Analysis of High-Frequency Thermoacoustic Instabilities in Lean-Premixed Gas Turbine Combustors

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Abstract: This paper presents a thermoacoustic analysis methodology of high-frequency instabilities in lean-premixed combustors, which centrally relies on COMSOL Multiphysics. The Linearized Euler Equations are discretized on a Finite Element mesh of a given combustor configuration. Then, a Reduced Order Model is derived from the resulting system matrices, which allows efficient thermoacoustic analyses in frequency and time domain. COMSOL simulations of the Linearized Euler systems serve as verification benchmarks for the Reduced Order Models. A demonstrative analysis is carried out and compared against experimental benchmark from a lab scale combustor that is operated in a self-sustained state of high-frequency oscillations.

Keywords: Thermoacoustic Instabilities, Stability Analysis, Reduced Order Modeling, Linearized Euler Equations

1. Introduction

Gas turbines play a crucial role in our future power generation landscape due to their high potentials in terms of operational flexibility, emissivity level and efficiency values. Lean-premixed combustion is commonly employed within the gas turbine combustor to meet these latter criteria, which is however associated with one significant downside: thermoacoustic instabilities. These instabilities arise due to a constructive coupling between the flame's unsteady heat release and the combustor's natural acoustics modes. Physically, these instabilities manifest as large amplitude pressure pulsations within the combustor, and may cause prohibitive emission levels associated with the combustion process or even exert detrimental mechanical stresses on hardware components. Avoidance of the instabilities are thus of top priority for the design of new and the retrofitting of existing gas turbine systems. The ability to analyze a given combustion system, and predict its underlying thermoacoustic performance represents an essential element within these foregoing engineering tasks. A representative lean-premixed, tubular combustor – which high-frequency thermoacoustic performance is used as the test case subject within this work – is shown in Fig. 1 revealing constituting components and functionality.

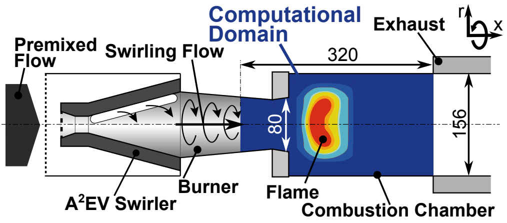

Figure 1: Lab-scale gas turbine combustor

2. Outline of Paper

The content of this paper is mainly retrieved from previous scientific publications [1, 5, 6, 7, 12] of the authors. Please consult these references, and the references provided therein, for specific explanations and elaborations on the upcoming paper's content. This paper's structure unfolds as follows. First, relevant theoretical background is presented. Then, the use of COMSOL Multiphysics is specfically outlined, followed by providing information on the test case benchmarks used. Finally, analysis results are presented and briefly discussed.

3. Theoretical Background

This section presents the relevant theoretical background of the following paper, and specifically unfolds into the presentation of governing equations, the discretization via a stabilized Finite Elements (FEM) scheme, and derivation of derivation of Reduced-Order Models (ROM) for thermoacoustic system analyses.

3.1. Governing Equations

Acoustic oscillations in the context of thermoacoustic instabilities are mathematically described by a first order approximation, i.e.

$$\phi(\mathbf{x}, t) = \bar{\phi}(\mathbf{x}) + \phi'(\mathbf{x}, t), \tag{1}$$

where the variable $\phi(\mathbf{x}, t)$ represents the conservation variables density (ρ), vectorial velocity ($\mathbf{u}$), and pressure (p) of the flow in the concerned system (i.e. the combustion chamber). In Eqn. 1, steady and unsteady acoustic quantities are denoted by bar $(\bar{.})$ and prime $(.')$ symbols, respectively. Substituting Eqn. 1 in the full Navier-Stokes Equation (cf. [2]) while neglecting viscous and thermal diffusion yields the Linearized Euler Equations (LEE):

