrical equivalent circuit is useful in understanding the resonance behavior and designing of matching circuits for SAW devices. This simulation can be further extended to study the equivalent circuit parameters of SAW resonator with finite number of IDT electrodes and other SAW devices.

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