Super-resolution Properties of the Maxwell Fish-Eye

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COMSOL Conference
Stuttgart, October 26-28, 2011

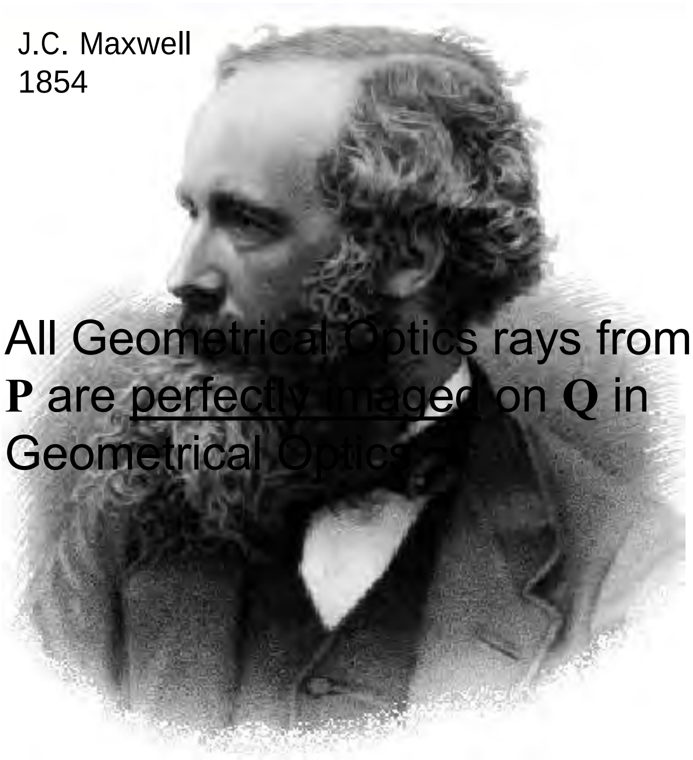
CeDInt POLITÉCNICA

Outline

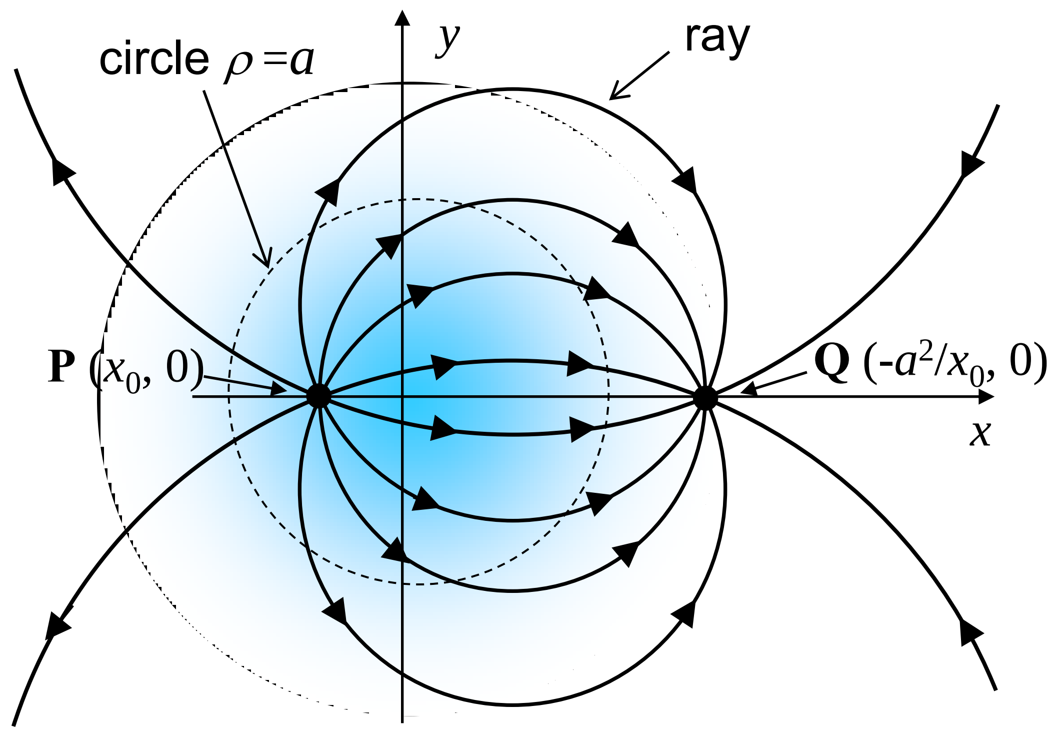
1. Introduction
2. Spherical Geodesic Waveguide
3. Simulations in COMSOL Multiphysics
4. Conclusions

