Analysis of D-Shaped Toroidal Superconductive Coils for Medium Size Fusion Experiment Facility

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Introduction

In magnetic confinement nuclear fusion the magnetic fields are utilized to confine an extremely hot gas (plasma) of hydrogen isotopes. Several devices design have been developed in the last decades but a particular kind of electromagnetic machine, called Tokamak (a Russian acronym), has showed the bigger capacity in order to gain high plasma performances and an acceptable efficiency in experiments fulfilment, even if it isn't capable of stationary operations but only pulsed. In a Tokamak the plasma is formed in a toroidal vacuum vessel that is surrounded by the coils generating the proper magnetic fields carrying out different functions and can be subdivided in three main groups:

- The toroidal magnet generating the main plasma confinement field action whose value ranges typically from 1 to 10 Tesla. It is constituted by a number of discrete coils varying usually from 16 to 24.
- The central solenoid, placed in the hole of the torus, whose role is to act as the primary of a transformer, where the plasma is the secondary, producing so a first strong heating of the plasma via Joule effect.
- The equilibrium coils installed around the torus, with the task of delivering additional magnetic fields of minor intensity, needed to control the position and the shape of the plasma inside the vacuum vessel preventing it by touching the walls that would be destroyed by the contact with the very hot plasma.

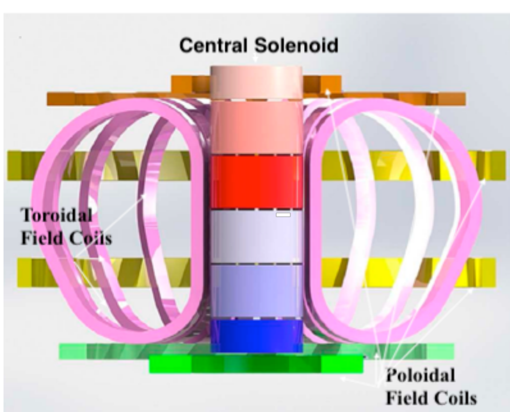

Figure 2. General view of the magnetic systems of a Tokamak.

In this paper are presented the analysis done, through modelling with Comsol Multiphysics, on the toroidal superconductor coils to start a preliminary design for a medium size tokamak for experimental activities aimed to study the behaviour of the divertor, one of the most important subsystem, in conditions near to those envisaged for a reactor. Avoiding of introducing excessive details about the role of the divertor, it's enough to underline that it operates as heat exhaust and fusion ashes sink so the tokamak has to reach conditions either of long pulses duration and of high plasma power density. For this reason the project provides for the use of superconductor coils with magnetic fields and current densities close to the limit of the actual low temperature superconductive materials. Moreover the overall dimensions of the device must be contained for obvious reasons of project cost adding so further complexity to the design requests.

Theory

The structural requirements imposed on toroidal magnet design are very stringent because of the electromagnetic loads. In particular on the inner straight leg due to the torus radial profile of the magnetic field given, for an ideal continuous thin winding (infinite number of circular current filaments), by [1]:

$$\vec{B} = \frac{\mu_0 NI}{2\pi r}\vec{i}_\phi \qquad \textbf{(1)}$$

So the body force distribution varies linearly from the maximum at the inner surface of the torus to zero on its outer one. The vertical and radial components of the total force are given by:

$$F_z = \frac{\mu_0 NI^2}{4\pi}\log_e\left(\frac{1+\gamma}{1-\gamma}\right) \qquad \textbf{(2)}$$

and

$$F_r = \frac{\mu_o NI^2}{2}\left[1-\frac{1}{\left(1-\gamma^2\right)^{1/2}}\right] \qquad \textbf{(3)}$$

where γ is the ratio between minor and major radii of the torus.

Of course the electromagnetic forces have a more complicated distribution in the case of a system of discrete thick coils toroidally equidistant. So it is necessary to optimize the geometrical structure of the coils for minimizing the maximum stress and making as uniform as possible the stress distribution [2].
Then a D-shape is normally adopted for the toroidal coils because such shape entails ideally a constant tension and moment-less configuration. A constant tension thin shell torus can be represented by:

$$r\frac{d^{2z}}{dr^2} = \frac{\pm 1}{k}\left[1+\left(\frac{dz}{dr}\right)^2\right]^{3/2} \qquad \textbf{(4)}$$

where

$$k = \frac{4\pi T}{\mu_0 N I^2} \qquad \textbf{(5)}$$

and T is the average tension.
For discrete thick coils systems numerical methods [3] have been proposed for solving equation (4), but based on specific hypothesis that don't make them of direct application for the superconductor coils case, where the arrangement and dimension of the superconducting cables inside the coil play an important role.