$$\begin{aligned} \frac{\partial p'}{\partial t} &+ \gamma p' \nabla \cdot \bar{\mathbf{u}} + \gamma \bar{p} \nabla \cdot \mathbf{u}' \\ &+ \bar{\mathbf{u}} \cdot \nabla p' + \mathbf{u}' \cdot \nabla \bar{p} = q' + f_e \end{aligned} \tag{2}$$

$$\begin{aligned} \frac{\partial \mathbf{u}'}{\partial t} &+ \bar{\mathbf{u}} \cdot \nabla \mathbf{u}' + \mathbf{u}' \cdot \nabla \bar{\mathbf{u}} \\ &+ \frac{\rho' \bar{\mathbf{u}} \cdot \nabla \bar{\mathbf{u}}}{\bar{\rho}} + \frac{\nabla p'}{\bar{\rho}} = 0 \end{aligned} \tag{3}$$

Acoustic isentropicity is assumed for this work, which allows to relate the acoustic pressure and density through $p' = \rho' \bar{c}^2$ with $\bar{c} = \bar{c}(\mathbf{x})$ being the local speed of sound. Besides spatial distribution, LEE systems as in Eqns. 2-3 carry information about damping (i.e. due to vortex shedding [8]) of the oscillations due to the inclusion of coupling terms between non-uniform mean flow and acoustic quantities. The terms q' and f_e denote source term functions due to heat release oscillations and external excitation (e.g. via a siren), respectively. The former is used to model thermoacoustic interactions between flame and acoustics via a general function of the acoustic pressure [1] given by

$$q'_s(\mathbf{x}_f, t) = F_L(p'_s(\mathbf{x}_f, t)) + F_{NL}(p'_s(\mathbf{x}_f, t)) + \xi(\mathbf{x}_f, t), \tag{4}$$

where F_L and F_{NL} describe linear and nonlinear thermoacoustic conversion processes, while ξ_s denotes emission of stochastic broadband fluctuations due to turbulent combustion. Note that $q'_s = q'_s(\mathbf{x}_f, t)$ varies spatially accross the flame volume $\mathbf{x}_f$. Details on the explicit form of the terms in Eqn. 4, physical implications and inclusion in the analyses of these functions are given below.

3.2. FEM Discretization

The first step towards system solutions of Eqns. 2-3 comprises the discretization via a stabilized FEM scheme under consideration of appropriate boundary conditions as well as a properly meshed combustor volume. The employed stabilization schemes is the so called Streamline Upwind Petrov Galerkin residual based approach, details of which are available in [11, 12, 3, 4].The resulting, spatially discrete system reads in state space form

$$\mathbf{E} \frac{\mathrm{d}\phi'}{\mathrm{d}t} = \mathbf{A}\phi' + \mathbf{B}u, \tag{5}$$

where $\mathbf{E}$ is the descriptor matrix, $\mathbf{A}$ is the system matrix, and ϕ' is the state vector hosting the temporal acoustic quantities at each mesh node. The input matrix $\mathbf{B}$ inserts multiple temporal signals u into the system that correspond to distinct source term prescriptions of heat release oscillations and/or external acoustic forcing. Multiple temporal outputs of acoustic oscillations are computed by

$$y = \mathbf{C}\phi', \tag{6}$$

where $\mathbf{C}$ is the output matrix, which is assembled from Q row vectors, each carrying an unity entry at the position of the desired output variable and location within the mesh, and zero elsewhere. The systems treated within this work are large-scale of order $N \approx 300,000$. Temporal integrations – which are required to model effects of stochastic forcing and nonlinear flame dynamics – of such large systems are vastly expensive in terms of computational resources, and are thus impractical for efficient thermoacoustic system simulations. For this reason, the system of Eqns. 5 and 6 is subjected to a Model Order Reduction (MOR) technique to derive a Reduced Order Model (ROM) which features a system order significantly lower than the large scale system. At the same time, the ROM is required to accurately reproduce the large-scale system's performance.