Maxwell fish-eye

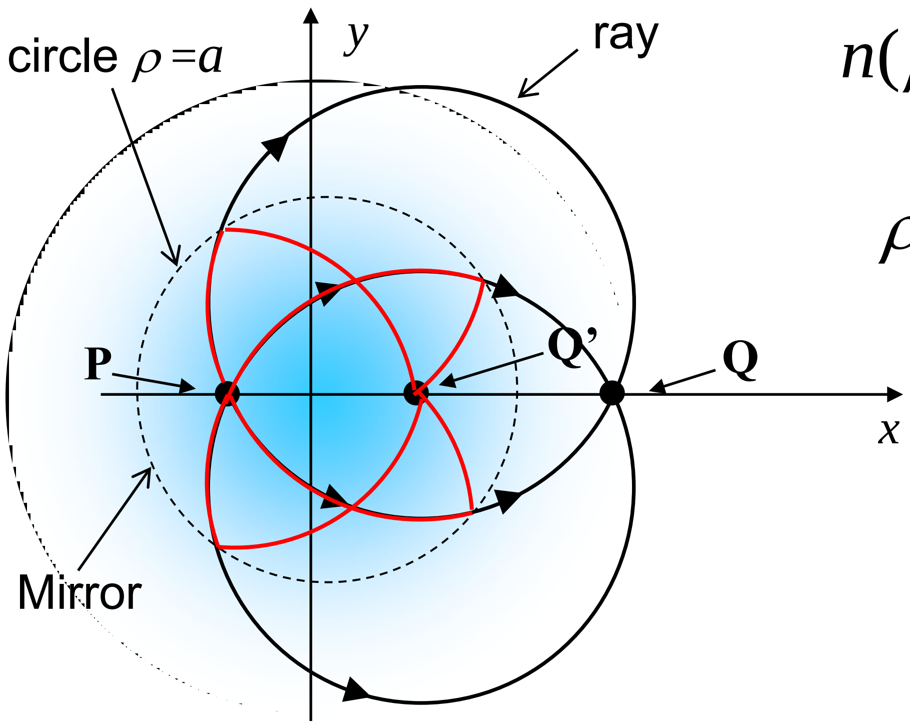
$$n(\rho) = \frac{2}{1+(\rho/a)^2} \quad \left(\rho^2 = x^2 + y^2\right)$$



All Geometrical Optics rays from **P** are perfectly imaged on **Q** in Geometrical Optics



Maxwell fish-eye



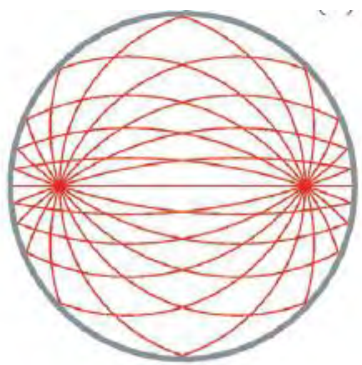
$$n(\rho) = \frac{2}{1 + (\rho / a)^2}$$

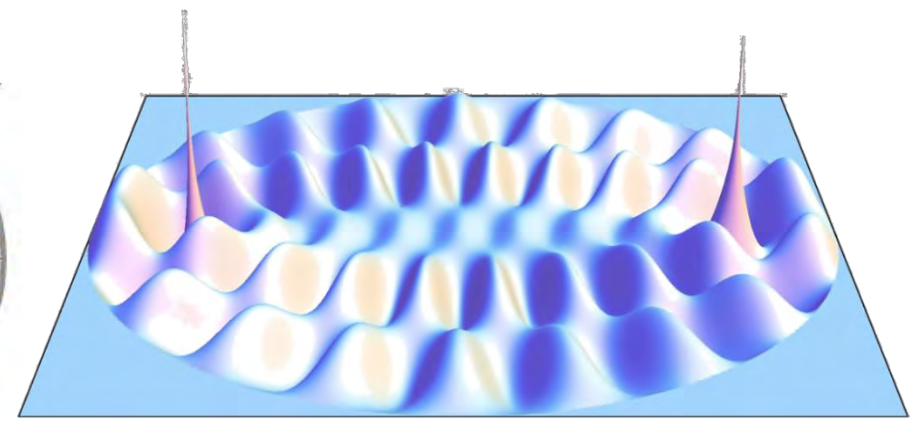
$$\rho > a, \quad n(\rho) < 1$$

Maxwell fish-eye

What happens in Wave Optics?