Method

To start the design of the toroidal coil on solid foundations an iterative multistep method based on the finite elements multi-physics simulation was set up. This method consists in three steps:

- In the first step the aim is to define the overall D-shape of the coil, sweeping some geometric parameters to optimize some magnetic and mechanical outputs of particular interest. In this phase the coils are modeled using smeared material properties to simulate the superconducting material because it's worth to ascertain the global behavior, disregarding the local effects.
- The second step is devoted to investigate the local response of the most solicited sections of the model, mapping the deformations obtained in the 2D model of the real coil section, *i.e.* drawn as a matrix of superconducting cables embedded in the insulating material, into the same section but with the same smeared material properties used in the previous step. So it's possible to get the stress intensification factors, with which is verified the coil strength.
- The third step is intended as validation of the first two. In this step an elementary 3D cell of the coils is submitted to a set of virtual test to determine the smeared properties of the given arrangement of the superconducting coil. Then the resultant stress intensification factors are compared with the ones obtained in the previous step.

Model Description

The methodology sketched above is implemented by a set of models, realized through the Magnetic Field and Solid Mechanics modules of Comsol Multiphysics, either in 3D and 2D geometry, relative to a toroidal magnet constituted by 16 coils, each with a 2.5 m major radius and a overall height of c.a. 4.5 m.
In the first step a 3D model was used to ascertain what is the optimal D-shape given the global size expected. The model consists of two components. The first component is made by of 5 superconducting coils, covering so an angle of 90° of the torus, and it's used to calculate the magnetic physical quantities, taking advantage of the symmetry through the periodic continuity conditions, reducing the dimensions of the model. The D shape geometry is made on a 2D work plane through the union of a rectangle with fillets in its corners and an ellipse whose dimensions are in relationship, so just one parameter is needed for changing the D shape (see table 1).

x_corner_rect	inner_radius
y_corner_rect	-height_rect/2
x_center_elli	x_corner_rect+width_rect/2
y_center_elli	y_corner_rect+height_rect/2
width_rect	2.0[m]
height_rect	4.6[m]
radius_fillet	width_rect/2
cond_thickness	0.25[m]
a_ellipse	height_rect/2
b_ellipse	outer_radius-width_rect/2-x_corner_rect
I_Toro	46.3[kA]
N_Toro	88
outer_radius	4[m]
inner_radius	1[m]
outer_coil_width	0.62[m]
alfa	90[deg]/8

Table 1. The geometry parameters used for the construction of the global model.

The second component is devoted to the structural mechanics simulation and is composed by only one

coil embedded in its steel casing. The coil winding pack material is supposed orthotropic and, as first guess, the smeared elastic material properties are taken equal to the ones already used for a previous project. The coil is loaded by the Lorentz's forces calculated in the first component and extruded on it by the genext operator. Then both the components are solved in stationary with a parametric sweep on the main geometric parameter that make it to vary continuously its D-shape in a range compatible with other general restrictions, due mainly to the relative positioning between plasma and equilibrium poloidal coils.
In the second step a 2D local analysis is carried out on the section taken on the equatorial plane of the coil inner leg, where the toroidal field is stronger, with the aim to verify the stress intensification factors. Also this model is constituted by two components. In the first a detailed model is used where the steel, epoxy glass insulation and resin filler are present with their real elastic properties. The second with the same geometry and mesh has for the winding package the smeared properties used in the first step. Extruding the displacements obtained in the first component on the second and forming a join data set is possible to take the ratio between the same physical quantities and so to obtain the stress intensification factors.
After this step the gained information could be sufficient for the coil design review. But to get a further refinement/validation of the design the third step has to be executed to check the correctness of the stress intensification factors calculation accuracy and to get data that will be compared with those coming from experimental tests. In this step an elementary cell of the real coil structure is submitted to a set of virtual tests, where on the cell are imposed displacements in the various direction while it is properly constrained to get the elastic constants

Simulation Results – Global Model

The magnetic model is constituted only by the winding packs of 5 coils, modelled in a simplified geometry without electric joints, helium inlet and other auxiliary equipment. It's simulated through numeric type homogenized multi-turn non solid material coils, because the electric properties change with the material strain is neglected in this step. The results obtained are shown in fig. 2 for the two extreme case of the geometric span.