3.3. Reduced Order Modeling

In order to derive the ROM, modal truncation is employed as MOR technique in this work. Within this method, the large-scale system is projected into a subspace spanned by a selected set of the given combustor configuration's acoustic eigenmodes. The number of considered eigenmodes directly translates into the ROM's system size. The selection of eigenmodes for the reduction occurs physics-based, i.e. based on the frequency range in which the dynamics are sought to be investigated/simulated. Please consult reference [7] for details on the derivation procedures. The final expression of the ROM

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[1] Neglection of velocity fluctuation dependence justified for high frequency modulation mechanisms [1, 5].

reads

$$\frac{d\phi'_R}{dt} = \mathbf{A}_R\phi'_R + \mathbf{B}_R u, \tag{7}$$

$$y = \mathbf{C}_R\phi'_R, \tag{8}$$

with a reduced system order N_R, where $N_R \ll N$. In Eqns. 7-8, the terms ϕ'_R, $\mathbf{A}_R$, $\mathbf{B}_R$, and $\mathbf{C}_R$ denote the ROM's state vector, system, input, and output matrix respectively. Notice that in- and outputs signals (u and y) are unaffected by the MOR. Finally, the quality of the ROM, i.e. its reproduction capabilities of the large-scale FEM-LEE system - is evaluated by the H_2-norm relative error

$$\text{REL.ERR.}(i\omega) = \frac{\|H(i\omega) - H_R(i\omega)\|_2}{\|H(i\omega)\|_2}, \tag{9}$$

where H and H_R denote the pressure responses due to an external excitation of the original and reduced system over a certain band of angular frequencies ω, respectively.

3.4. Thermoacoustic Feedback Modeling

High-frequency oscillations are typically associated with transversal, multidimensional modes, which feature acoustic length scales in the same order of magnitude as flame length scales (cf. Fig. 2).

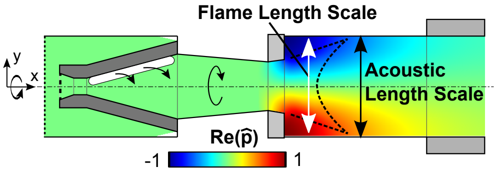

Figure 2: Transveral pressure mode and flame length scale relations

This implies that a local variability of thermoacoustic interactions between flame and acoustics needs to be accounted for. Such a so called thermoacoustically non-compact flame is handled by dividing the flame volume into several subregions, each of which can be rendered thermoacoustically compact, i.e. are geometrically small compared to the characteristic acoustic length scale of the mode of interest. Then, Eqn. 4 is used to model the thermoacosutic feedback for each subregion as schematically illustrated in Fig. 3, where the required acoustic reference quantities and the converted heat release oscillation resemble multiple output and input signals of the ROM state space system, respectively.

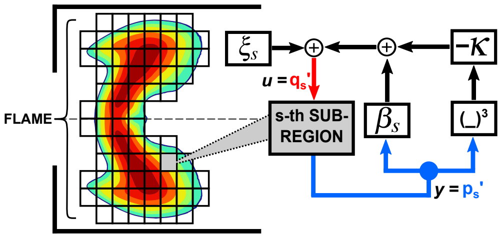

Figure 3: Non-compact feedback modeling approach

Due to the low order the state-space framework, system analyses can be easily conducted in frequency domain (e.g. for linear stability assessment) and time domain (e.g. for time domain limit cycle simulations) as is shown below. The last ingreillustratingdient required before such analyses can be conducted is an explicit functional description of the linear and nonlinear flame dynamics function in Eqn. 4, which is given by

$$F_L(p'_s) = \beta_s p'_s = \frac{1}{\gamma_s}\frac{\bar{q}_s}{\bar{p}_s}p'_s, \tag{10}$$

$$F_{NL}(p'_s) = -\kappa p'^3_s, \tag{11}$$

where the subscript s indicates the subregion. The quantities γ_s, $\bar{q}_s$ and $\bar{p}_s$ denote the mean flow's ratio of specific heats, mean heat release and pressure at subregion level, respectively. Physically, the linear function of Eqn. 10 describes the modulation of heat release due periodic flame shape deformations (cf. details in [1, 5]), whereas the nonlinear function in Eqn. 11 is empirical and originates from experimental observations (cf. details in [10, 9]). The parameter κ in the nonlinear function is an empirical coefficient used to reconstruct the experimentally observed limit cycle amplitude.