[Ulf Leonhardt, New Journal of Physics **11** (2009)]



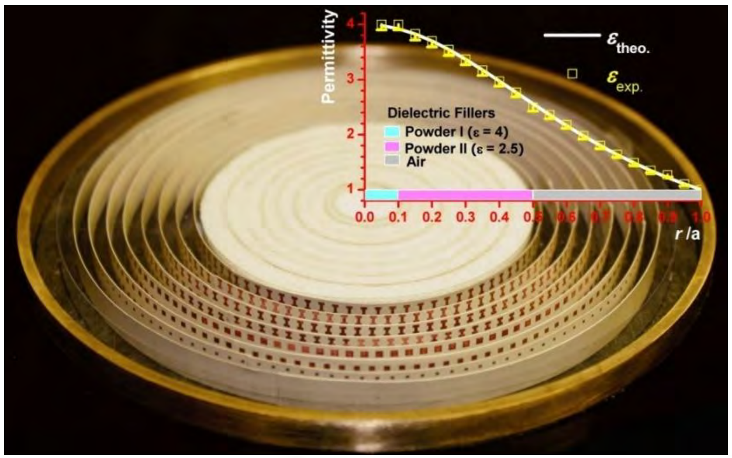


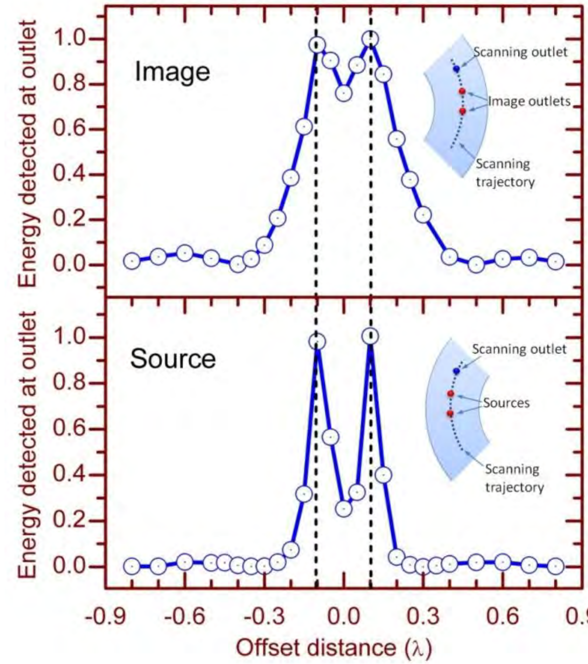
$$E = \frac{P_\nu\left(-\cos\theta\right) - e^{i\nu\pi} P_\nu\left(\cos\theta\right)}{4\sin(\nu\pi)}\mathbf{z}$$

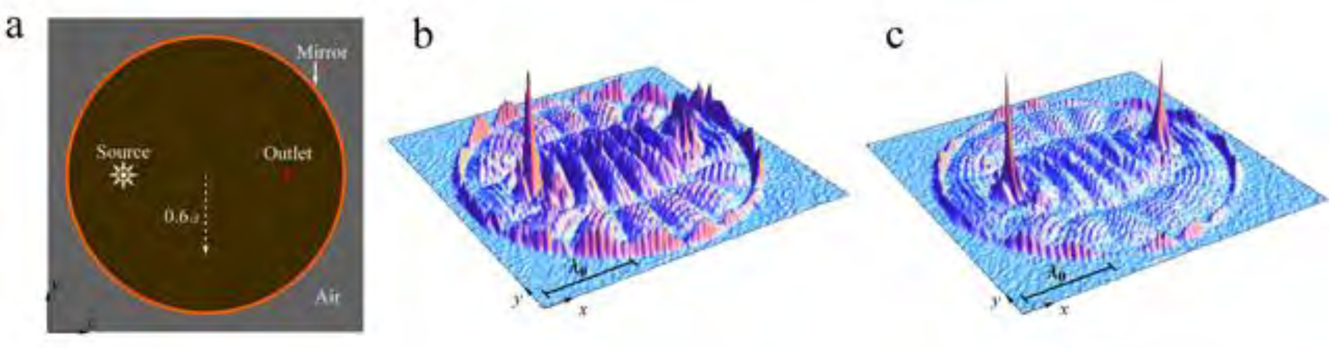
$$\nu(\nu+1) = (ak_0)^2$$

Experimental demonstration of λ/5 super-resolution

Super-resolution stands for the capacity of an optical system to resolve bellow by the diffraction limit







[Yungui Ma, Singapore]

