## Dielectric Properties of Heterogeneous Media with Inclusion of Fractal Media

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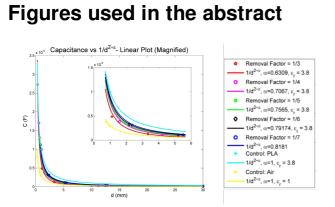
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## Abstract

The aim of this research is to study the electrostatic properties of heterogeneous media that have fractal geometries. Currently, composite dielectrics are assumed to have regular geometric structures (integer-based dimensions) and an isotopic, homogeneous dielectric permittivity; but, in reality, the dielectric permittivity is affected by the irregular interface between different materials. Fractal geometry can approximate these complicated irregularities. In this work, we developed a numerically and experimentally validated analytical model for describing the electrostatic properties of dielectric materials with fractal irregularities based on recently proposed fractional-dimensional electromagnetic models. We investigated the effects of the fractal irregularities on the capacitance of parallel-plate geometry in the following ways: (i) by making the distance between the two electrode plates fractal but keeping the plate area non-fractal; (ii) by making the plate area fractal but keeping the distance between the plates non-fractal; and (iii) by making both the plate area and the distance between the plates fractal. We employed COMSOL to perform simulations after inducing fractality in regular geometries like 3D cubes/cuboids and 2D squares/rectangle. We also performed practical experiments on 3D printed cubes. The fractal geometries, called Cantor plates, are constructed by dividing the region between the electrode plates into alternating parallel layers of dielectric and air, with the thickness of each dielectric layers set to the length of the components of a Cantor set of certain removal factors (3, 4, 5, 6, and 7) at the 4th iteration. Each removal factor resulted in different fractal dimension as determined by Hausdorff's formula. The results from the full-wave 2D COMSOL simulations (validated by the experiments) conformed to the analytical model we developed - an unconventional scaling relation between capacitance (C), plate distance (d) and the fractional dimension ( $\alpha$ ) to be C  $\propto 1/d^{(2-\alpha)}$  is revealed and is unambiguously confirmed by analytical model, COMSOL simulations as well as experimental measurements. This model now allows the characterizing the effective dielectric properties of heterogeneous fractal media from thickness and area scaling of its capacitance in a simple manner. For the next segments of our research, as well, we have used COMSOL simulations to validate the analytical models we theorized. Our results reveal a new viable direction towards the modelling of electromagnetic properties of complex media using non-integer dimension approach.

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## **Figure 1**: Capacitance (obtained from COMSOL simulations) as a function of plate distance satisfying the theoretical scaling