4. Use of COMSOL Multiphysics

A flow diagram illustrating the different work steps and highlighting the usage of COMSOL shows in Fig. 4. Specifically, thermoacoustic analyses as presented in this work start with the setup of the LEE description of the concerned combustion system. This requires COMSOL's CAD import module (due to using real 3D combustor geometries) as well as the Equation Based Modeling PDE Interface to directly prescribe the weak form of the LEE. Along with the PDE prescription, geometry meshing and boundary condition treatment need to be conducted. This entire problem setup is carried out in frequency domain, which provides – besides the discretization matrices $\mathbf{E}, \mathbf{A}, \mathbf{B}$ and $\mathbf{C}$ – the basis to perform reference solution for linear stability and frequency response studies. These study results serve as verification benchmarks

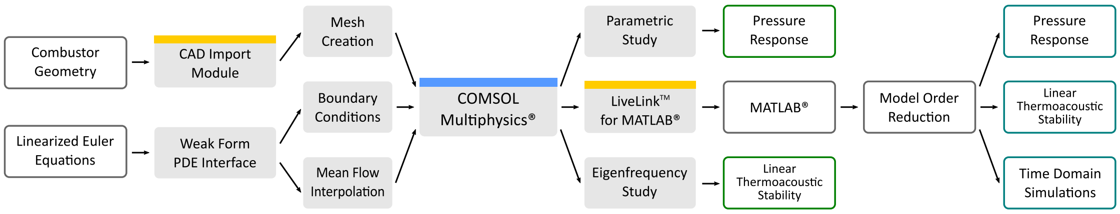

Figure 4: Schematic use of COMSOL Multiphysics

for the associated ROM of this work. Note that linear thermoacoustic analyses of high-frequency systems can be carried out directly in the COMSOL environment as demonstrated in [6, 12], while capturing of nonlinear and stochastic aspects needs to be executed in time domain with the ROM framework. The MOR reduction is performed in MATLAB, where access to COMSOL to retrieve system matrices, mesh information and eigenmode solutions are established via the MALAB LiveLink. All thermoacoustic analyses utilizing the ROM are then carried out within MATLAB/Simulink©.

5. Test Case Combustor

A swirl-stabilized lab scale combustor (cf. Fig. 1) that is operated under perfectly premixed conditions serves as the test case. This combustor exhibits self-sustained thermoacoustic oscillations of the first transversal mode, which provides suitable benchmark data for analyses and tool validation. Specifically, dynamic OH* and pressure measurements are readily available for the concerned operation point. The selected operation point's performance data are presented in Figure 5. More insight in terms of experimental characterization and analysis of the present combustor configuration can be retrieved from [1, 13].

Parameter	Value
Pressure amplitude	1250 Pa
Air excess ratio	1.1
Air mass flow	120 g/s
Fuel mass flow	7.0 g/s
Inlet temperature	673 K
Thermal power	350 kW

Figure 5: Operation point parameter

The non-uniform mean flow velocity distribution required for the LEE setup is obtained via isothermal CFD [2] simulations, while the mean pressure is assumed to be constant at atmospheric conditions. Mean temperature, density and speed of sound distributions are retrieved from time averaged OH* recordings. The mean temperature and heat release fields are illustrated in Fig. 6

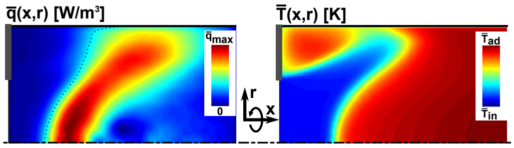

Figure 6: Mean fields of temperature and heat release [1]

The combustor domain is meshed using unstructured tetrahedrons on which the governing equations (stabilized LEE in frequency domain) are discretized using linear shape functions. Boundary conditions for combustor wall, inlet and outlet are prescribed to acoustic slip, energy neutral and pressure node, respectively. The mesh with employed boundary conditions is shown in Fig. 7. More details on boundary conditions and FEM treatment of the present configuration can be found in [6].