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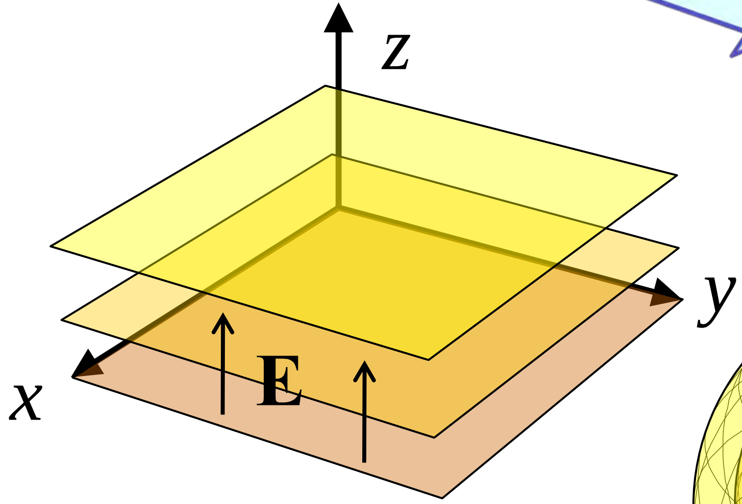
Spherical Geodesic Waveguide

Cylindrical 3D MFE lens

$$n(\rho) = \frac{2}{1+(\rho/a)^2}$$

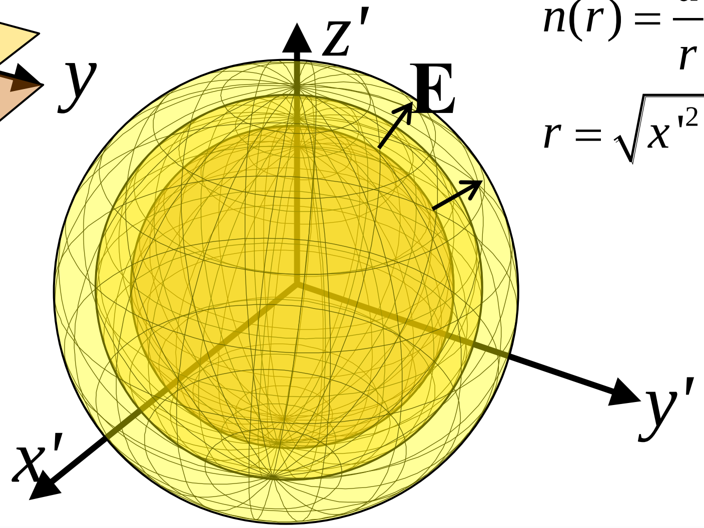


Each z=constant plane is mapped on a different sphere via stereographic projections