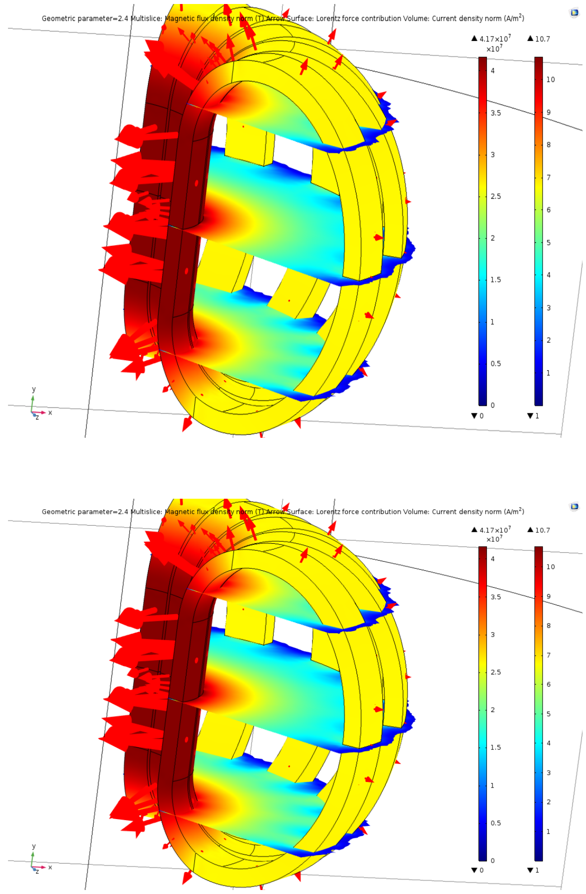

Figure 2. Lorentz forces (arrows) , Magnetic field (slice) and Current Density for the two extreme geometric configurations of the magnetic model.

The magnetic flux density norm slices show the magnetic field dis-homogeneity due to the discreteness of the coils, that has important consequences for the plasma operations, because of the influence on the charged particles trajectories. This fact is taken in account by a global parameter the toroidal magnetic field ripple.
The arrows are relative to the resultant centring forces, they are greater on the inner leg where current density and magnetic field are stronger.
The forces calculated in this model component are extruded to the second component that is constituted by only one coil (the central) but embedded in its steel casing. Also in this case the interest is put on the global behaviour of the magnet so some parts, such as outer inter-coil structure and gravity support of the casing aren't modelled. In fig. 3 are shown the Von Mises stresses either for the casing and for the winding pack in the lower value of the geometrical parameter case.

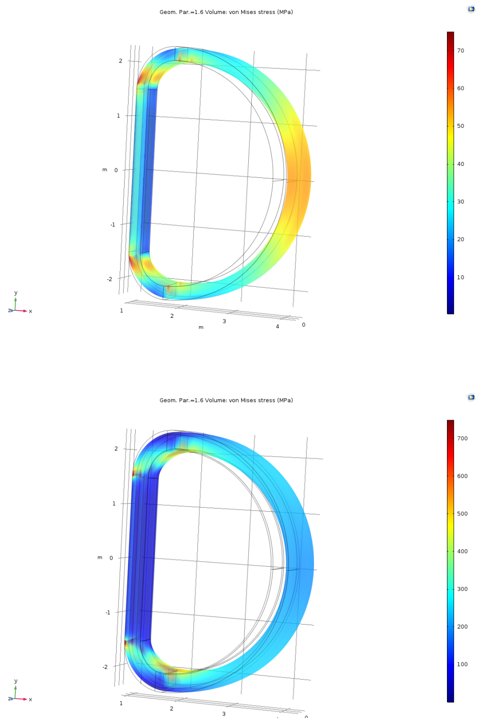

Figure 3. Von Mises stress for the winding pack (above) and the steel casing for the geometrical parameter = 1.6 case.

To assess which is the best geometry configuration to choose, the integral value of the elastic strain energy density is adopted. The results are plotted in fig. 4 either for the entire coil and separately for the winding pack and the casing.