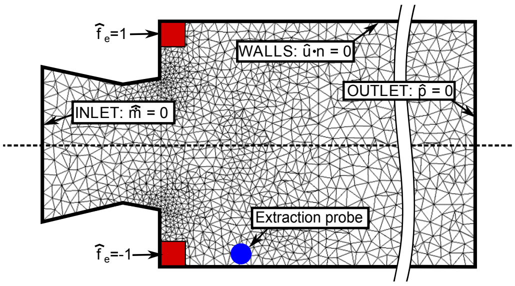

Figure 7: FEM mesh with boundary conditions, external excitation and pressure probe location [6]

The system size (i.e. number of degrees of freedom) is $N \approx 300,000$. The same figure indicates external excitation regions – prescribed as source terms of the energy equation – which are needed to compute the pressure response frequency sweep for verification of the ROM below. Time step and duration of the ROM integrations are selected accordingly to the experimental measurement data, which unfolds into $dt = 1x10^{-5}$s and $T_f = 4$s, respectively.

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[2] Computational Fluid Dynamics

6. Results

This section presents the results of the thermoacoustic analysis of the test case combustor, and is divided into three parts. The first part presents the verification of the ROM, i.e. its reproduction capabilities of the large-scale system it is based on, where frequency response of acoustic pressure of the LEE system is computed via COMSOL. Then, the linear and nonlinear analyses results obtained with the ROM are presented in subsequent order.

6.1. Pressure Response Frequency Sweep

The first step of the analysis is to verify the ROM's capability to reproduce the large scale FEM system accurately. Note that the required left and right eigenspaces for the MOR are computed with COMSOL's eigensolver. For this work, the first transversal mode is of primary interest (cf. Fig. 2) leading to the employment of 19 complex conjugate mode pairs for composition of the subspace [6]. The order of the ROM results in $N_r = 38$, which implies a quite large order reduction as the large scale system is of order $N \approx 300,000$. The amplitude and phase of the pressure response to a frequency sweep (of the open loop, i.e. with zero flame source term) are presented in Figs. 8-9. The required excitation source terms and pressure extraction location are displayed in Fig. 7. The large scale FEM system (i.e. the frequency domain version of Eqns. 2- 3) results are obtained by performing a parametric solution study with COMSOL, sweeping through a predefined set of frequencies (corresponding the considered range for the MOR) given by

$$2,000\ Hz \leq f_{MOR} \leq\ 5,000\ Hz \qquad (12)$$

while the ROM sweep results are computed with MATLAB. By visualization, the figure reveals proper agreement between ROM and FEM computation results. The relative error computes to less than 2%, while the computation time of the ROM against the FEM record at 10^{-5}s vs. 11h [3], which underlines the low order nature of the ROM and grants it suitability for low-order thermoacoustic analyses.

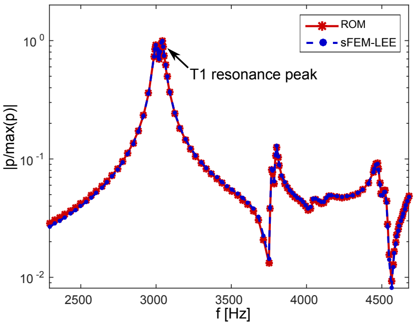

Figure 8: Amplitude of pressure response to frequency sweep (from [6])

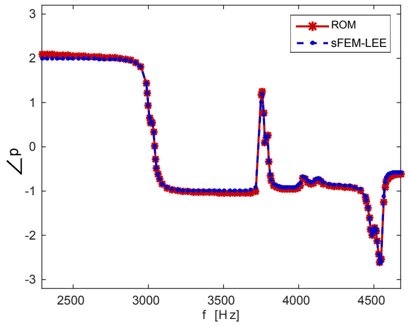

Figure 9: Phases of pressure response to frequency sweep (from [6])