Material with spherical symmetry

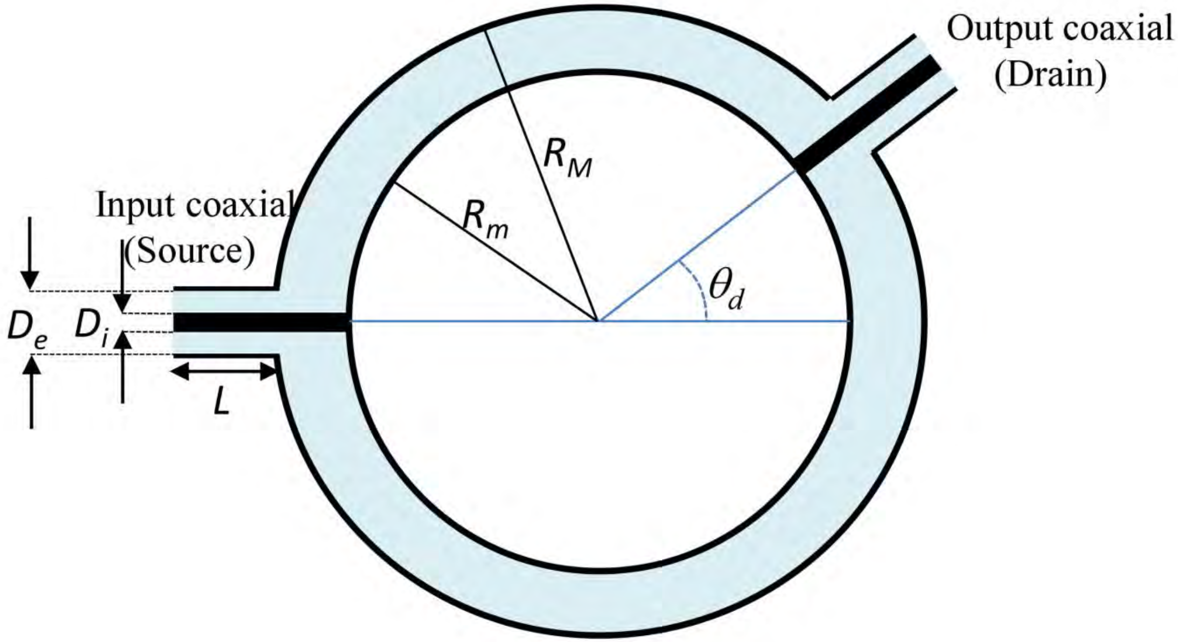
$$n(r) = \frac{a}{r}$$

$$r = \sqrt{x'^2 + y'^2 + z'^2}$$



[J.C. Miñano, P. Benítez, J.C. González, NJP, 12 (2010)]

Spherical Geodesic Waveguide

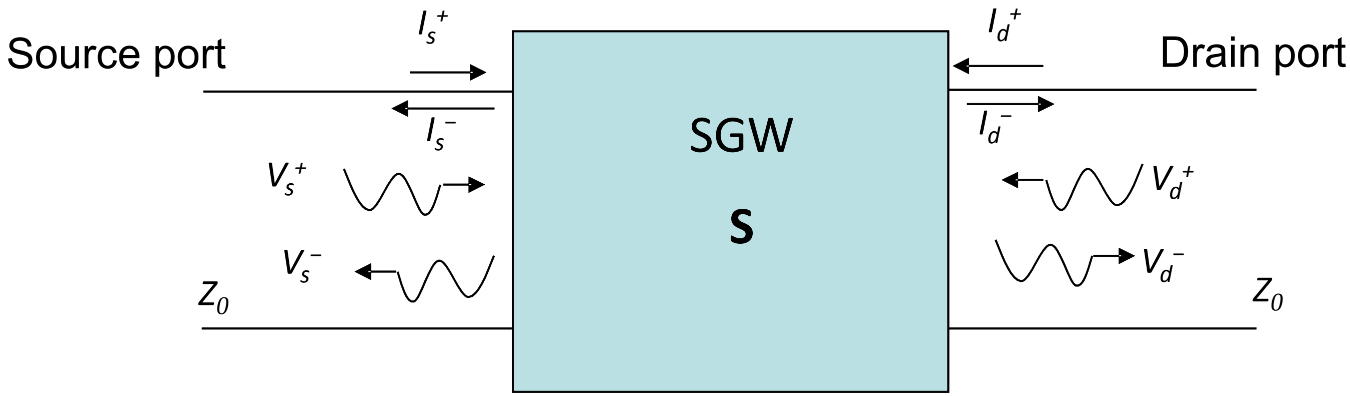


$$D_e = 10\ \text{mm} \quad D_i = 5\ \text{mm} \quad L = 20\ \text{mm}$$

$$R_M = 1005\ \text{mm} \quad R_m = 1000\ \text{mm}$$

$$n(r) = R_M / r \approx 1$$

Microwave circuit made up of the two ports and the spherical waveguide



When $V_d^+ = 0, I_d^+ = 0$

$$\begin{bmatrix} V_s^- \\ V_d^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} V_s^+ \\ V_d^+ \end{bmatrix}$$