The trend is monotone with the geometric shape variation, but is more contained in the winding pack. The elastic strain energy increase in the winding pack is only about 12%, while for the steel casing, that is more ductile, is about the double. (25%).

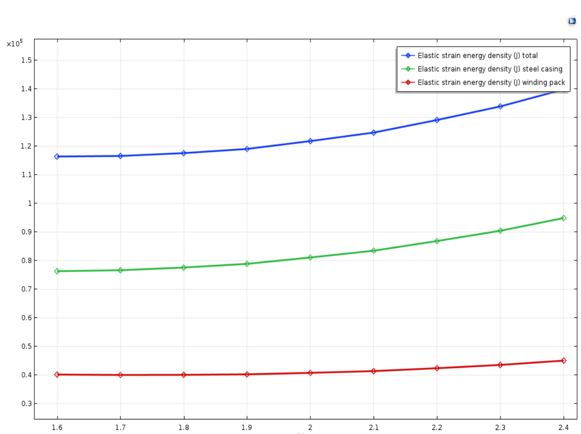

Figure 4. Elastic strain energy density integral for all the D shapes simulated.

The choice of the final D shape has to be done also on the basis of magnetic considerations. As sketched above, one of the most important magnetic parameter for a Tokamak is the toroidal field ripple defined as:

$$R_{\%} = 100 \bullet \frac{B_{\max} - B_{\min}}{B_{\max} + B_{\min}} \qquad \textbf{(6)}$$

along a toroidal circumference and at given poloidal angle. For the plasma operations is desiderable to have a ripple value as low as possible. Then, considering also this parameter that has an inverse monotone trend, see fig. 5, with the geometric parameter in the upper outer corner of the torus poloidal section (a critical area for the plasma), the D shape chosen is that with the intermediate value of 2 of the geometry parameter.

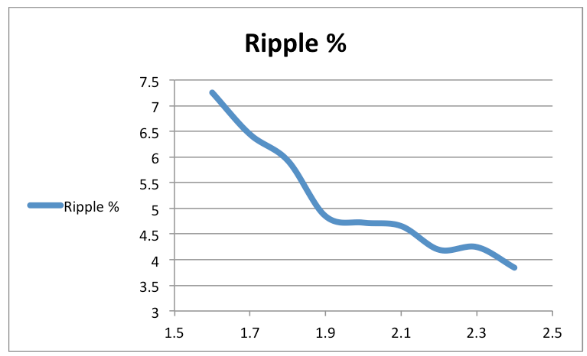

Figure 5. Magnetic field ripple calculated in the upper corner area of the poloidal section as function of the geometry parameter.

Simulation Results – Coil Equatorial Section

The mechanical quantities calculated with the global model relies on smeared material properties that can't give the detailed local pattern of the real stresses and strains occurring in the coil. So to satisfy this issue a 2D model of the section on the equatorial plane of the coil inner leg, where magnetic field and current density are stronger, is set up. The half of this section, for symmetry reason, is modeled first with the real winding pack structure made by the turns constituted by cables in conduit, the turn insulation and the resin filler, all contained in the steel casing. A second component with the winding pack modeled by a homogenized material with the same smeared properties used in the global model is added to the first. The displacements of the casing calculated in the first component (fig. 6) are imposed on the second using the genext operator and maintaining the same mesh adopted for the heterogeneous component.

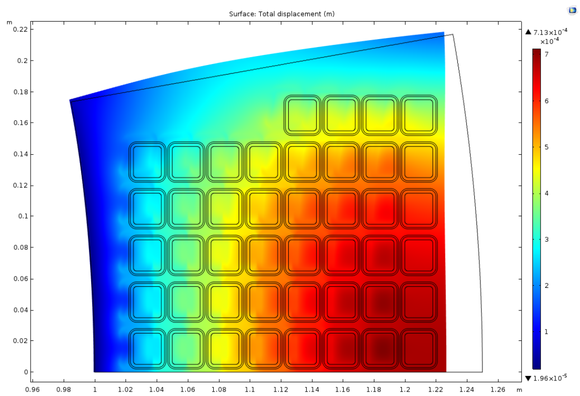

Figure 6. Total displacement in the heterogeneous winding pack poloidal section.