6.2. Linear Thermoacoustic Analyses

In order to model the thermoacoustic interaction between flame and acoustics, the non-compact feedback loop is closed using the previously created ROM with Multi-Inputs and Multi-Outputs (respectively located at the centers of the involved flame subregions) that are connected using the flame dynamics function in Eqn. 4. The required steady terms of the linear transfer function are retrieved from the readily available mean flow field. Linear stability is given by the complex eigenfrequencies of the closed system. These quantities are computed by only utilizing the linear contribution of the flame dynamics function for the feedback connections, and then simply solving for the eigenvalues of the closed loop ROM MIMO system (cf. [7]). Corresponding results of the large scale system are obtained by an eigenvalue analysis in COMSOL. For this, the heat release source term of the LEE in Eqn. 2 is again prescribed only by the linear part of the flame function, and then solved for eigenmodes and -frequencies. The complex eigenfrequency reads

$$\omega_n = 2\pi f_n + i\alpha_n, \qquad (13)$$

where f_n and α_n denote oscillation frequency and growth rate of the mode n, respectively. The

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[3] Computed with an Intel(R) Core(TM) i7-4770K CPU, 3.5GHz,32GB

sign of the growth rate determines linear stability, i.e. mode n is rendered thermoacoustically stable/unstable for $\alpha_n < 0/ > 0$. The resulting complex eigenfrequencies for the modes of interest show in Fig. 10.

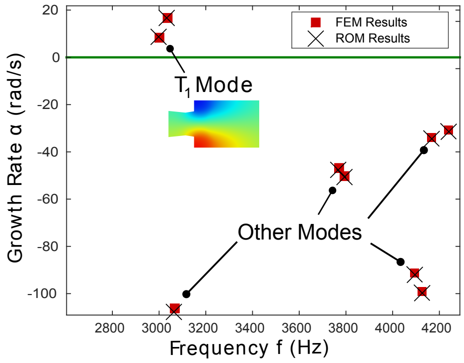

Figure 10: Complex eigenfrequency results

ROM-MIMO and FEM results agree acceptably, which strengthens the ROM's suitability for low order thermoacoustic analyses of high-frequency systems. Oscillation frequencies of the T1 mode (mode of interest) matches accurately with relative errors below 1% versus the experimental benchmarks (see [6] for details). Then, the growth rate signs indicate linear instability of the T1 mode as expected from the experimental observations. All other modes yield negative a growth rate and can be rated as strongly stable - as expected, too. The previous calculations are solely based on numerical and analytical frameworks. They proved capable for reconstructing the thermoacoustic stability behavior observed in an experiment. Technical relevance of such analyses is the usage to identify instability promoting and inhibiting factors in high-frequency thermoacoustic systems (as is done in [6]). Additionally, the approach can be employed to assess the stability of a combustor at development stage to ensure a (high-frequency) thermoacoustically robust and reliable final design.

6.3. Nonlinear Thermoacoustic Analyses

Time domain integrations of the ROM are executed and compared against experimental limit cycle data of the considered unstable operation point. The simulations are capable of considering two main physical effects occurring in real systems: (1) Nonlinear saturation of thermoacoustic energy conversion processes by the flame at high oscillation amplitudes. (2) Background noise due to turbulent combustion. Representative time signals g(t) are given in Figs. 11 - 12.

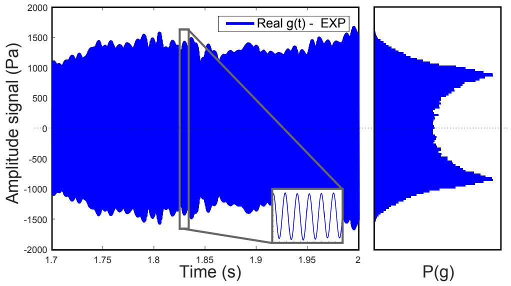

Figure 11: Limit cycle signal – experiment [6]