$$V_d^- = S_{21} V_s^+$$

$$V_s^- = S_{11} V_s^+$$

$$P_I = \frac{1}{2} \frac{|V_s^+|^2}{Z_o} \qquad P_T = P_I |S_{21}|^2$$

$$P_R = P_I (1 - |S_{21}|^2)$$

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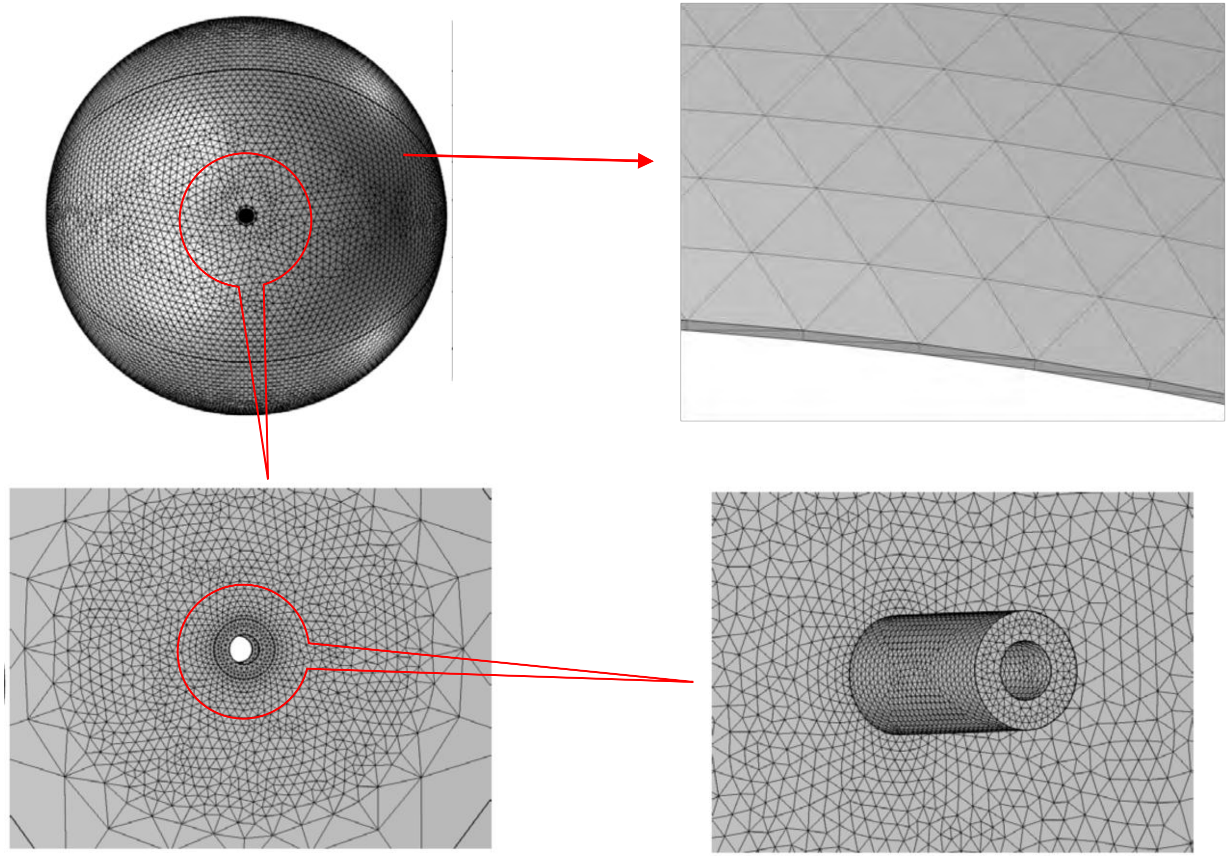
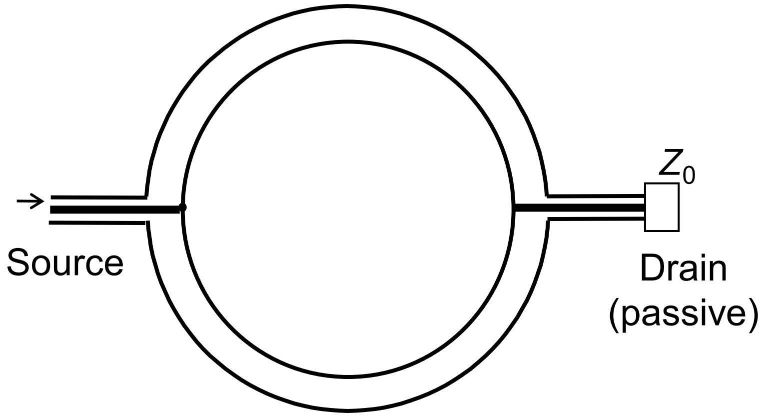
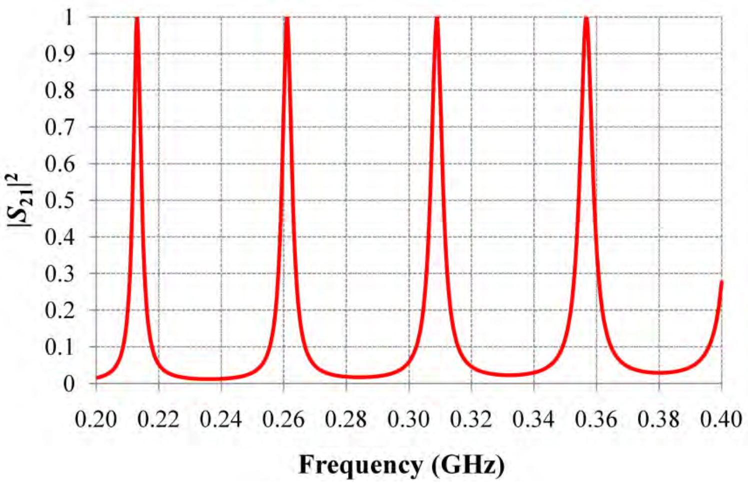
Meshing in Comsol