The resultant displacements in the homogenized component is shown in fig. 7.

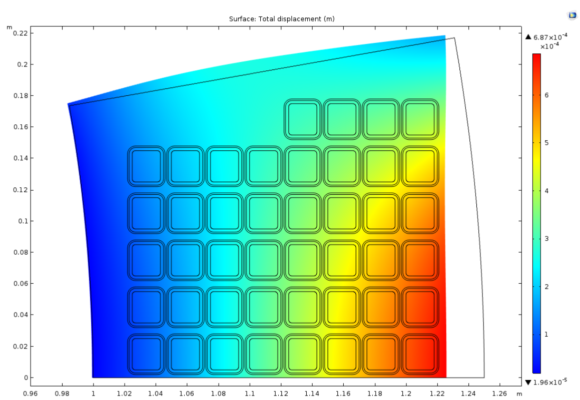

Figure 7. Total displacement in the homogeneous winding pack toroidal section.

Of course the displacement patterns for the two case are different since they are the result of completely different strains distributions, see fig.8.

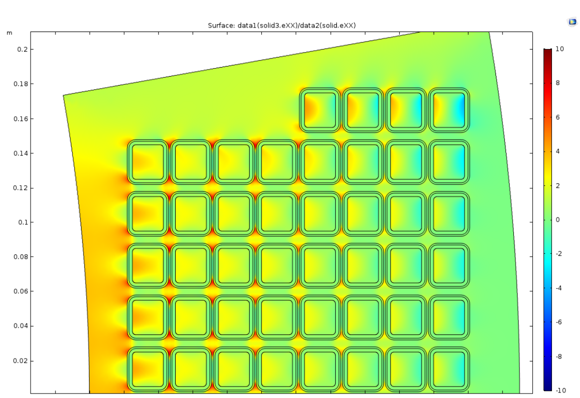

Figure 8. Ratio between the strain tensor XX components.

The Stress Intensification Factors (SIF's), needed to convert the stresses calculated in the 3D global model to the real ones, are obtained creating a join data set between the two components of the 2D model to do the ratio between the corresponding stresses, e. g. the xx component of the stress tensor in fig. 9.

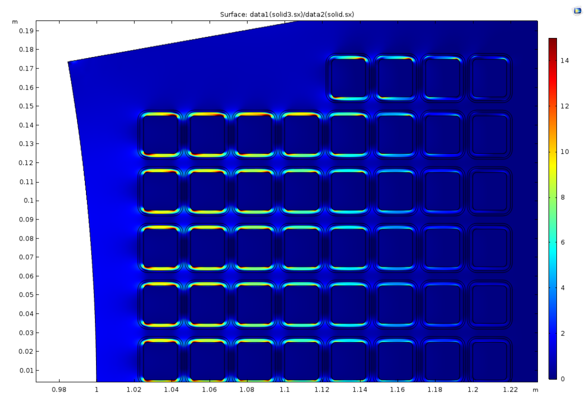

Figure 9. Stress Intensification Factor for the stress tensor x component.

In this step is possible also to compare various sizes and arrangements of cable-in-conduit to realize the coil, to ascertain if there is some enhancement in structural response varying the pattern and the geometry of the superconducting cable-in-conduit. For example in fig. 10 a SIF pattern of a different winding pack arrangement is shown,

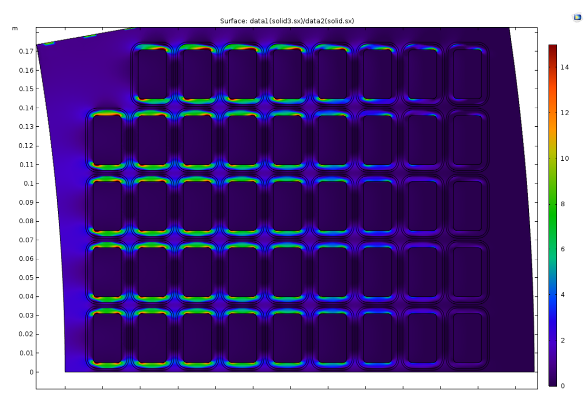

Figure 10. Stress Intensification Factor for the stress tensor x component for a different cables arrangement.