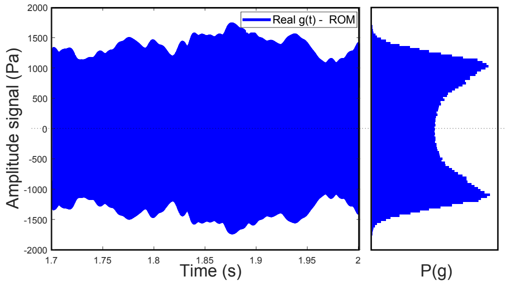

Figure 12: Limit cycle signal – ROM [6]

The limit cycle behavior of the system is effectively reproduced along with the clearly observable impact of noise, which stochastically modulates the oscillation amplitude. Especially the bi-modal shape of the probability distribution associated with the time series reveals the limit cycle nature under noisy conditions. Accurate agreement between experiments and simulations is achieved. Furthermore, the simulation reproduces the experimentally observed mode dynamics, i.e. a rotating transversal mode in limit cycle, which is shown in Figs. 13 - 14.

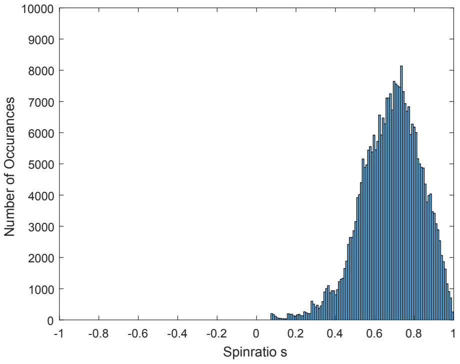

Figure 13: Spin ratio histogram – experiment [6]

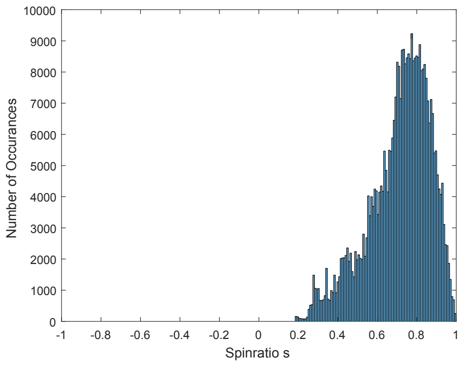

Figure 14: Spin ratio historgram – ROM [6]

These figures show the spin ratio S of the transversal mode which value indicates a standing mode if $s = 0$, a counterclockwise rotating mode $s = +1$, and clockwise rotating mode $s = -1$. Both, ROM and experimental data reveal predominantly counterclockwise rotating mode where the impact of stochastic forcing due to turbulent combustion noise induces the spread shown in the histogram plots. In practice, these time domain simulations are employed to study the combustor dynamics from a dynamical system theory perspective, i.e. fixed point analyses, their dependence on linear performance parameters, and possible bifurcation scenarios (especially under the presence of broadband combustion noise). A demonstration of such investigation can be found in [6]. Furthermore, the presented ROM approach is used as tool to generate realistic test case data for the development and validation of output only system identification methodologies as outlined in [10].

7. Conclusion

Linear stability and nonlinear dynamical simulation analyses in frequency and time domain of the high-frequency thermoacoustic performance of a lab scale combustor were executed. COMSOL Multiphysics represented the core tool to discretize the governing Linearized Euler Equations (LEE). COMSOL's equation based modeling interface was used to prescribe the weak form of the LEE (including a numerical stabilization extension for the convective terms). The resulting system matrices were then used to construct a Reduced Order Model of the large scale Linearized Euler System with which computationally efficient simulations – especially in time domain – can be carried out. COMSOL's MATLAB LiveLink was used to export the system matrices as well as required eigenmodes, which constitute the subspace for the order reduction technique. Then, the LEE in frequency domain was solved in COMSOL via the eigensolver and frequency sweep parametric studies for validation of the ROM results. The ROM results could successfully reproduce these FEM verification benchmarks, and were thus rendered suitable for further thermoacoustic system analyses. Subsequent analyses showed accurate agreement with experimental results.

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