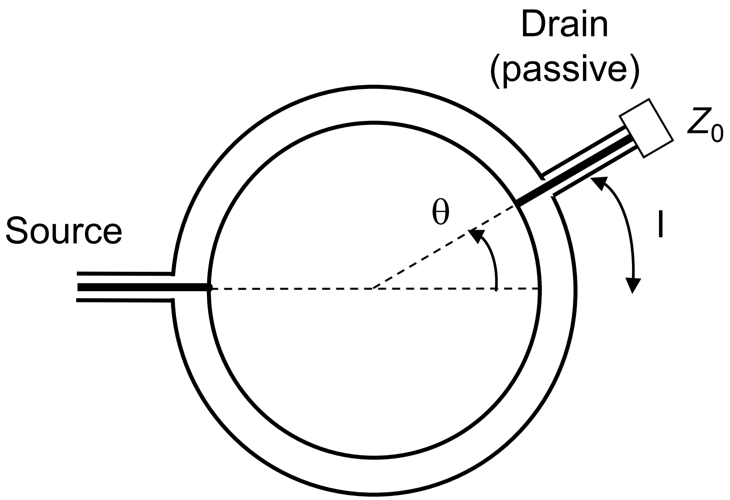


Transmitted power for different frequencies

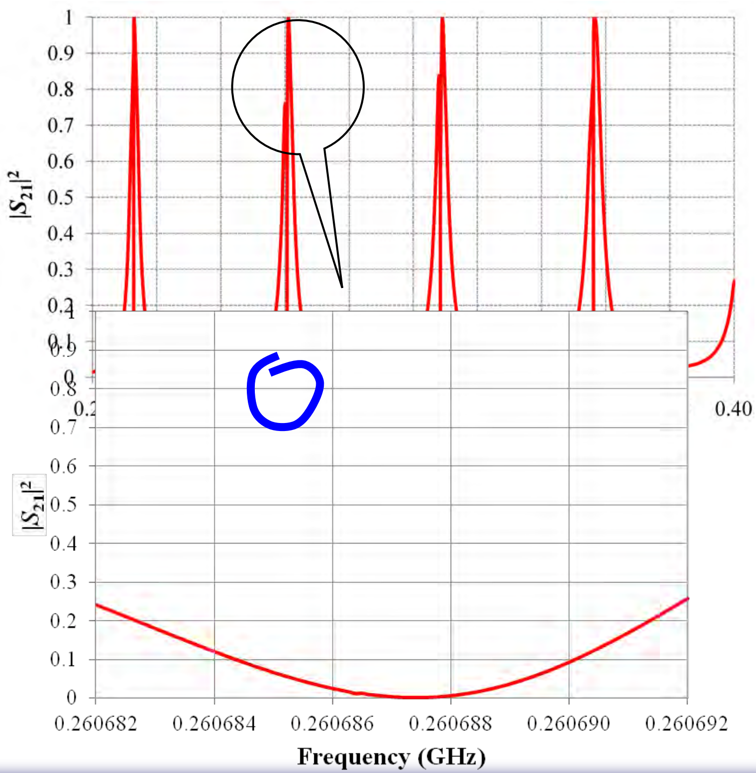




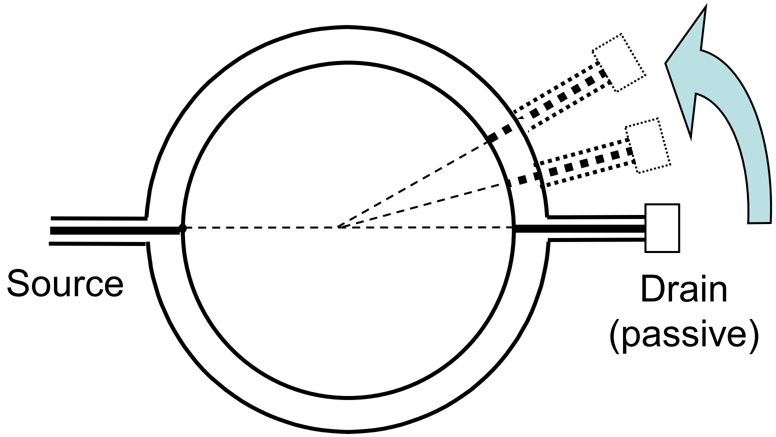
Transmitted power for different frequencies