Simulation Results – Virtual Test Sample

To verify the results until now obtained and to get data that is possible to compare with experimental data from tests on real material sample a 3D model of an elementary cell of the coil structure is set up. This simplified model is submitted to a prescribed displacement D_{test} and it is constrained so that it's possible to calculate the smeared properties of an equivalent homogenized orthotropic material. For example to get the Young's module in the x direction Exx one face perpendicular to x axis is displaced of a known amount while the opposite face is fixed constrained, see fig. 11.

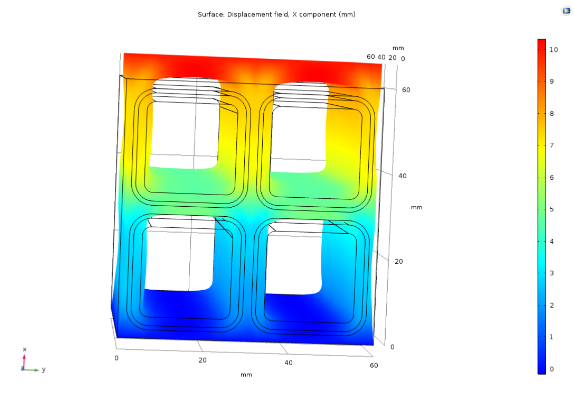

Figure 11. Displacement in x direction of the virtual test sample with the fixed constrain on the zy face.

The value of E_{xx} is calculated as:

$$E_{xx} = \frac{\sigma_x}{\varepsilon_x} = \frac{F_{react}}{Y_{spec} \bullet Z_{spec}} \bullet \frac{D_{test}}{X_{spec}} \qquad \textbf{(7)}$$

Similarly the other properties of an ortothopic material, for example G_{xy}:

$$G_{xy} = \frac{\tau_{xy}}{\gamma_{xy}} = \frac{Y_{spec}}{D_{test}} \bullet \frac{F_{react}}{X_{spec} \bullet Z_{spec}} \qquad \textbf{(8)}$$

The relative model, on which are imposed the proper displacement in x direction and constrained the face perpendicular to y axis, is shown in fig. 12.

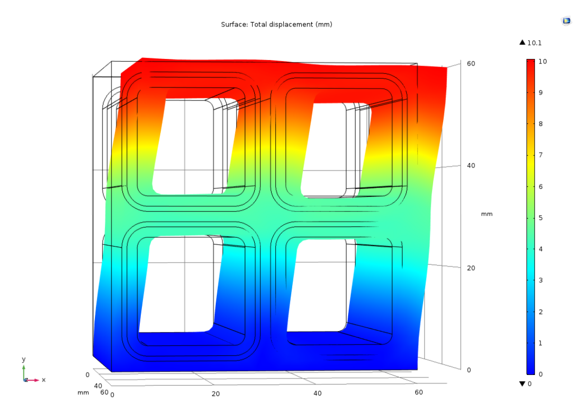

Figure 12. Displacement in x direction of the virtual test sample with the fixed constrain on the xz face.

Conclusions

The methodology proposed has to be validated with experimental test on specimens realized such as the final coils should be (see fig. 13).

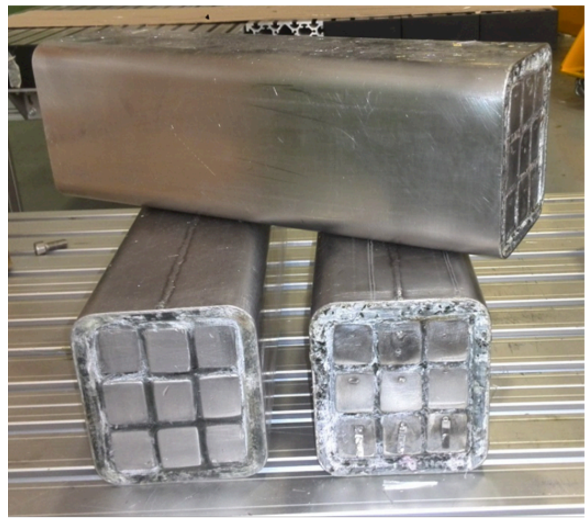

Figure 13 Picture of the three mock-ups for a successive experimental activity that will validate the results of the present analyses.

Of course the number of tests specimens to be tested shall be limited due to their costs. So it's important to simulate as well as possible the various possible arrangements of the cables in the winding pack and their dimensions so that the number of candidate specimens should be as limited as possible.

References

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