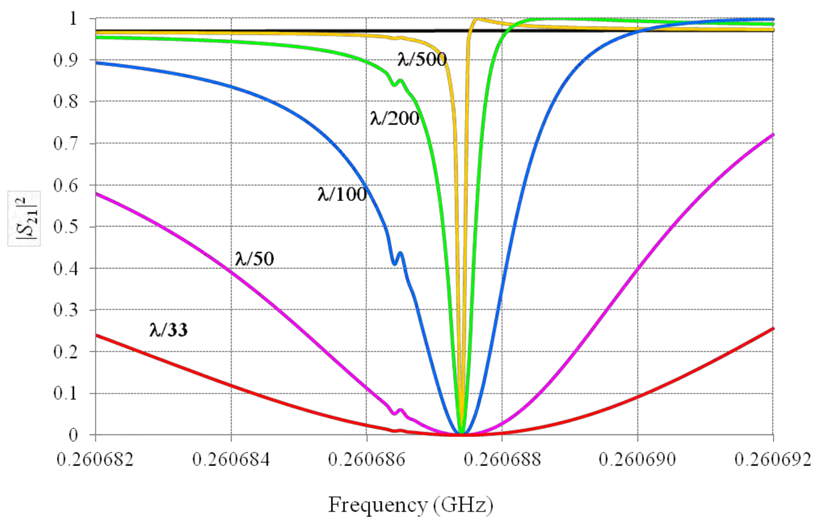


θ=2º, l=λ/33
λ=1.15m



Simulation of λ/500 super-resolution





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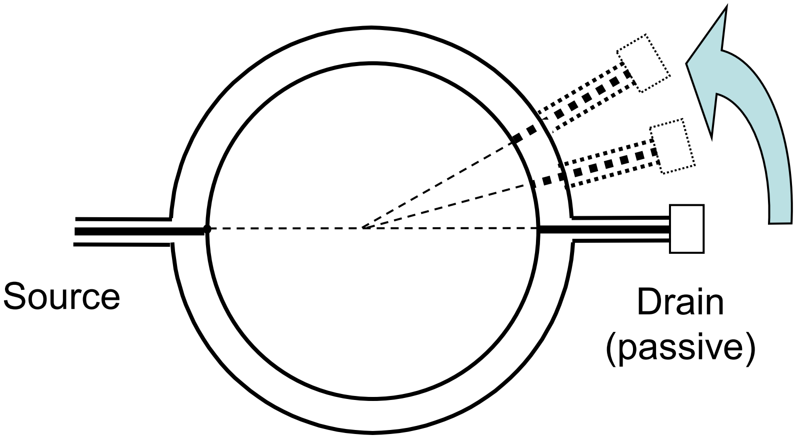

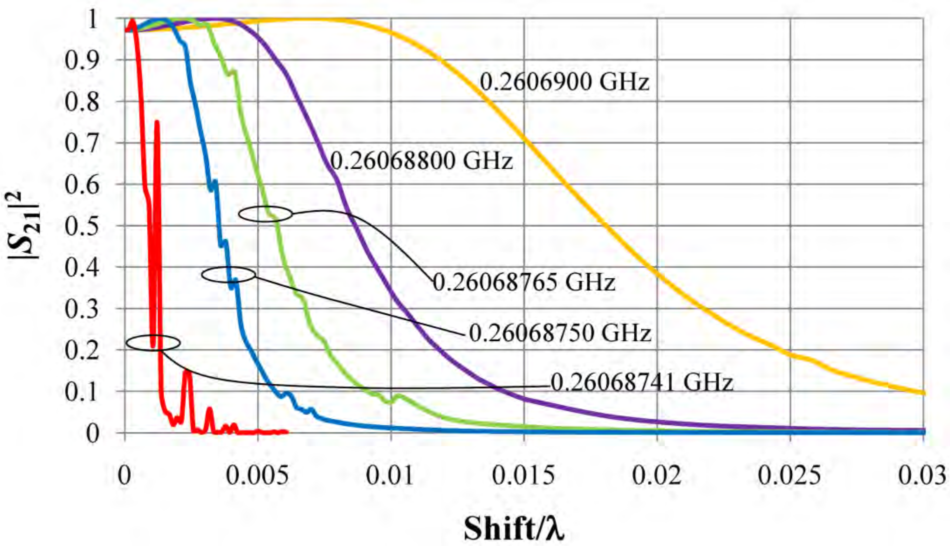

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Conclusions

- Super-resolution properties of the Maxwell Fish-eye are analyzed using Spherical Geodesic Waveguide (SGW).
- Simulations of the SGW show super-resolution up to λ /500 at microwave frequencies.
- The super-resolution is achieved using an approximate model of the SGW (the model having n=1 inside the waveguide) conveniente for manufactoring.